What makes a good student project?

Mincsovics Miklós E.

Farkas Miklós seminar
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Outline

- What is the meaning of "good" in the title? Possible answers.
- Examples. (Running trough quickly.)
- 2 examples, going into more deeply.
- Sources.

What is the meaning of "good" in the title? Possible answers.

- What is your intention with the project? What can you expect from the students?
- Problem solving, learning a subtopic not part of the course? Avoiding tests, exams?
- Complexity of the problem, degree of freedom.
- For engineers: applications $\rightarrow$ theory.

Shuttlecock

- Mark Peastrel, Rosemary Lynch, and Angelo Armenti Jr.: Terminal velocity of a shuttlecock in vertical fall, American Journal of Physics 48, 511 (1980); doi: 10.1119/1.12373
- Kurt Bryan: Falling Shuttlecocks and Parameter Estimation (talk);

Model

$$\dot{v}(t) = g - F\left(v(t)\right)$$

Options for air resistance: $F(v) = 0$, $F(v) = kv$, $F(v) = kv^2$, $F(v) = k_1 v + k_2 v^2$, $F(v) = kv^r$

Solution

numerical ODE solving, modelfit, $\rightsquigarrow$ model selection

High altitude free fall

- Mohazzabi P. and Shea J.H.: High-altitude free fall, Am. J. Phys. 64, 1242–1246 (1996), https://doi.org/10.1119/1.18386;
- Baumgartner's jump
    - Statistics
    - Video

Model

$$\dot{v}(t) = -g + k(x)v^2(t)$$

Solution

numerical ODE solving, modelfit goes a bit off

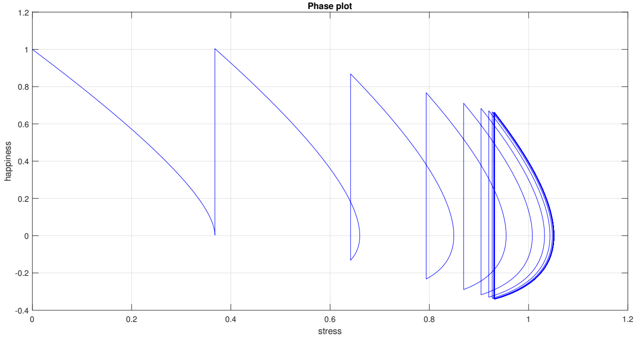
Addiction

- Héctor Mera Couto: Harmonic oscillators and addictive behaviours (talk);
- J. C. Sprott: Dynamical Models of Happiness, Nonlinear Dynamics, Psychology, and Life Sciences, Vol. 9, No. 1, January, 2005. (paper);

Model

$$\ddot{x}(t) + 2\mu\omega\dot{x}(t) + \omega^2 x(t) = F(t)$$

- $\dot{x}(t)$ - happiness level
- $\mu = 1$ - with critical damping
- $F(t) = c\sum_{n=0}^{N}\delta(t - n\hat{t})$ - periodic drug intake


Phase plot
happiness
stress
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
0
0.2
0.4
0.6
0.8
1
1.2

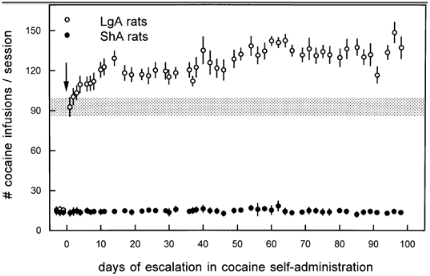
Reality

Psychopharmacology (1999) 146:303–312 © Springer-Verlag 1999

ORIGINAL INVESTIGATION

Serge H. Ahmed · George F. Koob

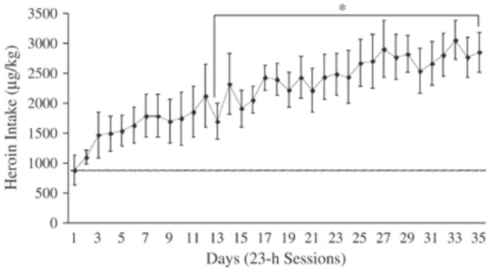
Long-lasting increase in the set point for cocaine self-administration after escalation in rats



Unlimited Access to Heroin Self-Administration: Independent Motivational Markers of Opiate Dependence

Scott A Chen, Laura E O'Dell, Michael E Hoefer, Thomas N Greenwell, Eric P Zorrilla & George F Koob

Neuropsychopharmacology **31**, 2692–2707 (2006) | Cite this article



How to modify the model?

...

