Modelling human balancing tasks

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MŰEGYETEM 1782

joint work with Balázs A. Kovács, Gergely Buza, László Bencsik, Ambrus Zelei, Csenge A. Molnár, John Milton, Gábor Stépán

Contents

Motivation

Stick balancing

stick length

reaction time delay

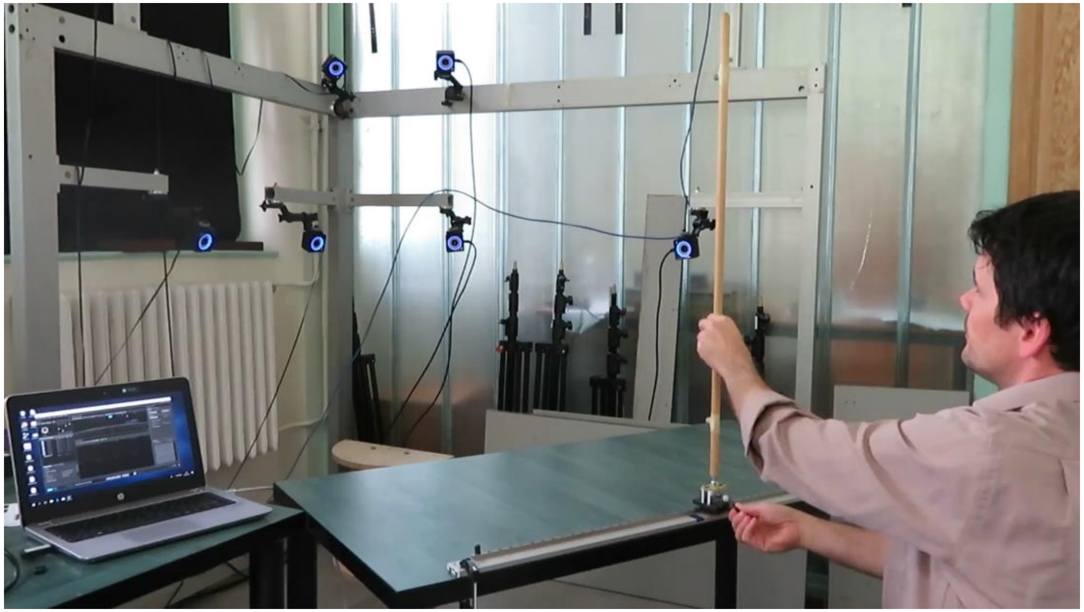
sensory uncertainty

~ critical length?

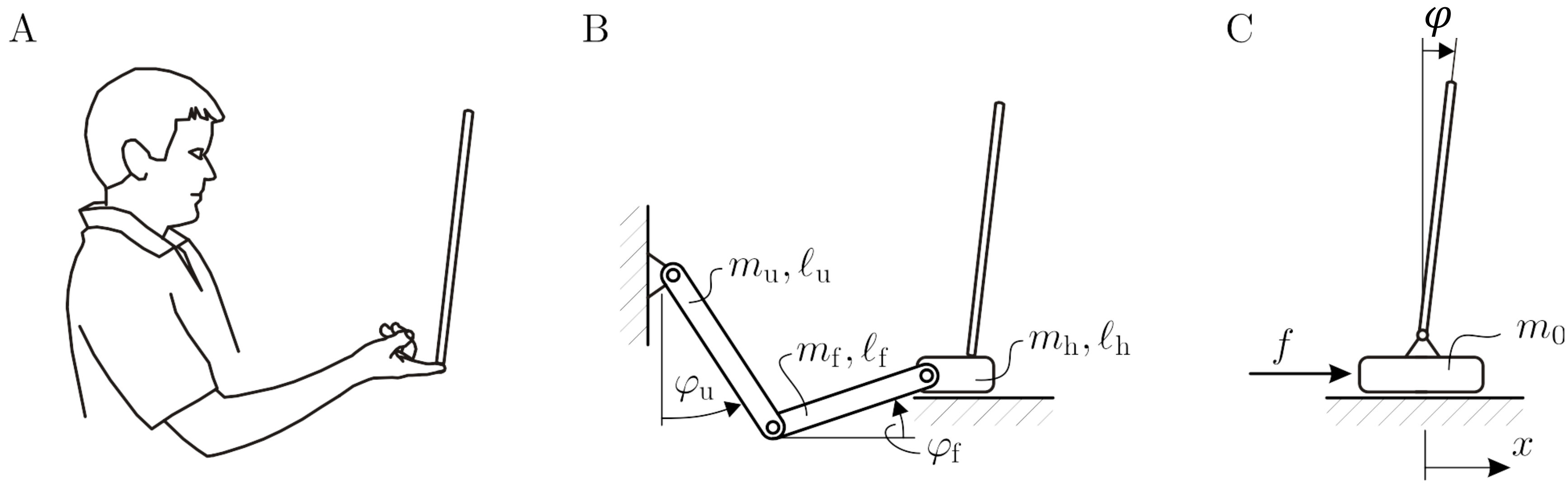
Stick balancing on fingertip

Stick balancing on pingpong racket

Balancing a linearly driven stick



Stick balancing on the fingertip

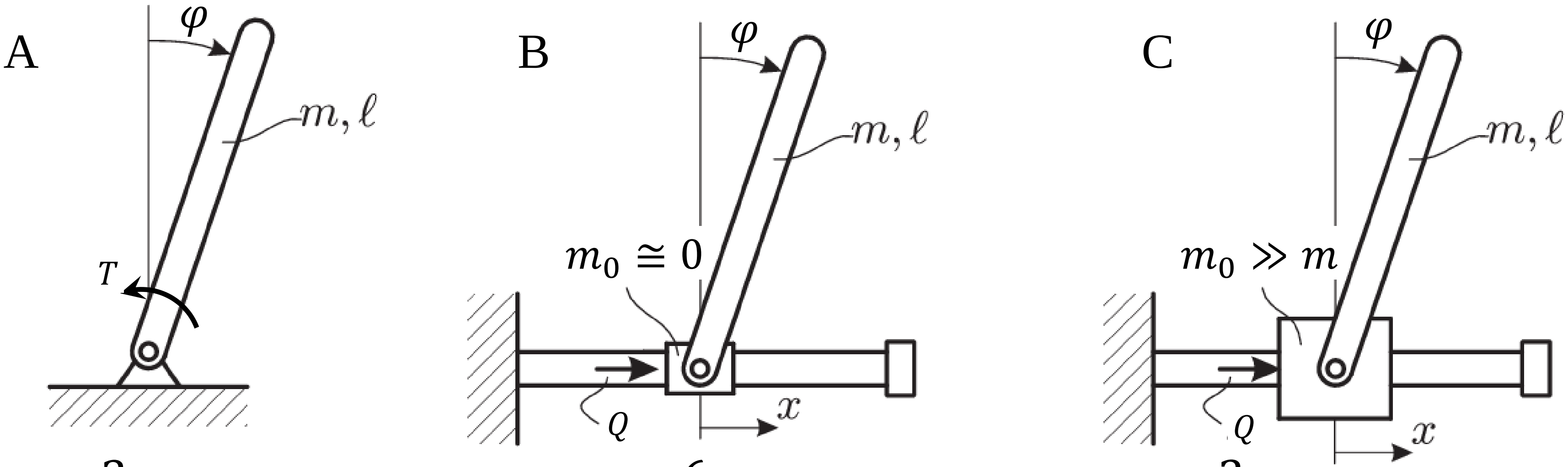


segment	mass	length
upper arm	$m_u = 1.775$kg	$\ell_u = 0.2874$m
forearm	$m_f = 1.015$kg	$\ell_f = 0.2666$m
hand	$m_h = 1.015$kg	$\ell_h = 0.0821$m

$\Rightarrow \quad m_0 \cong 2.3$kg

(de Leva, 1996)

Stick balancing model

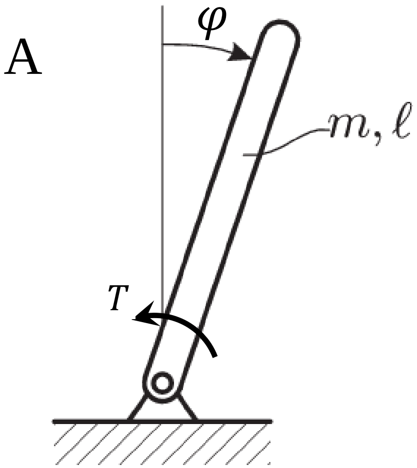


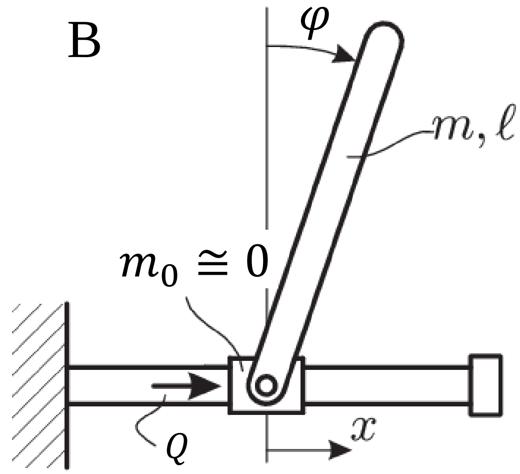
$$\ddot{\varphi} - \frac{3g}{2l}\varphi = c_A T$$

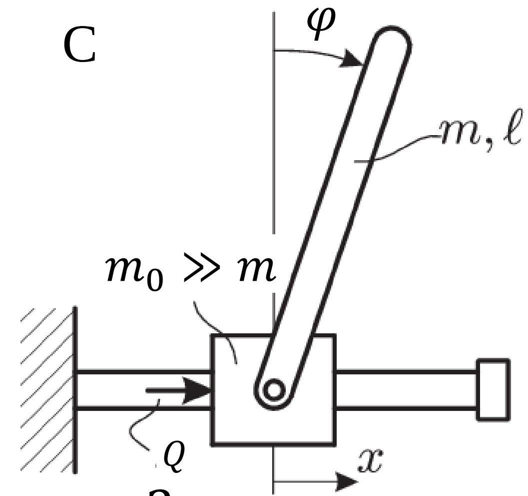
$$\ddot{\varphi} - \frac{6g}{l}\varphi = c_B Q$$

$$\ddot{\varphi} - \frac{3g}{2l}\varphi = c_C Q$$

Stick balancing model







$$\ddot{\varphi} - \frac{3g}{2l}\varphi = c_A T$$

$$\ddot{\varphi} - \frac{6g}{l}\varphi = c_B Q$$

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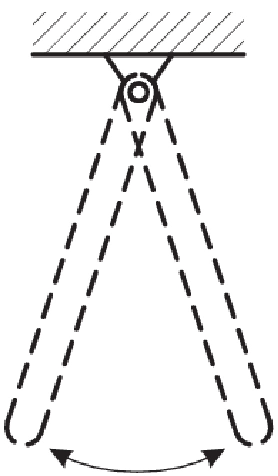
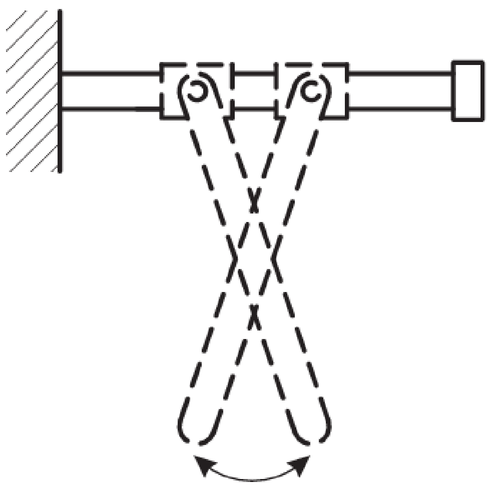
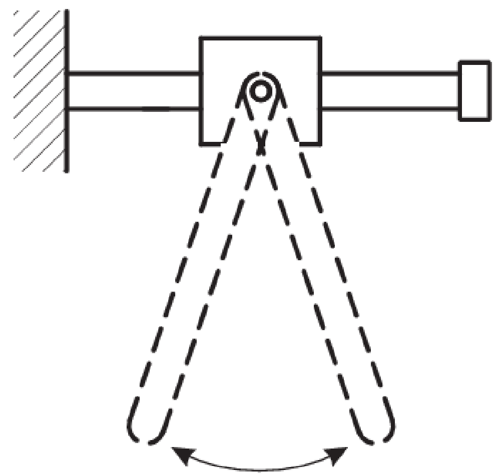


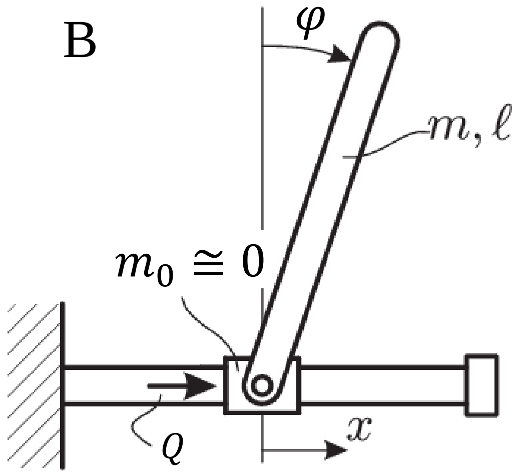


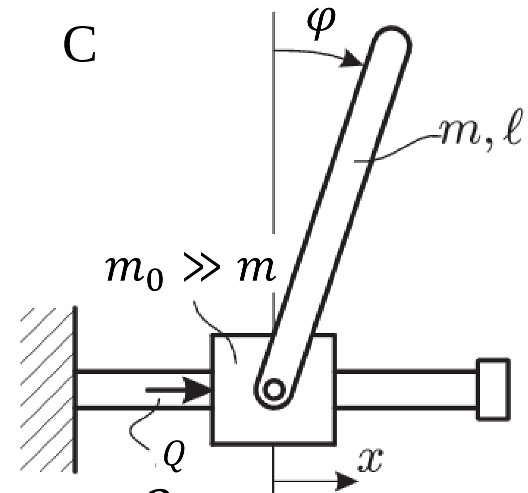


$$T_{\mathrm{A}} = 2\pi\sqrt{2l/(3g)}$$

$$T_{\mathrm{B}} = 2\pi\sqrt{l/(6g)} = T_{\mathrm{A}}/2$$

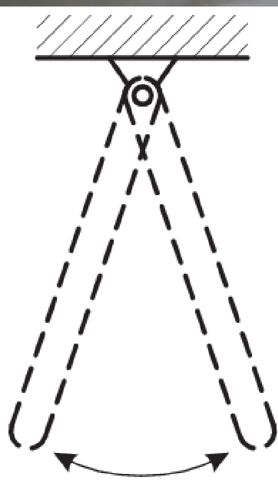
$$T_{\mathrm{C}} = 2\pi\sqrt{2l/(3g)} = T_{\mathrm{A}}$$

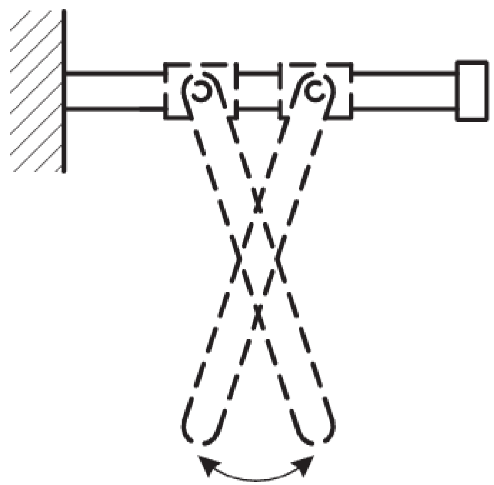


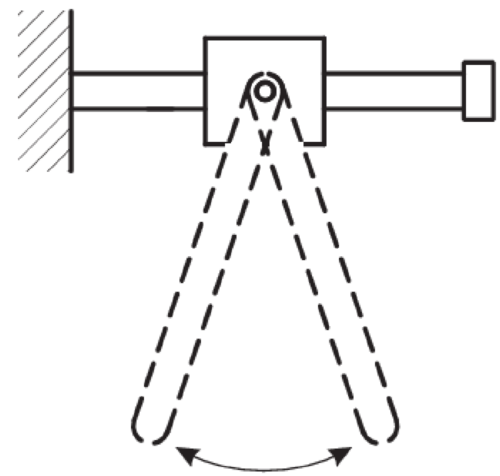


$$\ddot{\varphi} - \frac{6g}{l}\varphi = c_B Q$$

$$\ddot{\varphi} - \frac{3g}{2l}\varphi = c_C Q$$







$$T_{\mathrm{A}} = 2\pi\sqrt{2l/(3g)}$$

$$T_{\mathrm{B}} = 2\pi\sqrt{l/(6g)} = T_{\mathrm{A}}/2$$

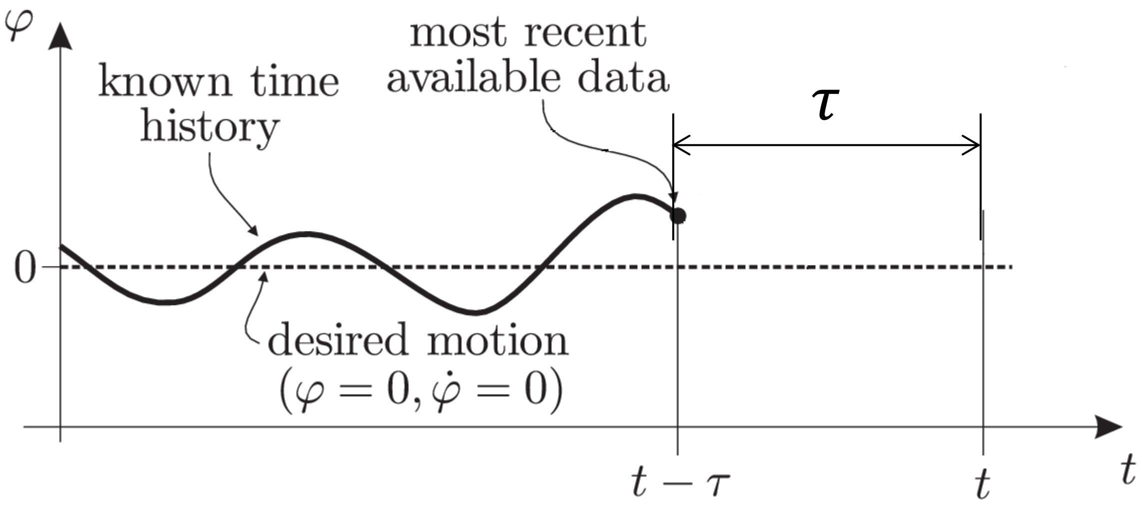
$$T_{\mathrm{C}} = 2\pi\sqrt{2l/(3g)} = T_{\mathrm{A}}$$

Reaction delay

$$\ddot{\varphi}(t) - \frac{3g}{2l}\varphi(t) = -\frac{6}{ml}Q(t)$$

$$Q(t) = f(\varphi(\vartheta), \dot{\varphi}(\vartheta), \ddot{\varphi}(\vartheta)) \qquad \vartheta \in [0, t-\tau]$$

For example $Q(t) = -k_\mathrm{p}\varphi(t-\tau) - k_\mathrm{d}\dot{\varphi}(t-\tau)$

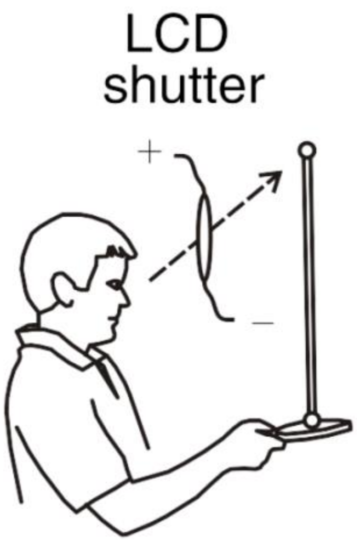


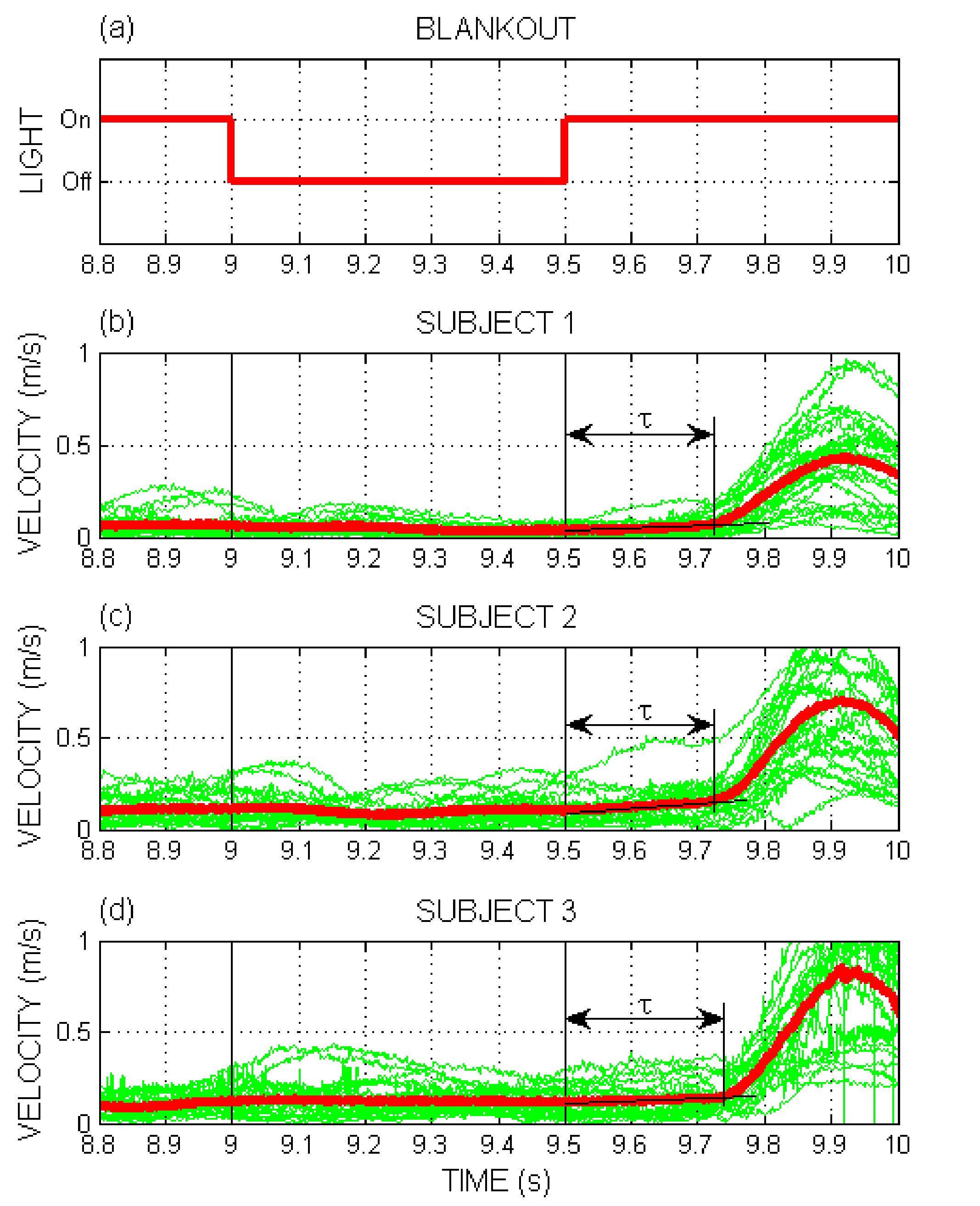
delay for visual tracking
Nasher (1976): 150~250ms
Miall (1993): 200~250ms
Jordan (1996): 100~200ms
Kawato (1999): 150~250ms

delay for stick balancing using cross-correlation:
Cabrera, Milton (2004): 80~200ms

Reaction delay

blankout tests:
Milton (2011):
$\tau \approx 230\text{ms}$





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$$\ddot{\varphi}(t) - a\varphi(t) = -k_{\mathrm{p}}\varphi(t-\tau) - k_{\mathrm{d}}\dot{\varphi}(t-\tau)$$

$$a = \frac{3g}{2l}$$

(system parameter)

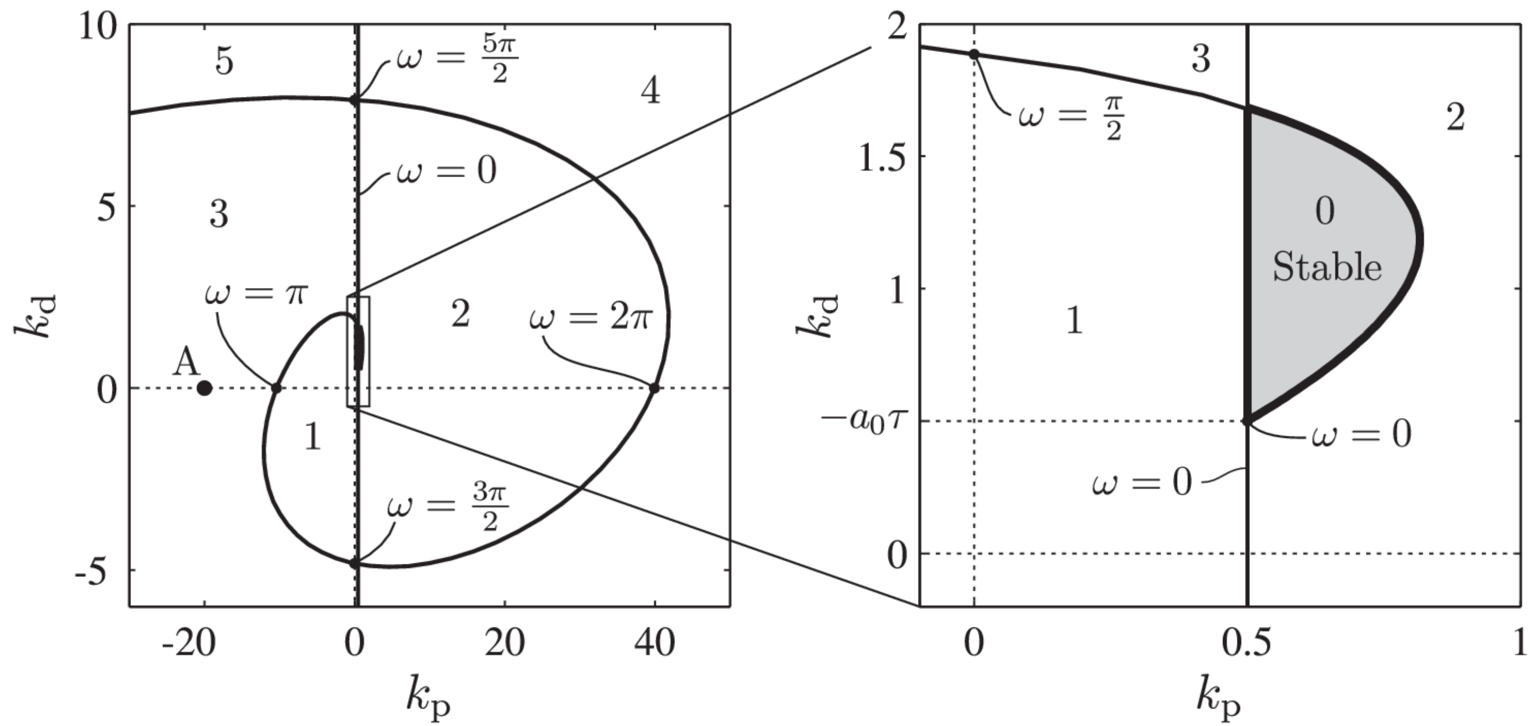
D-subdivision:

$\omega = 0$: $k_{\mathrm{p}} = a$

$\omega \neq 0$: $k_{\mathrm{p}} = (\omega^2 + a)\cos(\omega\tau)$

$$k_{\mathrm{d}} = \frac{\omega^2 + a}{\omega}\sin(\omega\tau)$$

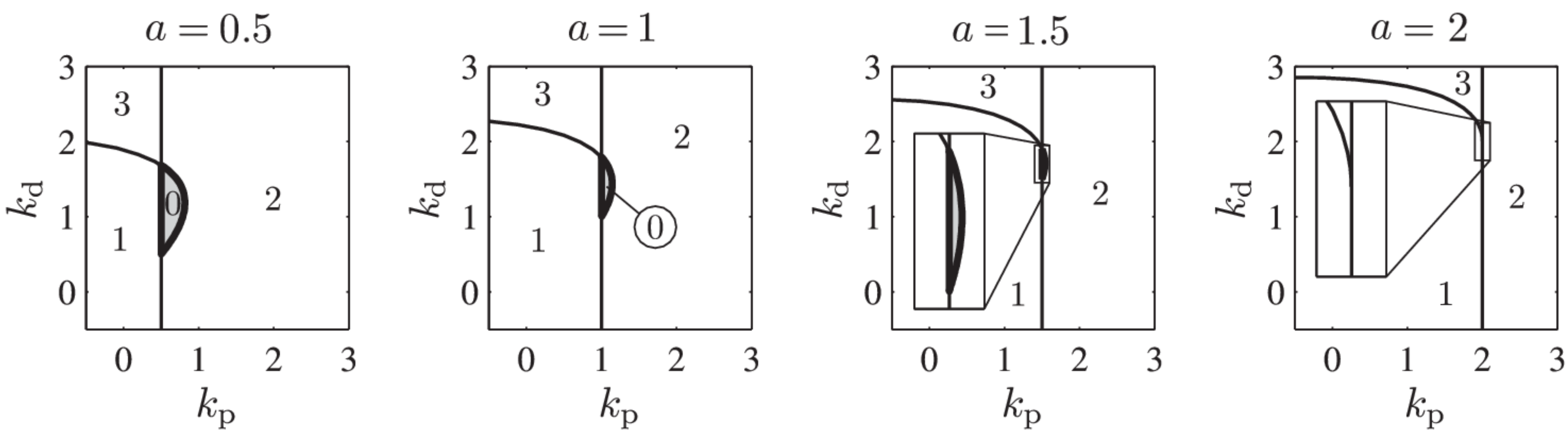
$\tau = 1, a = 0.5$



Delayed PD feedback

$$\ddot{\varphi}(t) - a\varphi(t) = -k_\mathrm{p}\varphi(t-\tau) - k_\mathrm{d}\dot{\varphi}(t-\tau)$$

$\tau = 1$



$$a_\mathrm{crit} = \frac{2}{\tau^2}$$ (Schürer, 1948)

Or, for fixed a, $\tau_\mathrm{crit} = \sqrt{\frac{2}{a}} = \frac{T_\mathrm{p}}{\pi\sqrt{2}}$, T_p: downward oscillation period

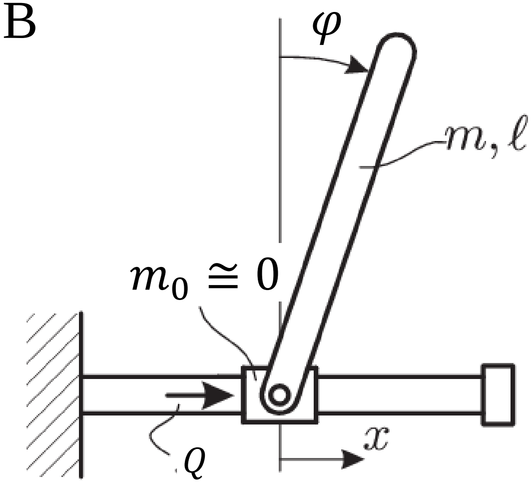
(Stepan, 2009)

Stick balancing model

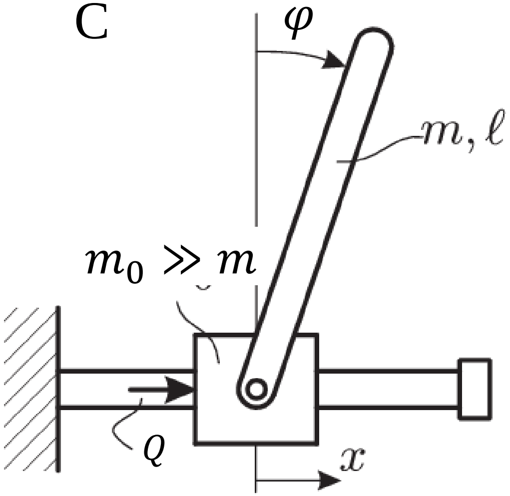
$$\ddot{\varphi}(t) - a\varphi(t) = -k_{\mathrm{p}}\varphi(t-\tau) - k_{\mathrm{d}}\dot{\varphi}(t-\tau)$$

$$\tau = 230\mathrm{ms}$$

B



C



$$a_{\mathrm{crit}} = \frac{2}{\tau^2}$$

$$a = \frac{6g}{l}$$

$$a = \frac{3g}{2l}$$

$$l_{\mathrm{crit-B}} = 3g\tau^2 = 156\mathrm{cm}$$

$$l_{\mathrm{crit-C}} = \frac{3}{4}g\tau^2 = 39\mathrm{cm}$$

Experiments: $l_{\mathrm{crit}} = 25\sim30\mathrm{cm}$ (Milton et al., 1990–)

Stick balancing model

$$l_{\text{crit-C}} = \frac{3}{4} g\tau^2 = 39\text{cm}$$

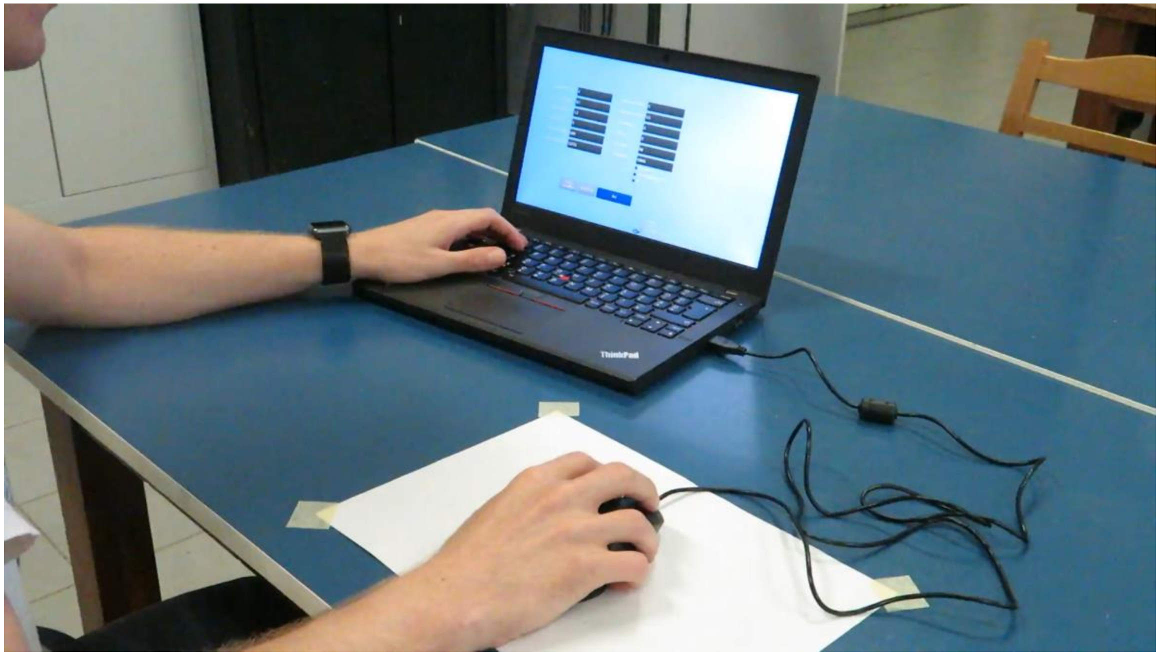
Experiments: $l_{\text{crit}} = 25\sim30\text{cm}$ (Milton et al., 1990–)

Another video

$$l_{\text{crit-c}} = \frac{3}{4} g \tau^2 = 39\text{cm}$$

Experiments: $l_{\text{crit}} = 25 \sim 30\text{cm}$ (Milton et al., 1990–)

Virtual stick balancing



Virtual stick balancing

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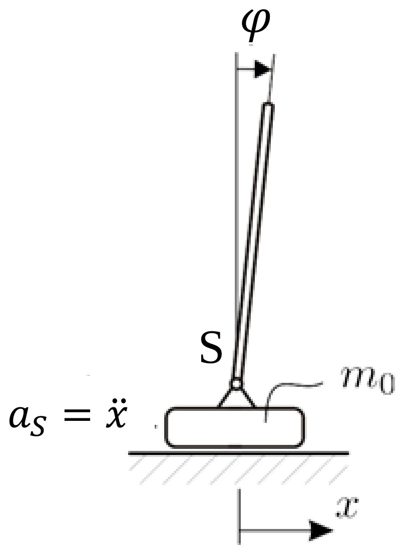


$$\ddot{\varphi}(t) - \frac{3g}{2l}\varphi(t) = -\frac{3}{2l}a_S(t)$$

a_S: acceleration of stick's bottom

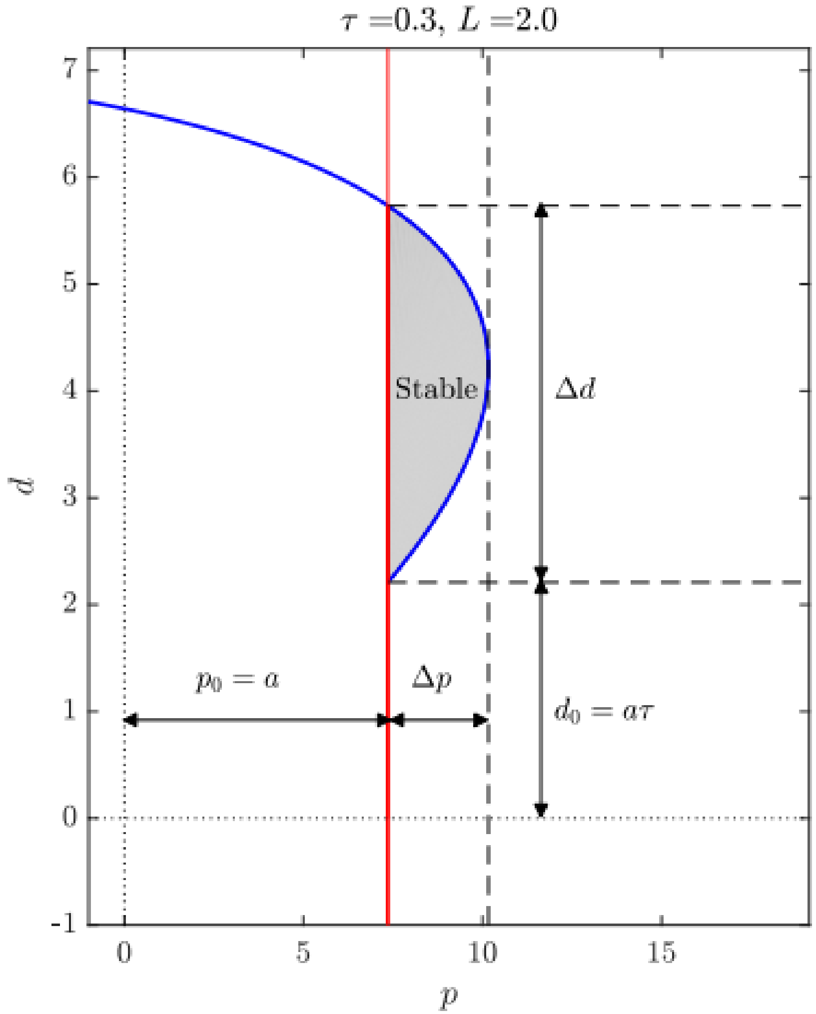
$$a_S(t) = -k_\mathrm{p}\varphi(t-\tau) - k_\mathrm{d}\dot{\varphi}(t-\tau)$$

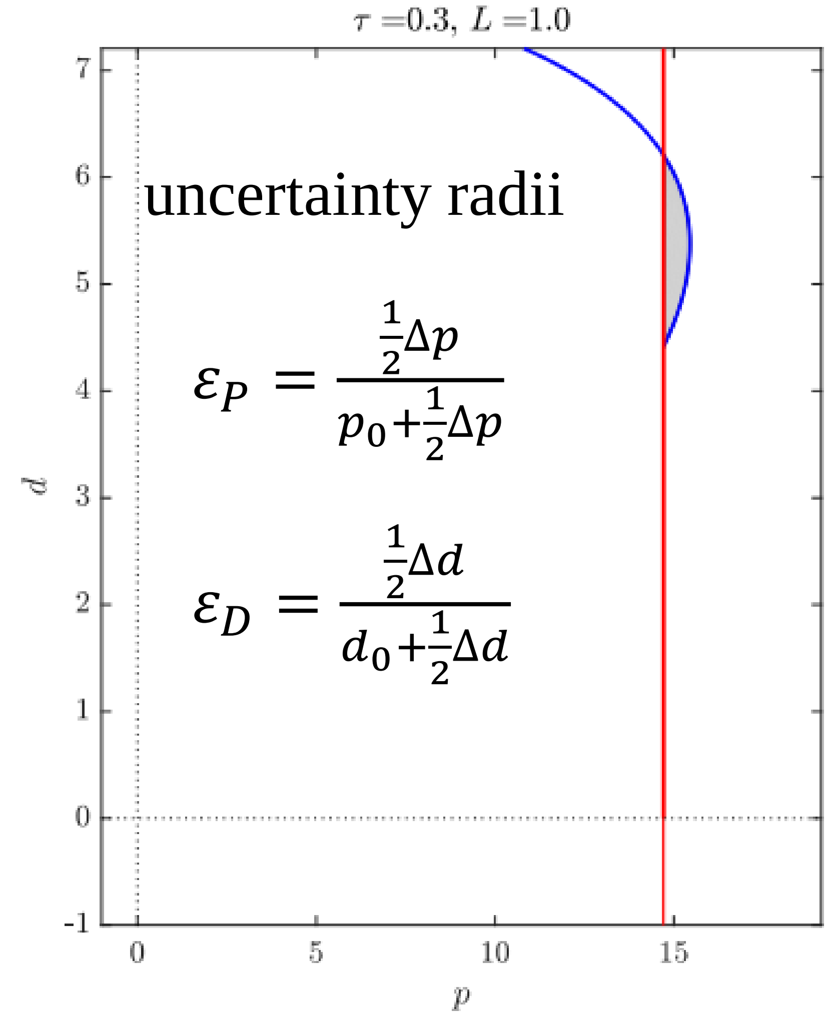
$$\ddot{\varphi}(t) - a\varphi(t) = -p\,\varphi(t-\tau) - d\,\dot{\varphi}(t-\tau)$$

$$a = \frac{3g}{2l}$$

Virtual stick balancing

$$\ddot{\varphi}(t) - a\varphi(t) = -p\,\varphi(t-\tau) - d\,\dot{\varphi}(t-\tau)$$





uncertainty radii

$$\varepsilon_P = \frac{\frac{1}{2}\Delta p}{p_0 + \frac{1}{2}\Delta p}$$

$$\varepsilon_D = \frac{\frac{1}{2}\Delta d}{d_0 + \frac{1}{2}\Delta d}$$

Virtual stick balancing

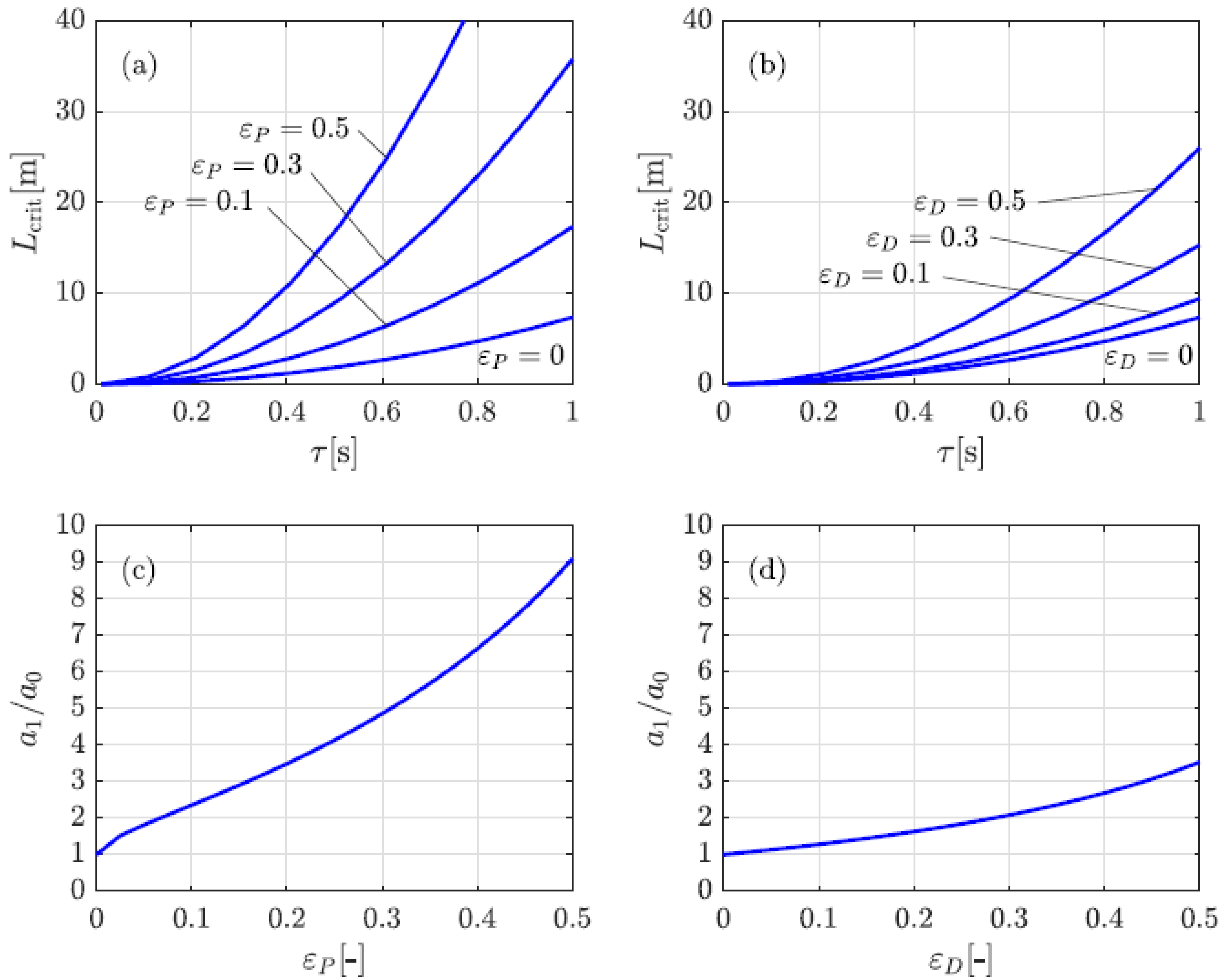
$$\ddot{\varphi}(t) - a\varphi(t) = -p\,\varphi(t-\tau) - d\,\dot{\varphi}(t-\tau)$$

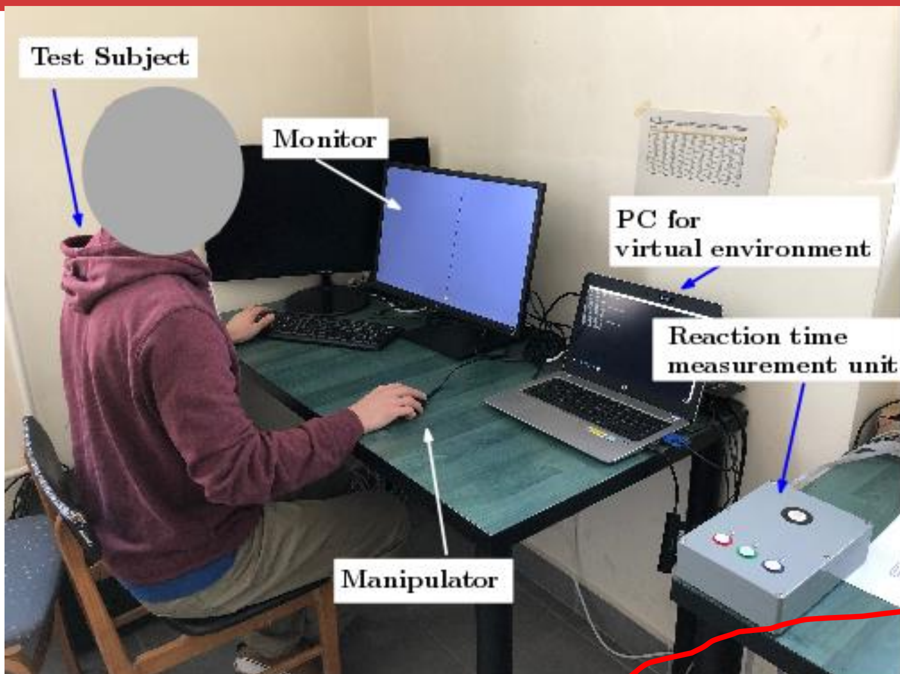
$$l_{\text{crit}}\Big|_{\varepsilon=0} = a_0\tau^2$$

$$a_0 = \frac{3}{4}g$$

$$l_{\text{crit}}\Big|_{\varepsilon>0} = a_1\tau^2$$

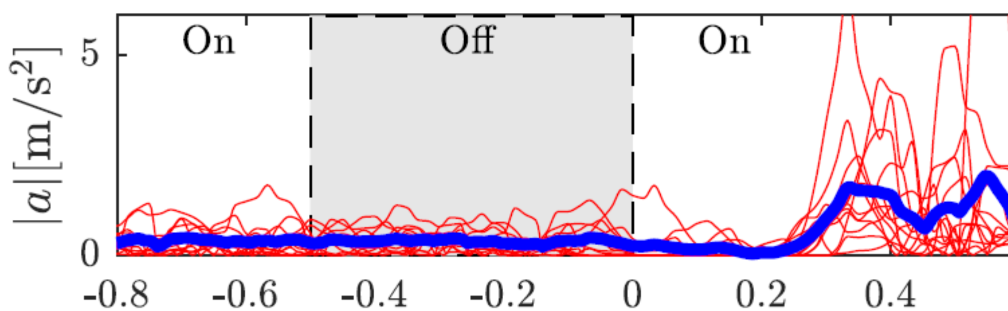
$$\frac{a_1}{a_0} = ?$$



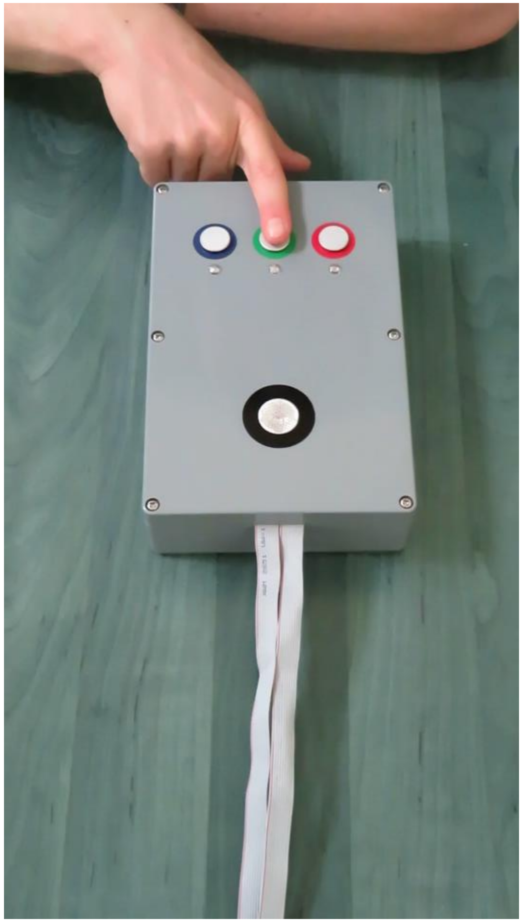


$$\tau = \tau_{\text{Machine}} + \tau_{\text{Neural}} + \tau_{\text{Added}}$$

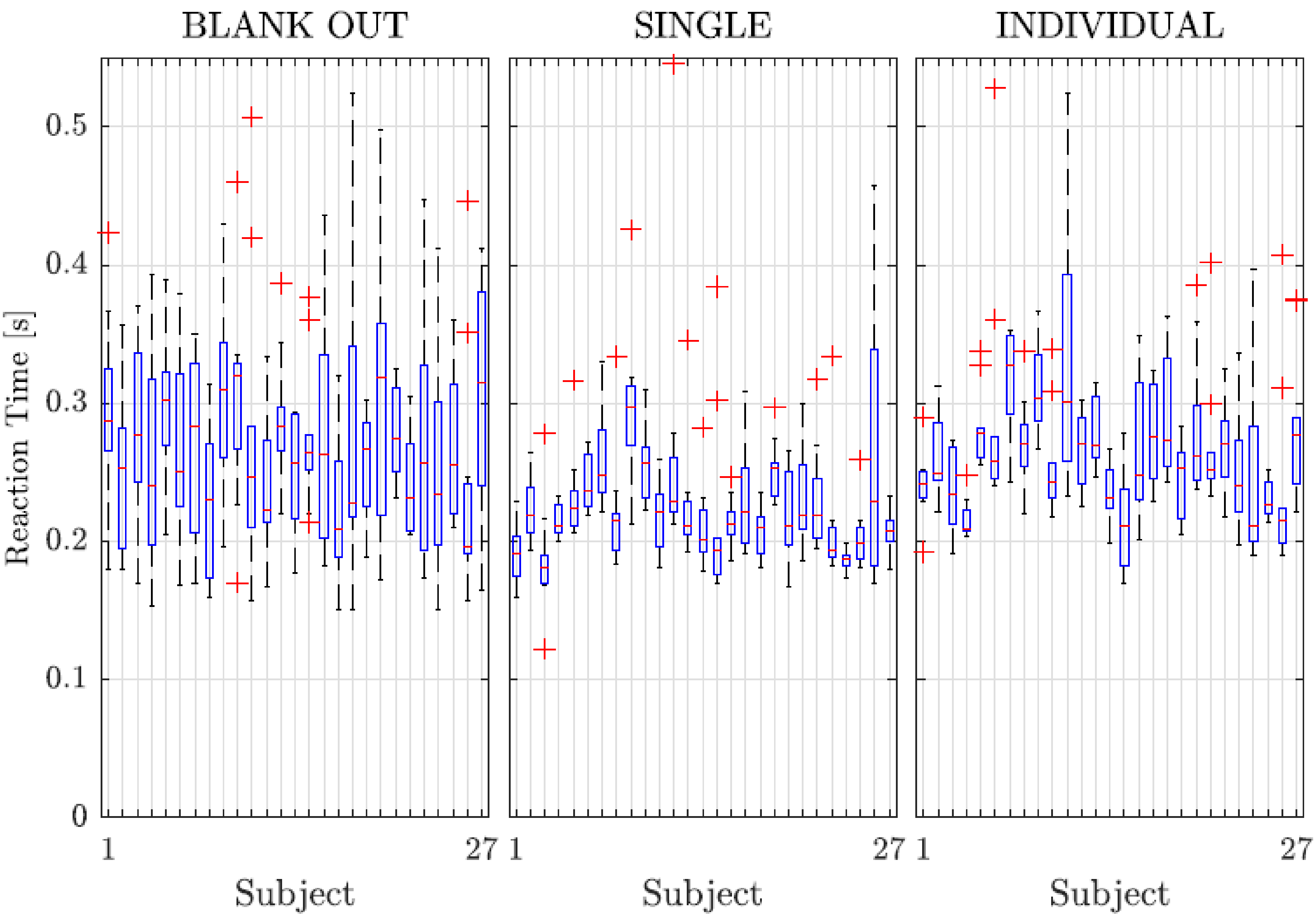
122 ms ~250 ms $k \times 50$ ms



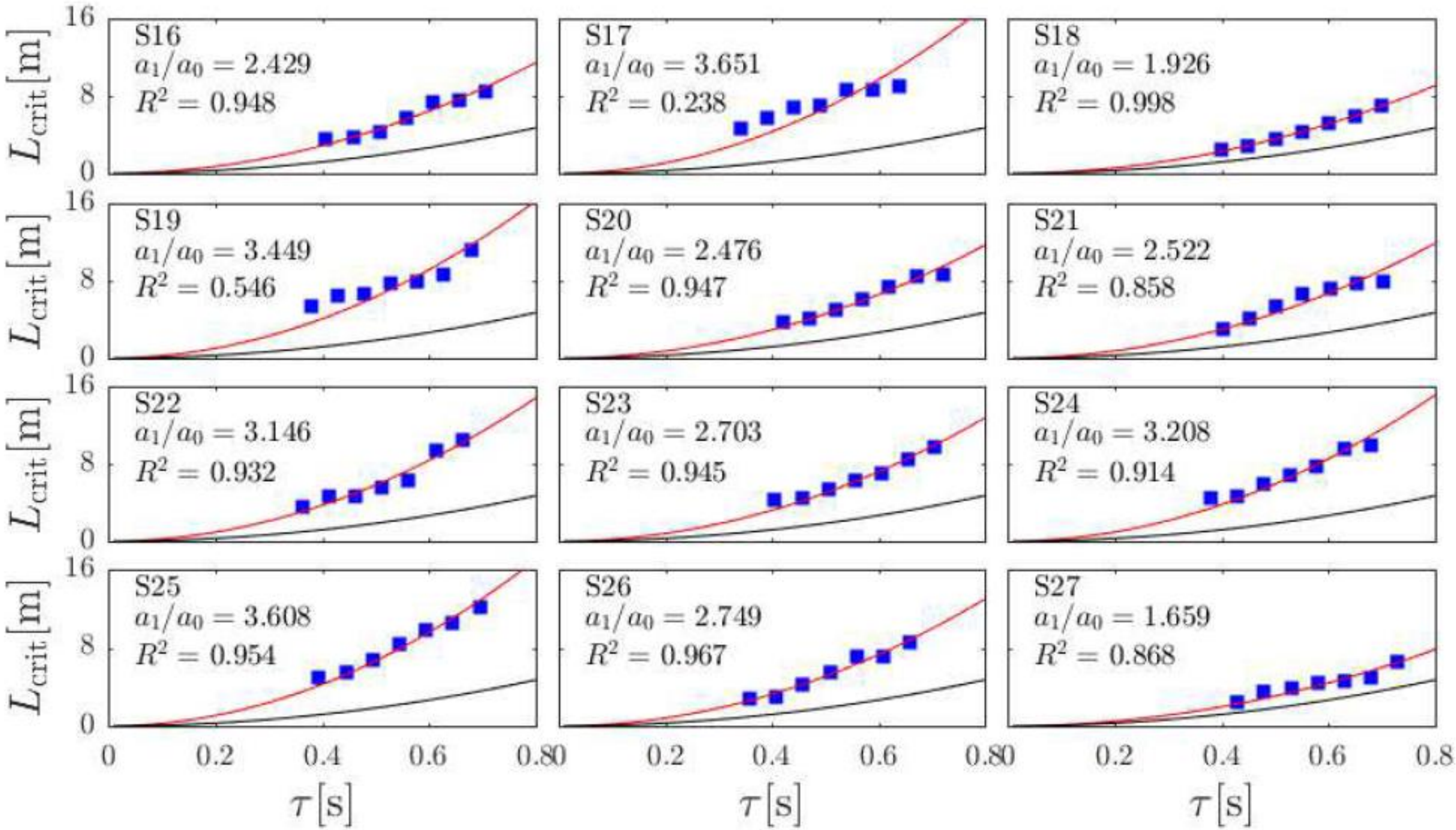
SINGLE



INDIVIDUAL


BLANK OUT
SINGLE
INDIVIDUAL
Reaction Time [s]
0
0.1
0.2
0.3
0.4
0.5
1
27
Subject
1
27
Subject
1
27
Subject

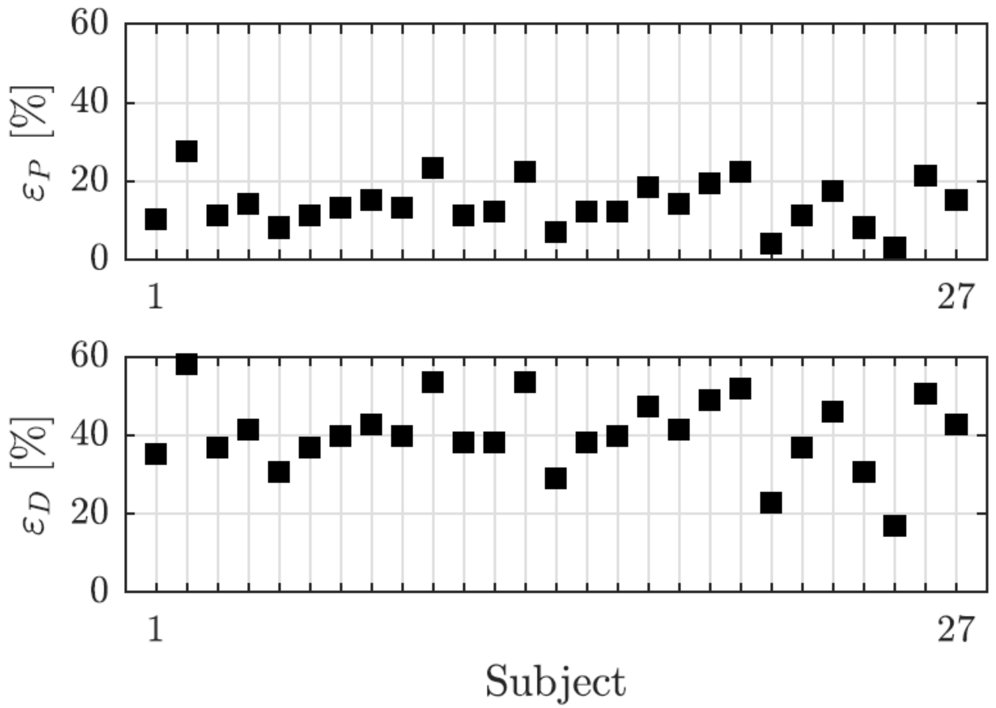
Virtual stick balancing



$$l_{\mathrm{crit}}\Big|_{\varepsilon=0} = a_0\tau^2$$

$$l_{\mathrm{crit}}\Big|_{\varepsilon>0} = a_1\tau^2$$

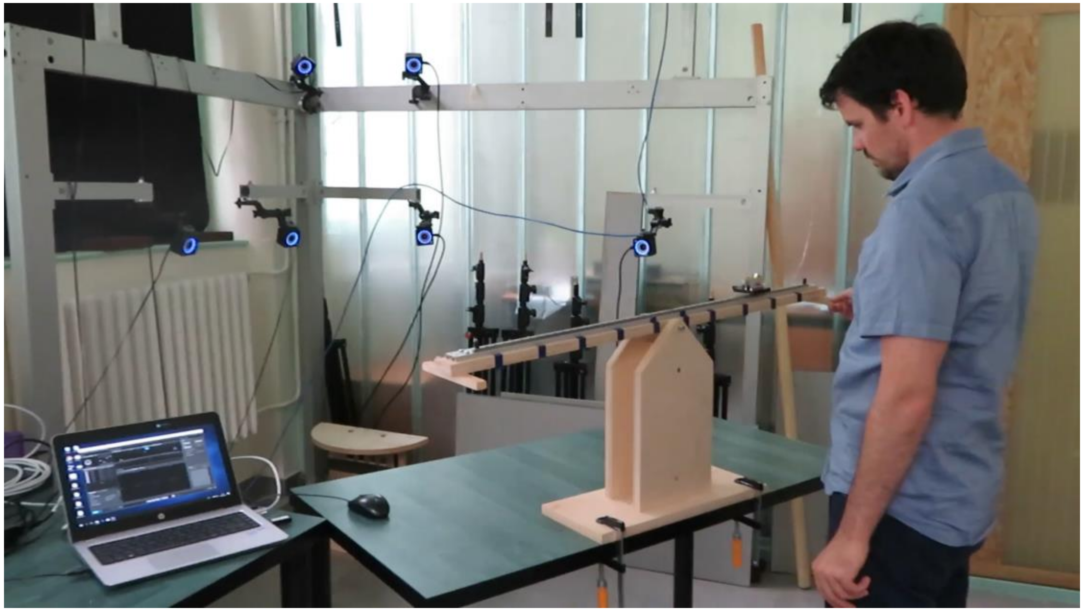
Uncertainty radii



3.1~27.6%
mean: 14.1%

16.8~58.2%
mean: 40.3%

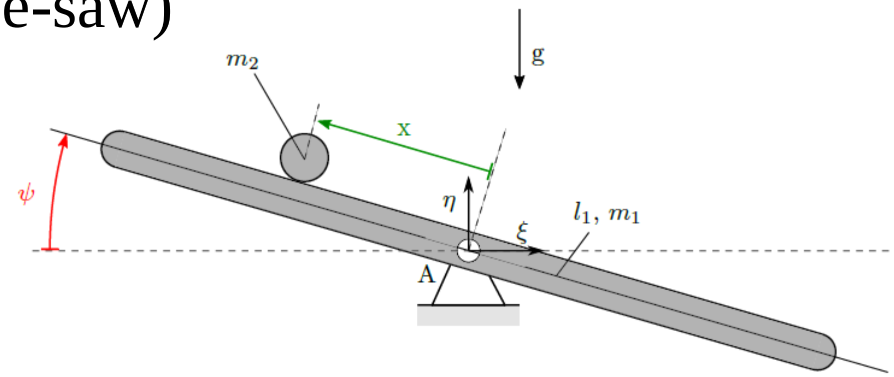
Ball and beam (rolling cart on a see-saw)



Pendulum-cart and beam (pendulum-cart rolling on a see-saw)

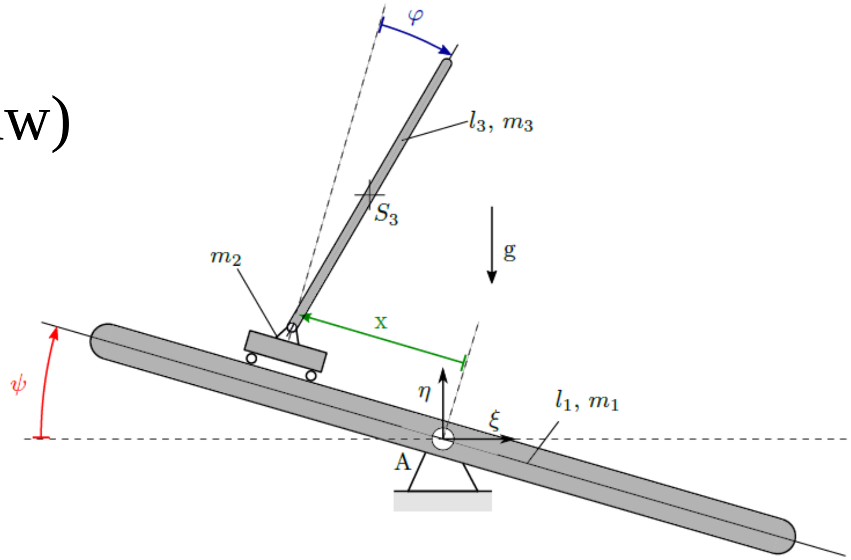
Two balancing tasks

1, Ball and beam (rolling cart on a see-saw)



Manipulated variable:
1.1. angle
1.2. angular velocity
1.3. angular acceleration
1.4. torque

2, Pendulum-cart and beam (pendulum-cart rolling on a see-saw)



Manipulated variable:
2.1. angle
2.2. angular velocity
2.3. angular acceleration
2.4. torque

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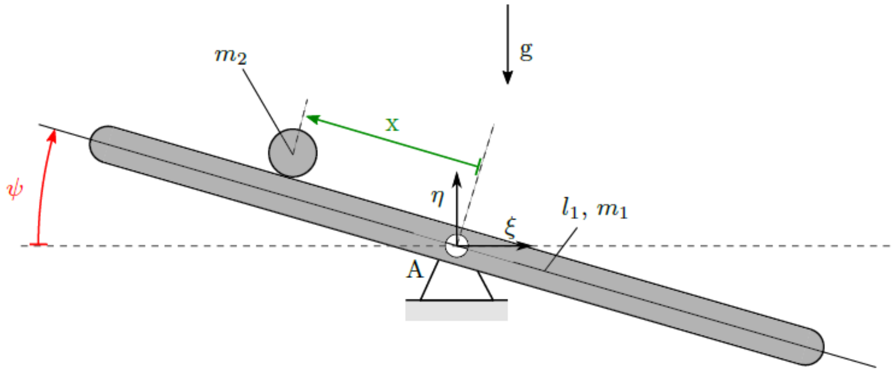
Ball and beam ~ angle

$$\ddot{x}(t) = -g\psi(t) \qquad \psi(t) = P_x x(t) + D_x \dot{x}(t)$$

$$\ddot{x}(t) + gD_x\dot{x}(t) + gP_x x(t) = 0$$

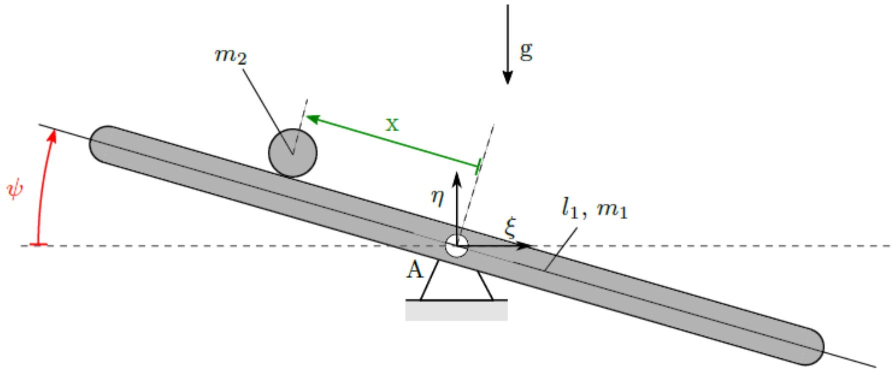


Ball and beam ~ angle

with reaction delay

$$\ddot{x}(t) = -g\psi(t) \qquad \psi(t) = P_x x(t-\tau) + D_x \dot{x}(t-\tau)$$

$$\ddot{x}(t) + gD_x\dot{x}(t-\tau) + gP_x x(t-\tau) = 0$$

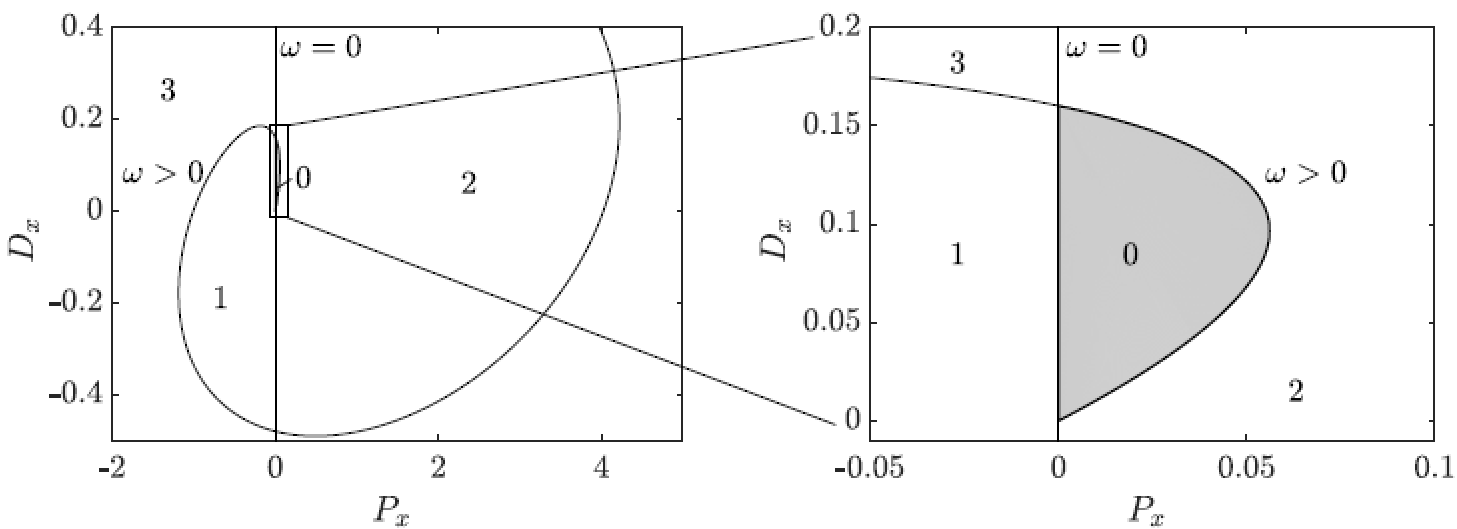


Ball and beam ~ angle

with reaction delay

$$\ddot{x}(t) = -g\psi(t) \qquad \psi(t) = P_x x(t-\tau) + D_x \dot{x}(t-\tau)$$

$$\ddot{x}(t) + gD_x\dot{x}(t-\tau) + gP_x x(t-\tau) = 0$$



$\tau_{\text{crit}} = \infty$

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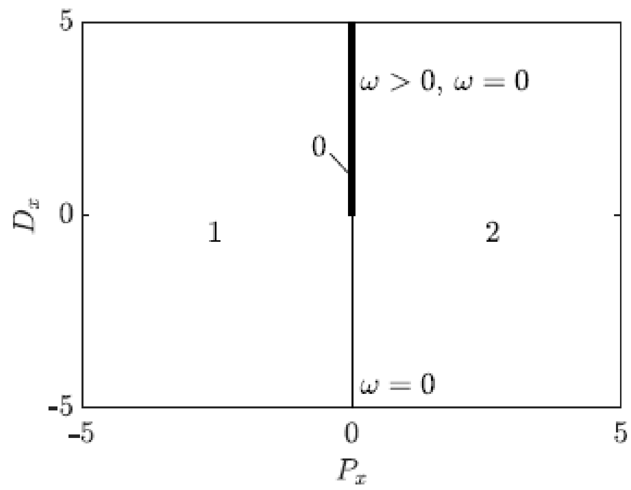
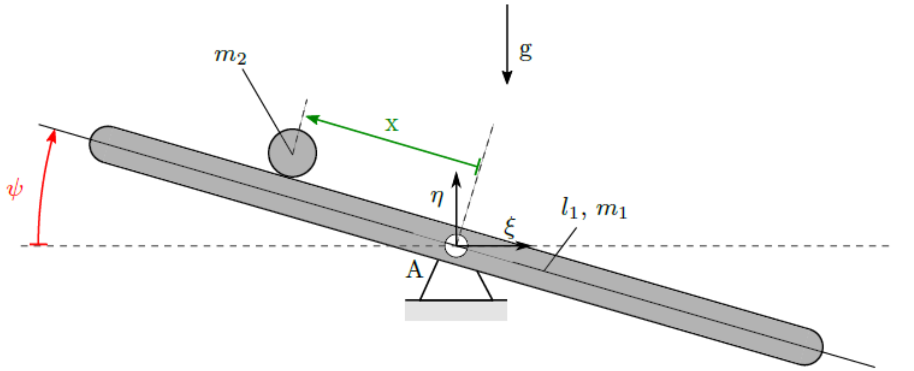
Ball and beam ~ angular velocity

$$\ddot{x}(t) = -g\dot{\psi}(t) = -g\omega(t)$$

$$\omega(t) = P_x x(t) + D_x \dot{x}(t)$$

$$\ddot{x}(t) + gD_x\dot{x}(t) + gP_x x(t) = 0$$



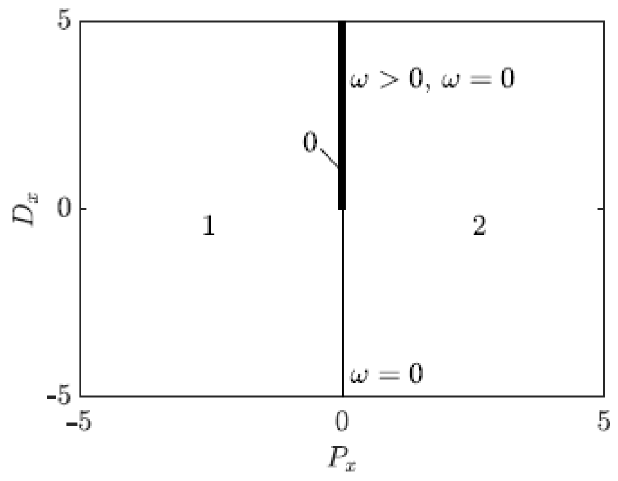


Ball and beam ~ angular velocity

$$\ddot{x}(t) = -g\dot{\psi}(t) = -g\omega(t)$$

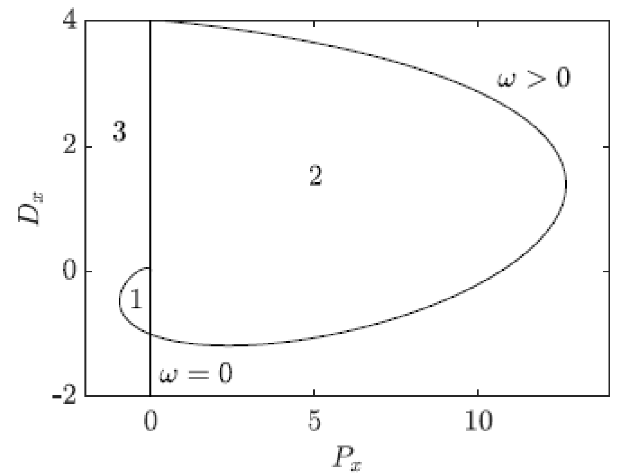
$$\omega(t) = P_x x(t) + D_x \dot{x}(t)$$

$$\ddot{x}(t) + gD_x\dot{x}(t) + gP_x x(t) = 0$$



with reaction delay

$$\dddot{x}(t) + gD_x\dot{x}(t-\tau) + gP_x x(t-\tau) = 0$$

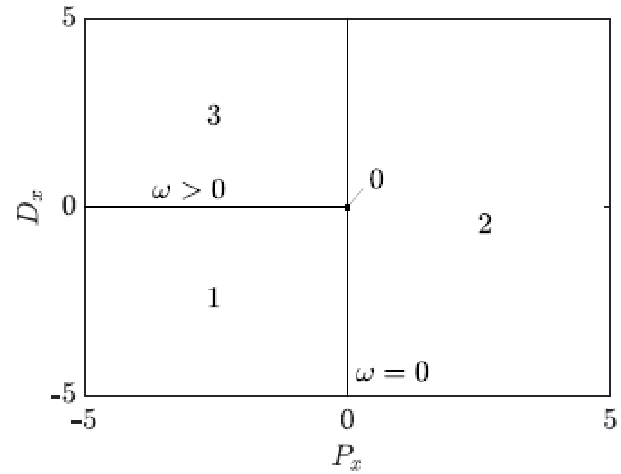


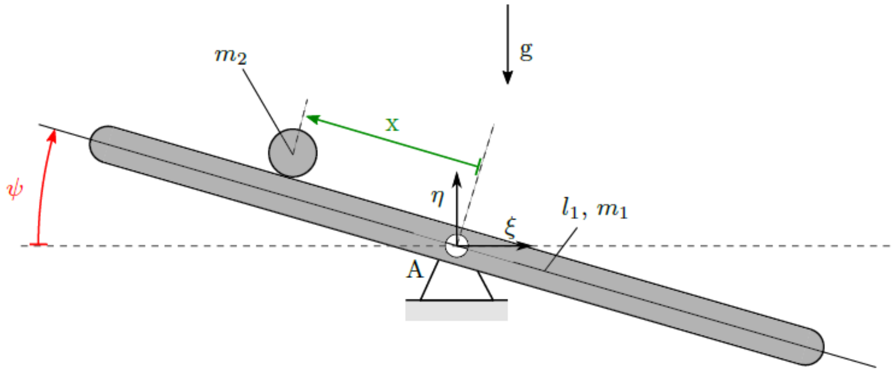
Ball and beam ~ angular acceleration

$$x^{(\mathrm{iv})}(t) = -g\ddot{\psi}(t) = -g\varepsilon(t)$$

$$\varepsilon(t) = P_x x(t) + D_x \dot{x}(t)$$

$$x^{(\mathrm{iv})}(t) + gD_x\dot{x}(t) + gP_x x(t) = 0$$



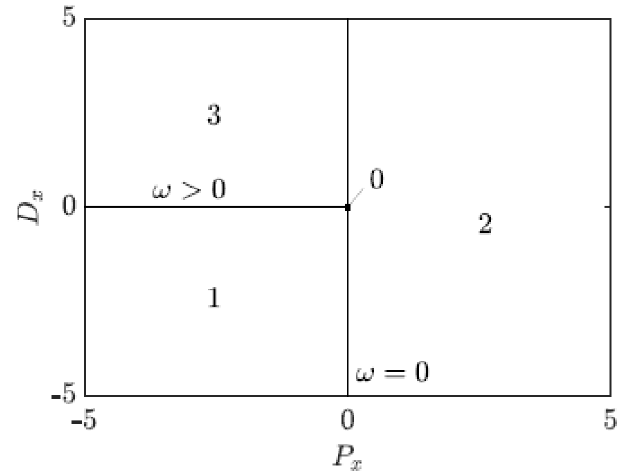


Ball and beam ~ angular acceleration

$$x^{(\mathrm{iv})}(t) = -g\ddot{\psi}(t) = -g\varepsilon(t)$$

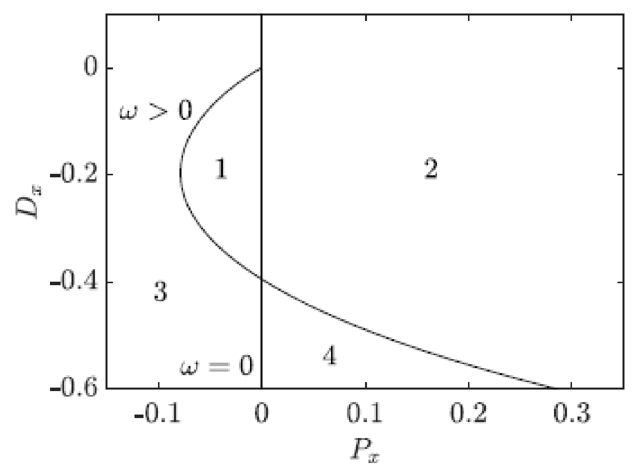
$$\varepsilon(t) = P_x x(t) + D_x \dot{x}(t)$$

$$x^{(\mathrm{iv})}(t) + gD_x\dot{x}(t) + gP_x x(t) = 0$$



with reaction delay

$$x^{(\mathrm{iv})}(t) + gD_x\dot{x}(t-\tau) + gP_x x(t-\tau) = 0$$

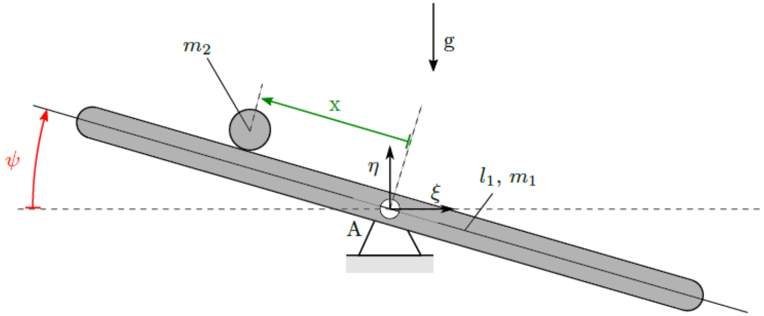


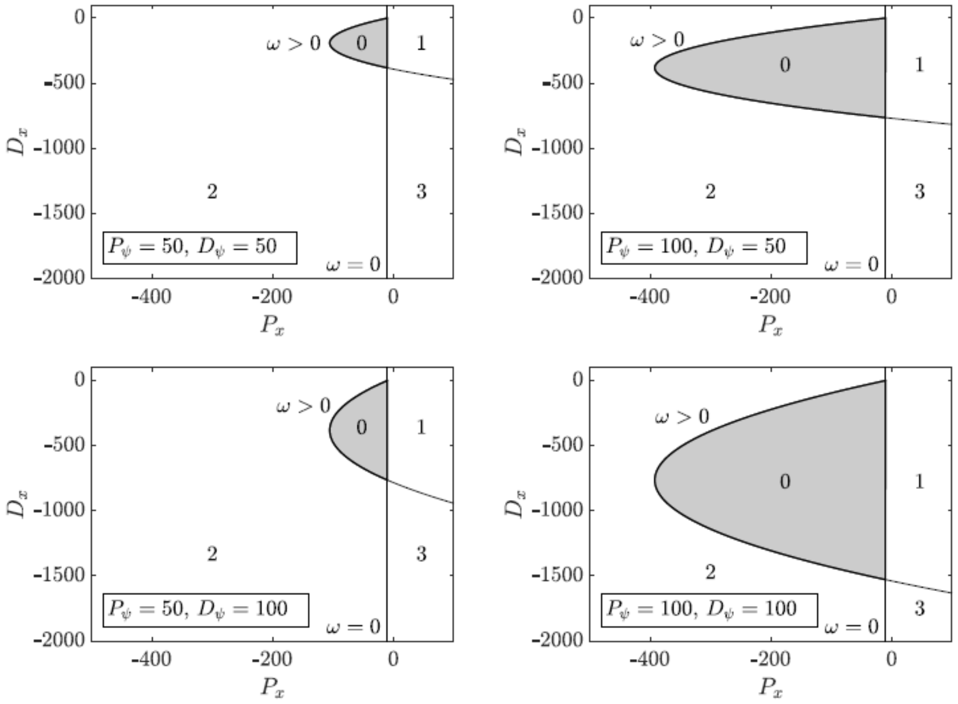
Ball and beam ~ torque (2 DoF)

$$\mathbf{q}(t) = \left(x(t), \psi(t)\right)^T$$

$$\begin{pmatrix} m_2 & 0 \\ 0 & \theta_1 \end{pmatrix} \ddot{\mathbf{q}}(t) + \begin{pmatrix} 0 & m_2 g \\ m_2 g & 0 \end{pmatrix} \mathbf{q}(t) = \begin{pmatrix} 0 \\ -Q(t) \end{pmatrix}$$

$$Q(t) = P_x x(t) + D_x \dot{x}(t) + P_\psi \psi(t) + D_\psi \dot{\psi}(t)$$





Model 1.4

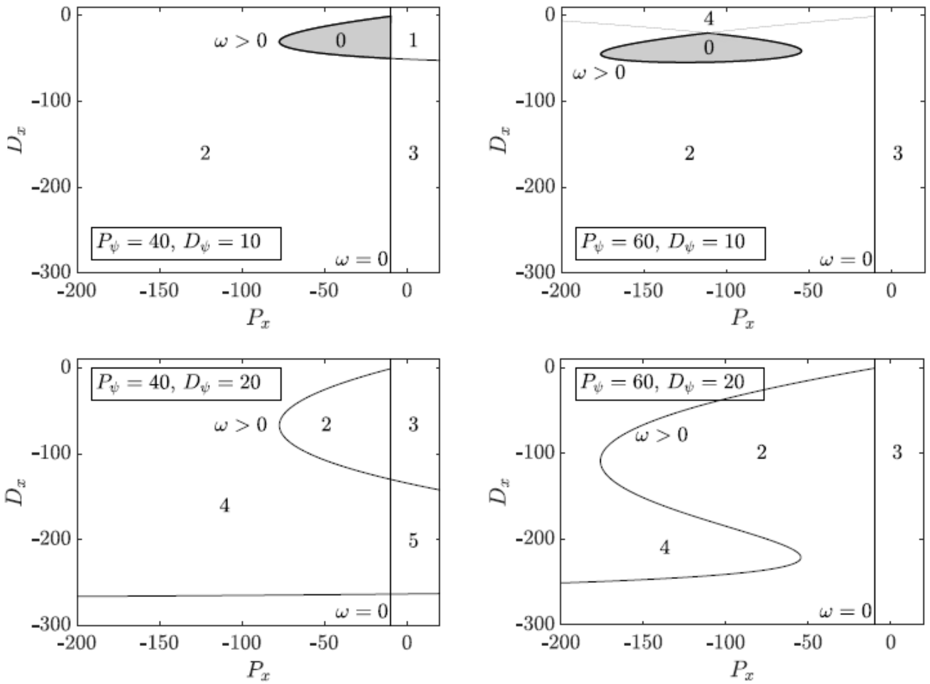
Ball and beam ~ torque (2 DoF)

$$\mathbf{q}(t) = \left(x(t), \psi(t)\right)^T$$

$$\begin{pmatrix} m_2 & 0 \\ 0 & \theta_1 \end{pmatrix} \ddot{\mathbf{q}}(t) + \begin{pmatrix} 0 & m_2 g \\ m_2 g & 0 \end{pmatrix} \mathbf{q}(t) = \begin{pmatrix} 0 \\ -Q(t) \end{pmatrix}$$

with reaction delay

$$Q(t) = P_x x(t-\tau) + D_x \dot{x}(t-\tau) + P_\psi \psi(t-\tau) + D_\psi \dot{\psi}(t-\tau)$$



$\tau_{\text{crit}} = 180$ ms

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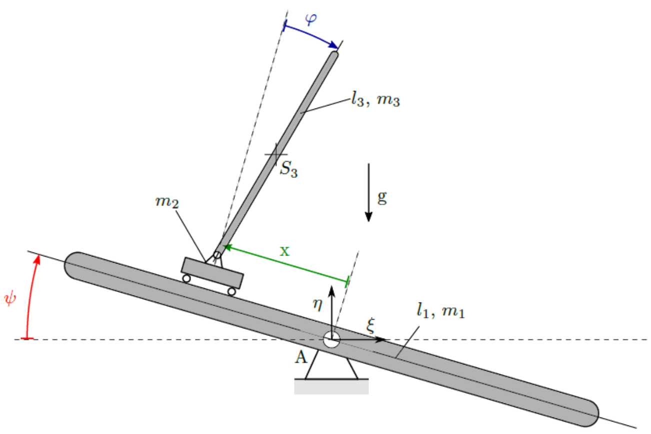
Pendulum-cart and beam ~ angle (2 DoF)

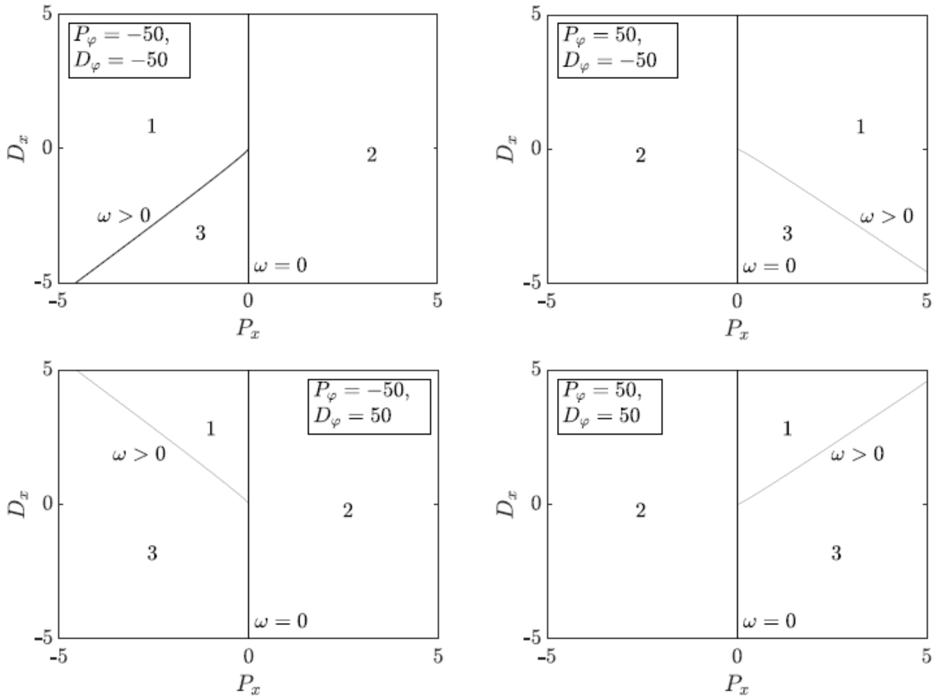
$$\ddot{x}(t) = a\varphi(t) - g\psi(t)$$

$$\ddot{\varphi}(t) = b\varphi(t) - \ddot{\psi}(t)$$

$$\psi(t) = P_x x(t) + D_x \dot{x}(t) + P_\varphi \varphi(t) + D_\varphi \dot{\varphi}(t)$$

$$a = \frac{3m_3 g}{m_3 + 4m_2}, \quad b = \frac{6g(m_2 + m_3)}{l_3(m_3 + 4m_2)}$$





Pendulum-cart and beam ~ angle (2 DoF)

$$\ddot{x}(t) = a\varphi(t) - g\psi(t)$$
$$\ddot{\varphi}(t) = b\varphi(t) - \ddot{\psi}(t)$$

$$a = \frac{3m_3 g}{m_3 + 4m_2}, \quad b = \frac{6g(m_2 + m_3)}{l_3(m_3 + 4m_2)}$$

with reaction delay

$$\psi(t) = P_x x(t-\tau) + D_x \dot{x}(t-\tau) + P_\varphi \varphi(t-\tau) + D_\varphi \dot{\varphi}(t-\tau)$$

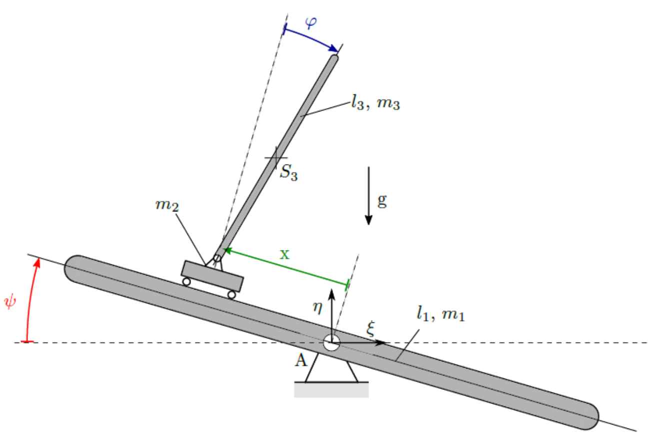
Advanced functional differential equation

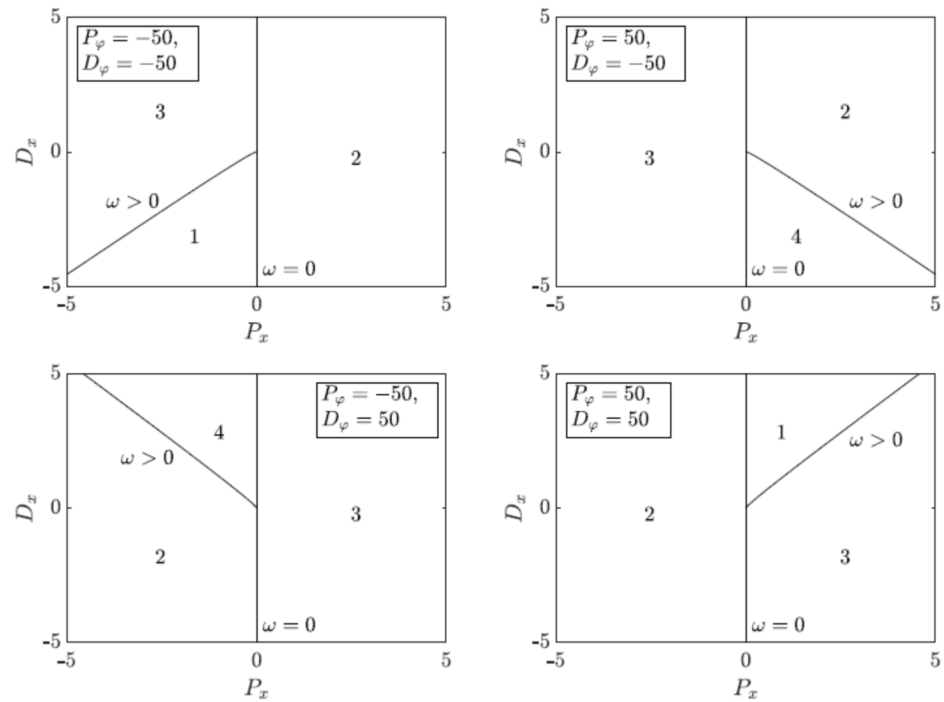
$$D(\lambda) = D_\varphi \lambda^5 e^{-\lambda\tau} + (P_\varphi e^{-\lambda\tau} + 1)\lambda^4 + (a+g)D_x \lambda^3 e^{-\lambda\tau} + \left[(a+g)P_x e^{-\lambda\tau} - b\right]\lambda^2 - bgD_x\lambda e^{-\lambda\tau} - bgP_x e^{-\lambda\tau}$$

Pendulum-cart and beam ~ angular velocity (2 DoF)

$$\ddot{x}(t) = a\dot{\varphi}(t) - g\omega(t)$$

$$\ddot{\varphi}(t) = b\varphi(t) - \dot{\omega}(t)$$





$$\omega(t) = \dot{\psi}(t) = P_x x(t) + D_x \dot{x}(t) + P_\varphi \varphi(t) + D_\varphi \dot{\varphi}(t)$$

Pendulum-cart and beam ~ angular velocity (2 DoF)

$$\dddot{x}(t) = a\dot{\varphi}(t) - g\omega(t)$$

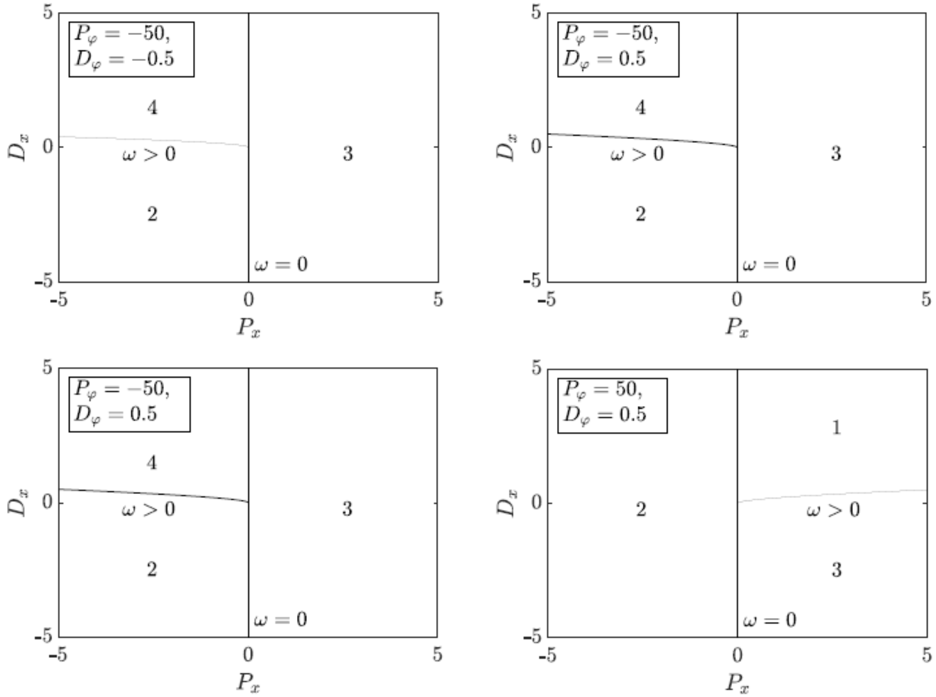
$$\ddot{\varphi}(t) = b\varphi(t) - \dot{\omega}(t)$$

with reaction delay

$$\omega(t) = P_x x(t-\tau) + D_x \dot{x}(t-\tau) + P_\varphi \varphi(t-\tau) + D_\varphi \dot{\varphi}(t-\tau)$$

Neutral functional diff. eq.

strong stability: $|D_\varphi| > 1$



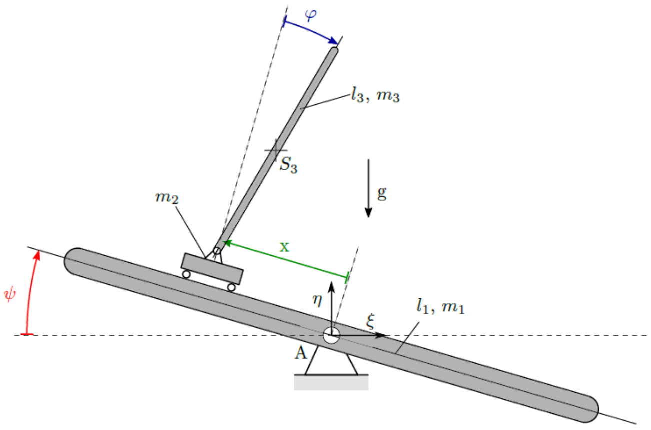
$$D(\lambda) = (1 + D_\varphi e^{-\lambda\tau})\lambda^5 + P_\varphi \lambda^4 e^{-\lambda\tau} - b\lambda^3 + \left[(a+g)\lambda^2 - bg\right](P_x + \lambda D_x)e^{-\lambda\tau}$$

Pendulum-cart and beam ~ angular acceleration (2 DoF)

$$x^{(\mathrm{iv})}(t) = a\ddot{\varphi}(t) - g\varepsilon(t)$$

$$\ddot{\varphi}(t) = b\varphi(t) - \varepsilon(t)$$

Not controllable!

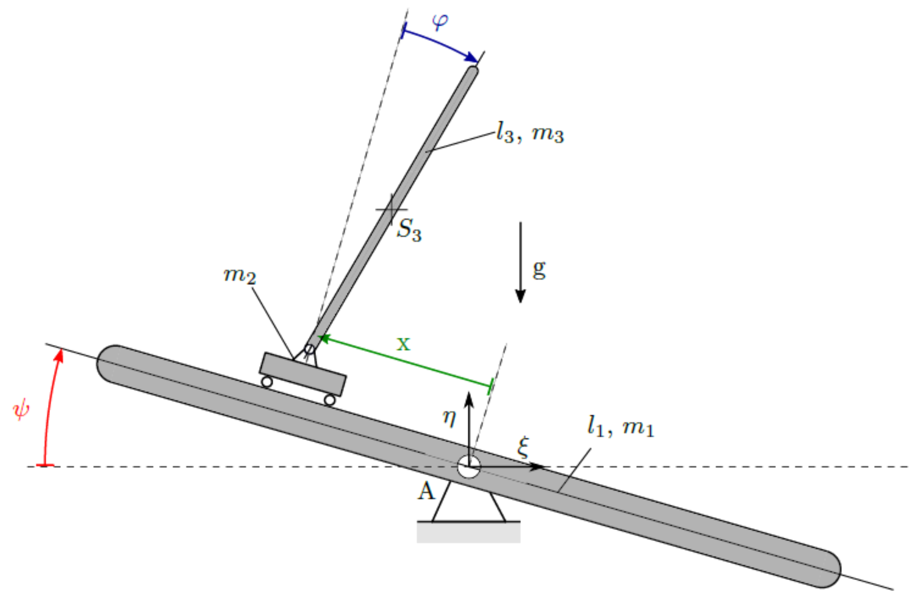


Pendulum-cart and beam ~ torque (3 DoF)

$$\mathbf{M}\ddot{\mathbf{q}}(t) + \mathbf{S}\mathbf{q}(t) = \mathbf{Q}^*(t)$$

$$\mathbf{Q}^*(t) = \left(0, -Q(t), 0\right)^T$$

$$\mathbf{q} = \left(x(t), \psi(t), \varphi(t)\right)^T$$



$$\mathbf{S} = \begin{pmatrix} 0 & g(m_2+m_3) & 0 \\ g(m_2+m_3) & -\frac{1}{2}gl_3m_3 & -\frac{1}{2}gl_3m_3 \\ 0 & -\frac{1}{2}gl_3m_3 & -\frac{1}{2}gl_3m_3 \end{pmatrix}$$

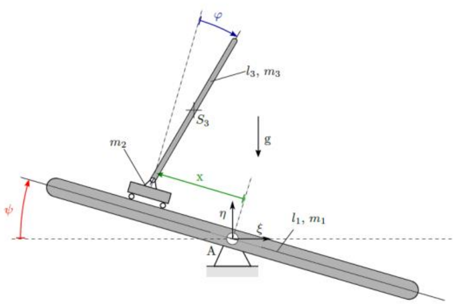
$$\mathbf{M} = \begin{pmatrix} m_2+m_3 & -\frac{l_3m_3}{2} & -\frac{l_3m_3}{2} \\ -\frac{l_3m_3}{2} & \frac{1}{12}(l_1^2m_1+4l_3^2m_3) & \frac{l_3^2m_3}{3} \\ -\frac{l_3m_3}{2} & \frac{l_3^2m_3}{3} & \frac{l_3^2m_3}{3} \end{pmatrix}$$

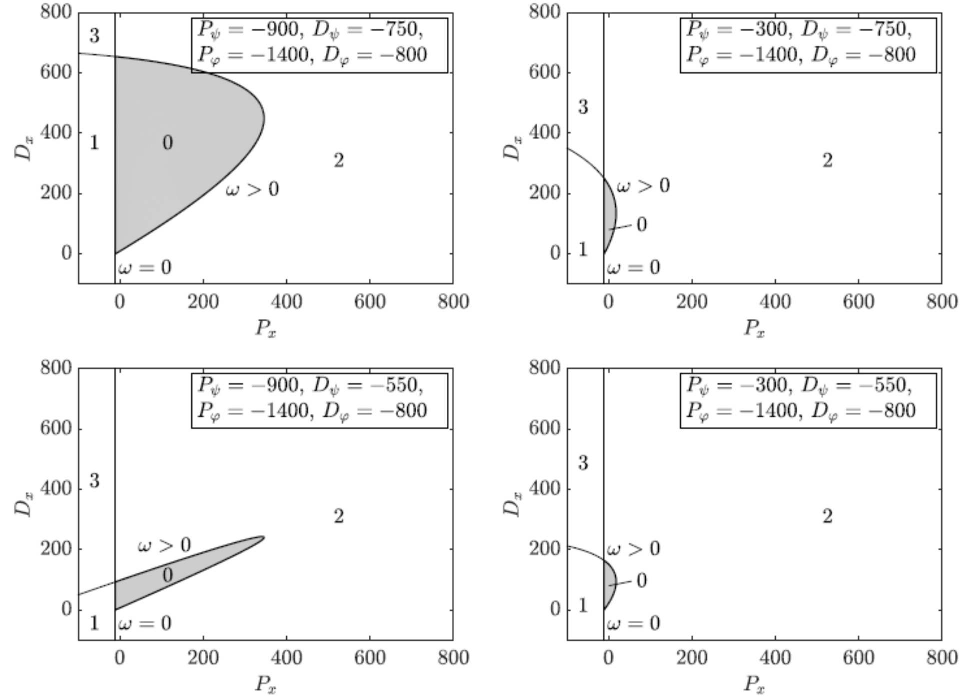
Pendulum-cart and beam ~ torque (3 DoF)

$$\mathbf{M}\ddot{\mathbf{q}}(t) + \mathbf{S}\mathbf{q}(t) = \mathbf{Q}^*(t)$$

$$\mathbf{Q}^*(t) = (0, -Q(t), 0)^T$$

$$\mathbf{q} = (x(t), \psi(t), \varphi(t))^T$$





$$Q(t) = P_x x(t) + D_x \dot{x}(t) + P_\psi \psi(t) + D_\psi \dot{\psi}(t) + P_\varphi \varphi(t) + D_\varphi \dot{\varphi}(t)$$

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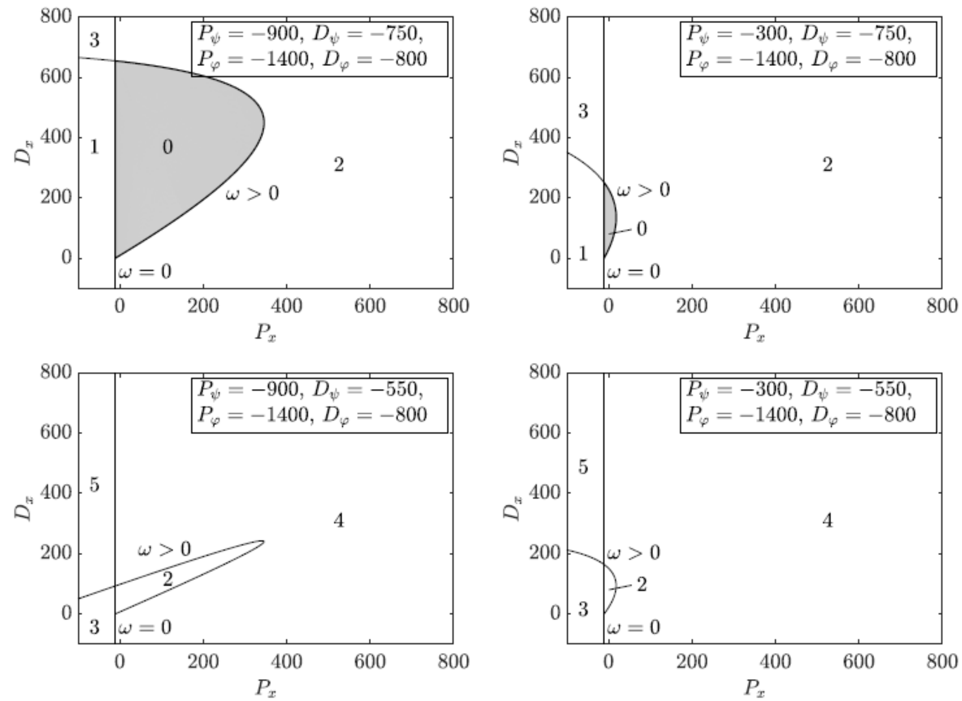
Pendulum-cart and beam ~ torque (3 DoF)

$$\mathbf{M}\ddot{\mathbf{q}}(t) + \mathbf{S}\mathbf{q}(t) = \mathbf{Q}^*(t)$$

$$\mathbf{Q}^*(t) = (0, -Q(t), 0)^T$$

$$\mathbf{q} = (x(t), \psi(t), \varphi(t))^T$$

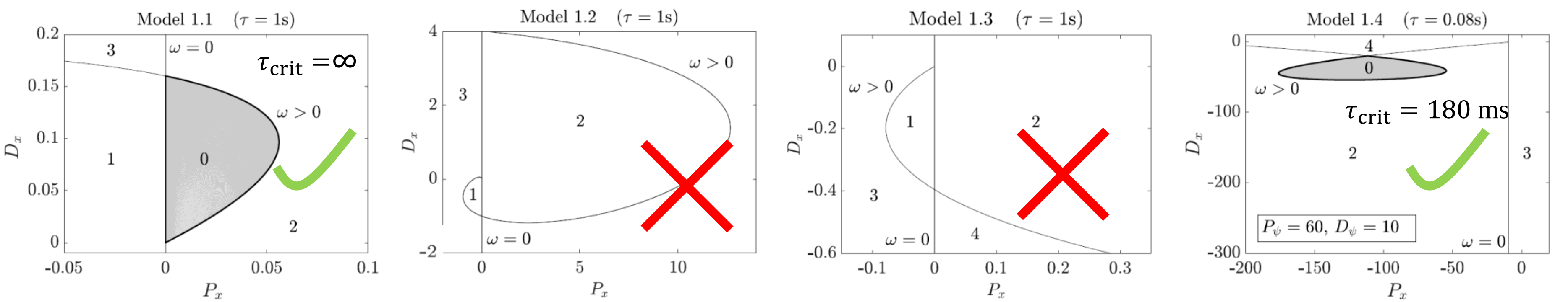


with reaction delay

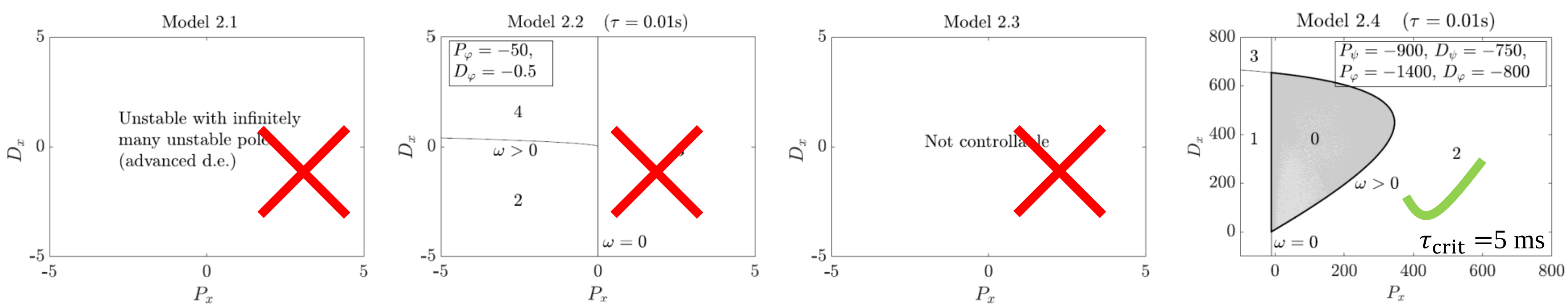
$$Q(t) = P_x x(t-\tau) + D_x \dot{x}(t-\tau) + P_\psi \psi(t-\tau) + D_\psi \dot{\psi}(t-\tau) + P_\varphi \varphi(t-\tau) + D_\varphi \dot{\varphi}(t-\tau)$$

τ_{crit} =5 ms

Ball and beam

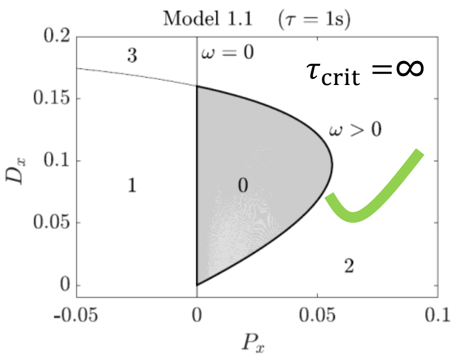


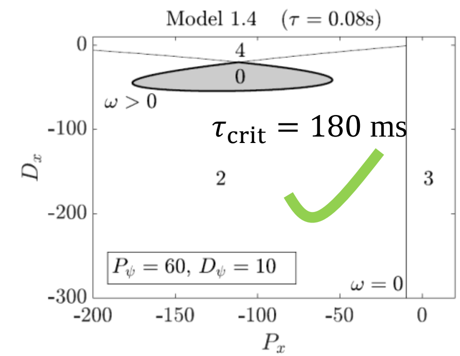
Pendulum-cart and beam



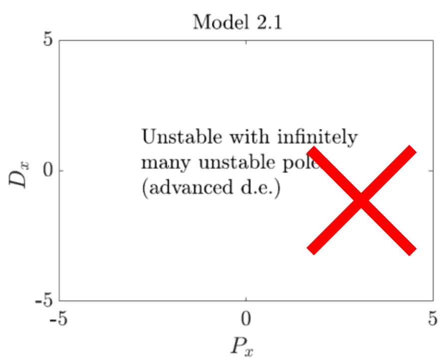
Comparison

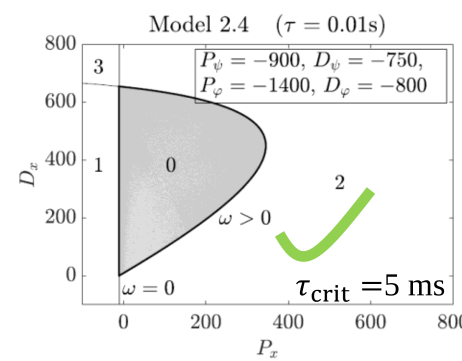
Ball and beam





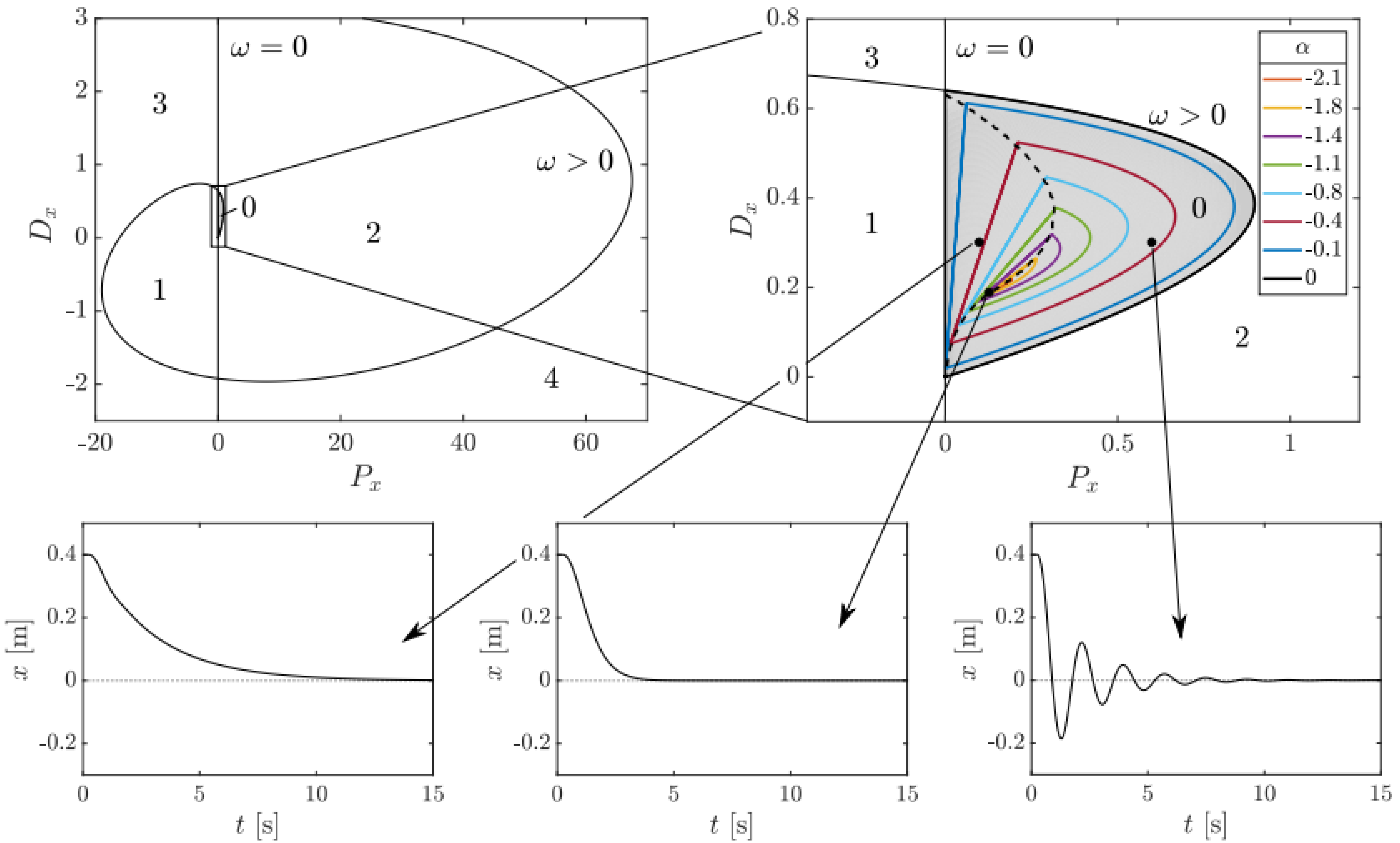
Pendulum-cart and beam





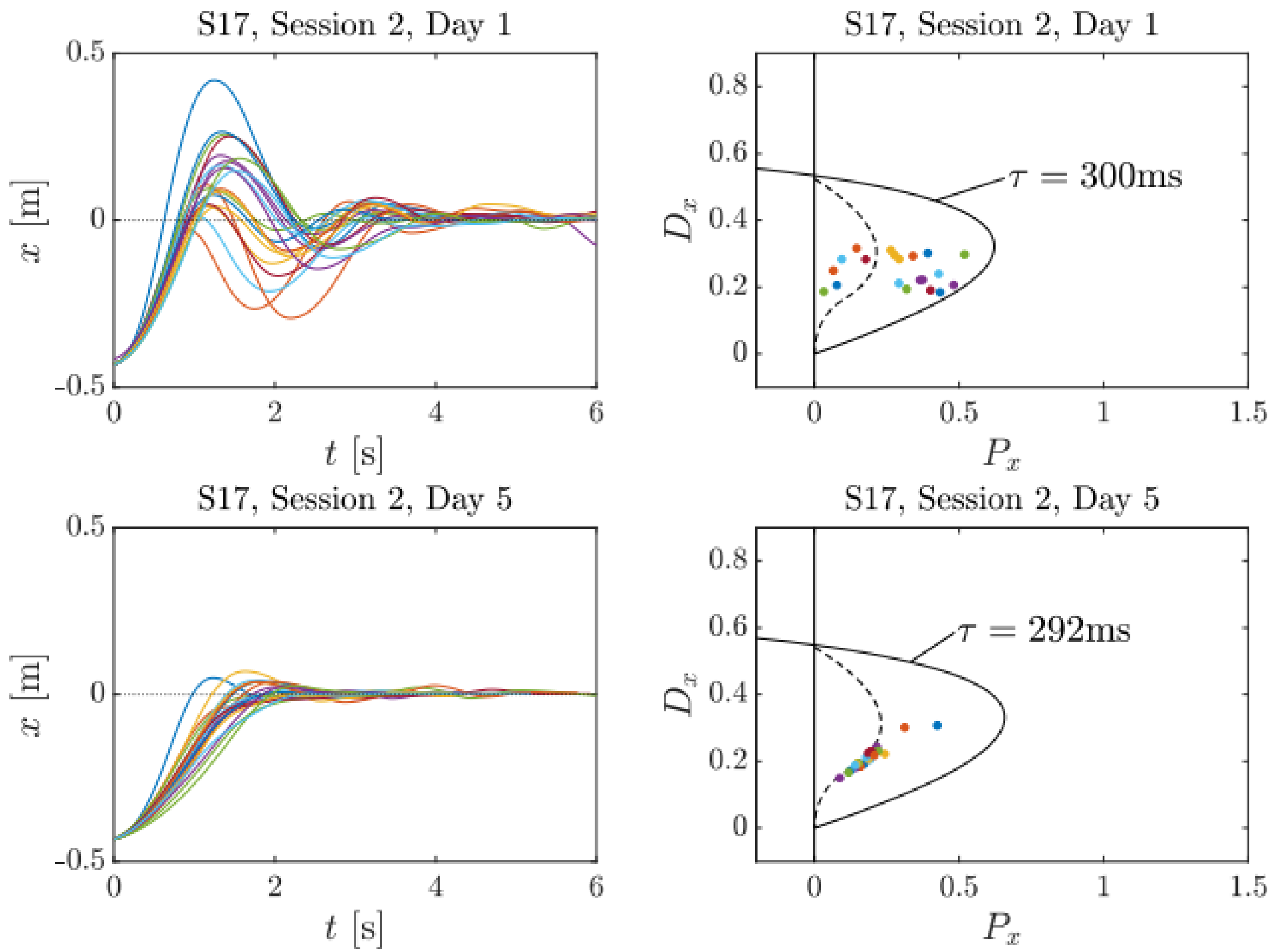
Model 1.1

$$\ddot{x}(t) + gD_x\dot{x}(t-\tau) + gP_x x(t-\tau) = 0$$

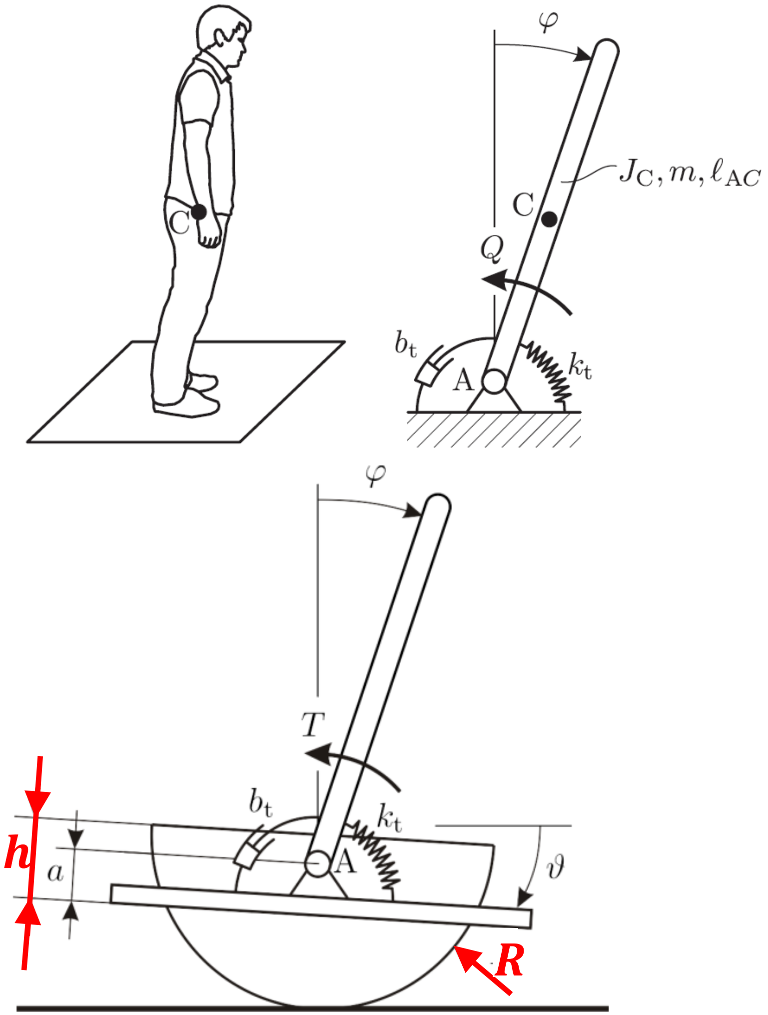


Experiments

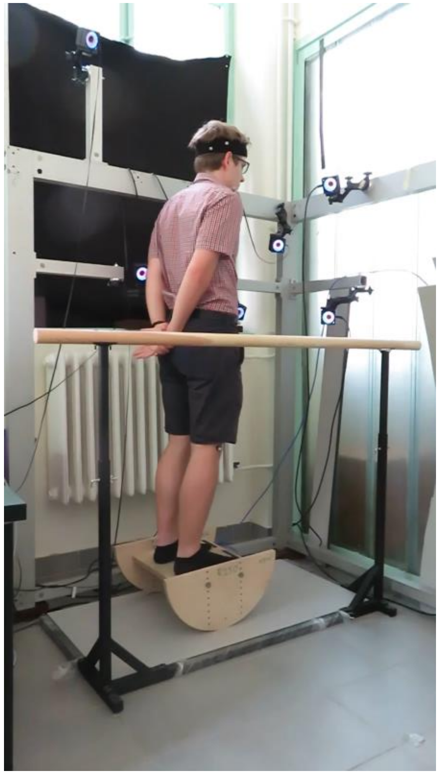
- Tests by 22 subjects
- 5-day test series
- 20 trials per day
- Settling time decreases
- Overshoot decreases



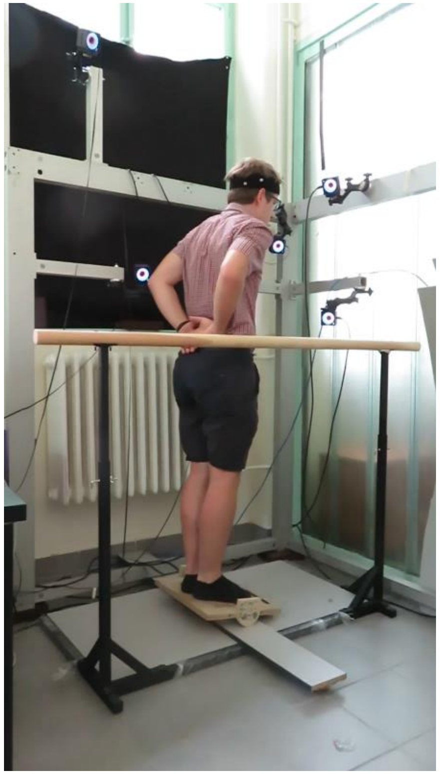
Balance Board



„easy"



„difficult"



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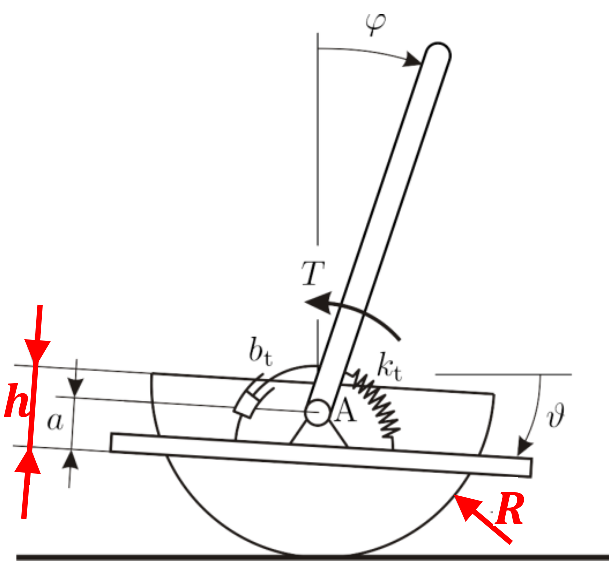
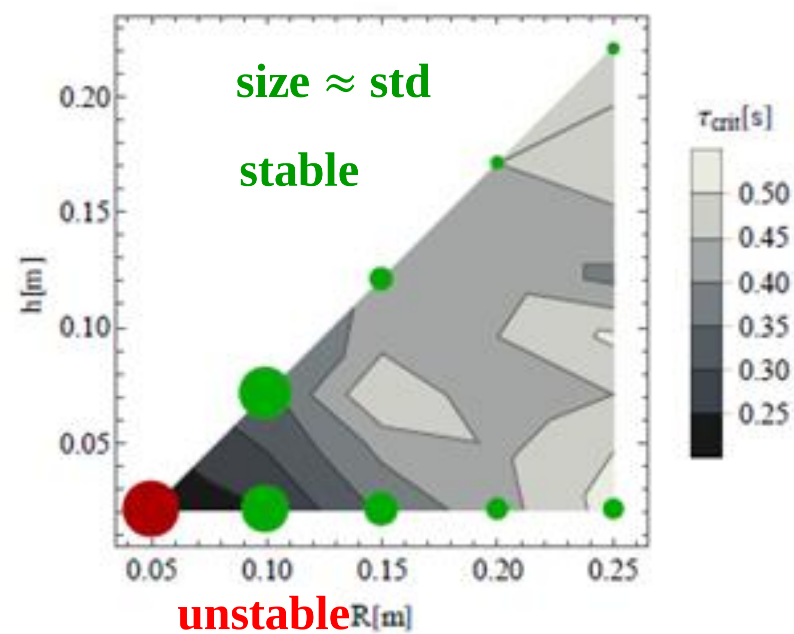
$$\left(\frac{1}{4}l^2 m_h + J_h\right)\ddot{\varphi} + b_t\dot{\varphi} + \left(k_t - \frac{1}{2}glm_h\right)\varphi + \left(\frac{1}{2}lm_hR + \frac{1}{2}alm_h - \frac{1}{2}hlm_h\right)\ddot{\vartheta} - b_t\dot{\vartheta} - k_t\vartheta = -T$$

$$\left(\frac{1}{2}lm_hR + \frac{1}{2}alm_h - \frac{1}{2}hlm_h\right)\ddot{\varphi} - b_t\dot{\varphi} - k_t\varphi + (m_bR^2 + J_b + m_hR^2 + a^2m_h - 2ahm_h + h^2m_h$$
$$+ {l_b}^2 m_b + 2am_hR - 2hm_hR)\ddot{\vartheta} + b_t\dot{\vartheta} + (-agm_h + ghm_h - gl_bm_b + gm_bR + k_t)\vartheta = T$$

$$T = P_\varphi\varphi(t-\tau) + D_\varphi\dot{\varphi}(t-\tau) + P_\vartheta\vartheta(t-\tau) + D_\vartheta\dot{\vartheta}(t-\tau)$$





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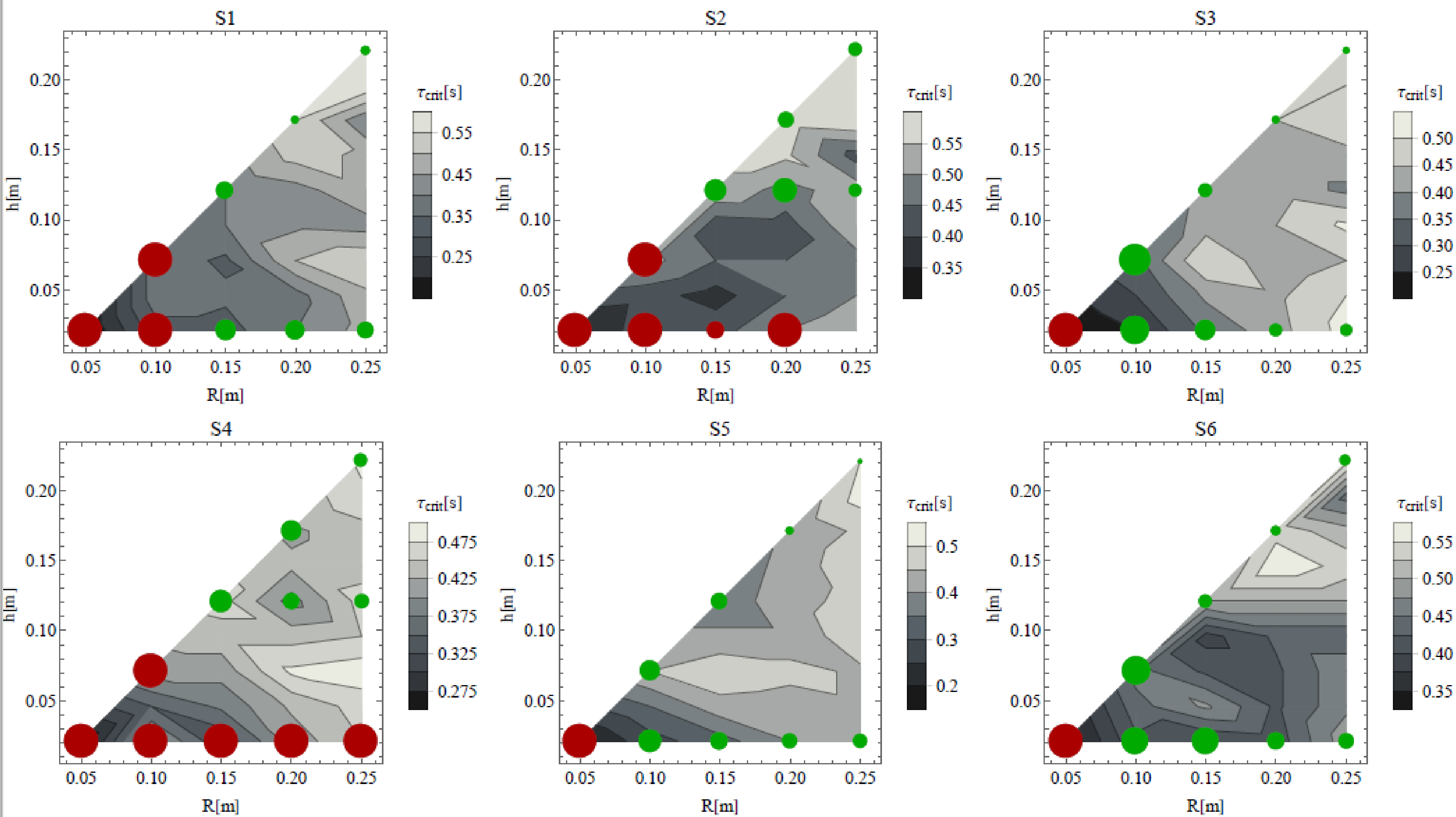
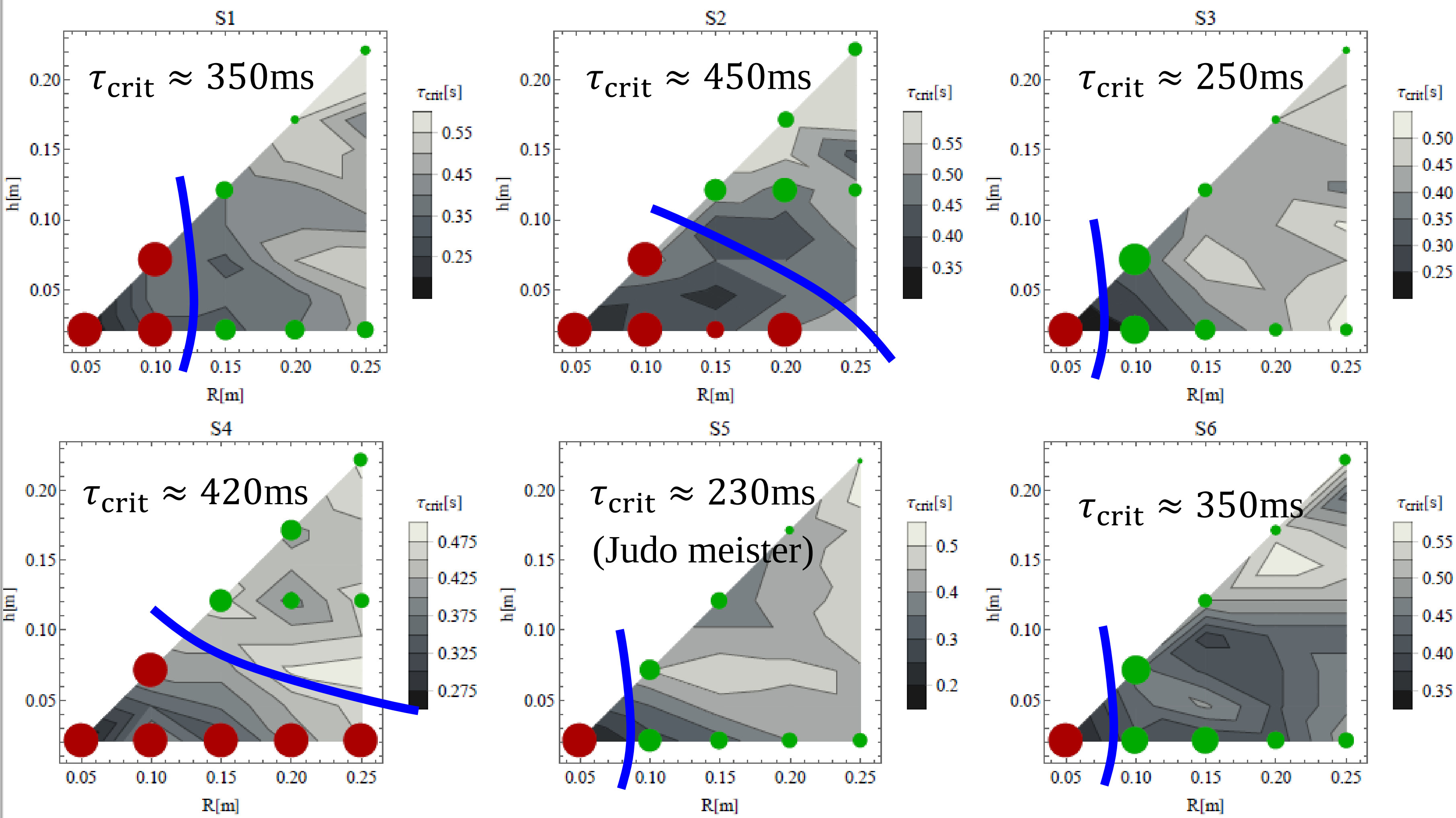


Balance Board



Thank you!