Results: periodic intake

Theorem

Independently of the starting point we have

$$\lim_{N\to\infty}\begin{pmatrix}x_N\\ v_N\end{pmatrix}=\frac{ce^{-\omega\hat{t}}}{1-e^{-\omega\hat{t}}}\begin{pmatrix}\dfrac{\hat{t}}{1-e^{-\omega\hat{t}}}\\ 1-\dfrac{\omega\hat{t}}{1-e^{-\omega\hat{t}}}\end{pmatrix}.$$

Results: happiness level dosing

Lemma

("Getting off the train")
We assume that the tolerance level is θ.

$$A = \frac{\theta}{c} + 1\,, \quad B(A) = \begin{cases} \dfrac{A}{W_0\left(-\frac{1}{e}\frac{A}{A-1}\right)} & , \textit{if} \quad 1 > A \neq 0 \\ e & , \textit{if} \quad A = 0 \end{cases} .$$

If

(A) $A \geq 1$, *then there will be always a next dose.*

(B) $A < 1$ *and iff*

$$X_n = \frac{\omega}{c} x_n + A < B(A)$$

then there will be no more dose, the trajectory will tend to $(0,0)'$ *according to the homogeneous solution.*

Theorem

We introduce the constant

$$K = \frac{4 - 2W_0\left(-\frac{2}{e^2}\right)\left(\log\left(-\frac{1}{2}W_0\left(-\frac{2}{e^2}\right)\right) - 1\right)}{\left(W_0\left(-\frac{2}{e^2}\right) + 2\right)^2} \approx 0.7449\,,$$

and if

(a) *"Being safe"*

$$A \leq \frac{1}{2}\,,$$

then there is no periodic orbit. The dosing rate will decrease to 0 and the trajectory will tend to $(0,0)'$ *independently of* x_0.

(b) "Getting hooked or not"

$$\frac{1}{2} < A \leq K \,,$$

then there is a unique $\bar{x}$ that, if $x_0 = \bar{x}$, then the trajectory is an unstable periodic orbit. If $x_0 < \bar{x}$ then the dosing rate will decrease to 0 and the trajectory will tend to $(0,0)'$. If $x_0 > \bar{x}$, then the dosing rate will increase.

(c) "Thin ice"

$$K < A < 1 \,,$$

then there is no periodic orbit. If x_0 is not big enough, then there will be no second dose, and the trajectory will tend to $(0,0)'$ according to the homogeneous solution. If x_0 is big enough, then the dosing rate will increase.

(d) "Without any willpower it is hopeless"

$$A \geq 1\,,$$

then independently of x_0 the dosing rate will increase.

Remark

At $A = \frac{1}{2}$ an unstable periodic orbit is emerging and at $A = K$ it disappears. Strictly speaking neither of it is a subcritical Hopf bifurcation, since it is not emerging/disappearing around a stationary point.

There is a problem ...

If the dosing rate will increase, then it tends to ∞.

Results: dosing at 0 happiness level

Theorem

Independently of the starting point and the dosing amount c, the dosing rate $r_n := \frac{1}{t_n}$ tends to infinity, moreover, the slope of the dosing rate (against time) curve tends to $\frac{\omega^2}{2}$. Similarly, the stress grows rate tends to $\frac{c}{2}$.

$\rightsquigarrow$ modification of the model ...

Results: dosing with square signals

Periodic case: similar to the Dirac-delta case

Theorem (Dosing at 0 happiness level)

We assume that the dosing "intensity" is $\frac{c}{\varepsilon}$ and the length of it is ε. Independently of the starting point we have

$$\lim_{N\to\infty} \begin{pmatrix} x_N \\ v_N \end{pmatrix} = \begin{pmatrix} \frac{c}{\varepsilon\omega^2} \\ 0 \end{pmatrix} .$$

Interpretation

Chain-smoker limit.

But!

Dosing with Dirac-deltas including periodic inactive periods!

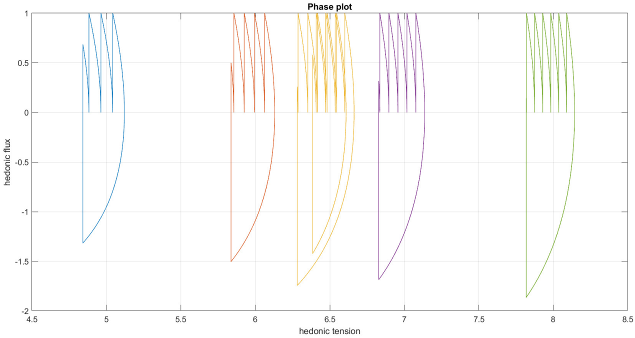


Figure: $\omega = 1$, $c = 1$, $T_1 = 0.6$, $T_2 = 0.4$. Stable periodic orbits. Note that the one in the middle has two loops.

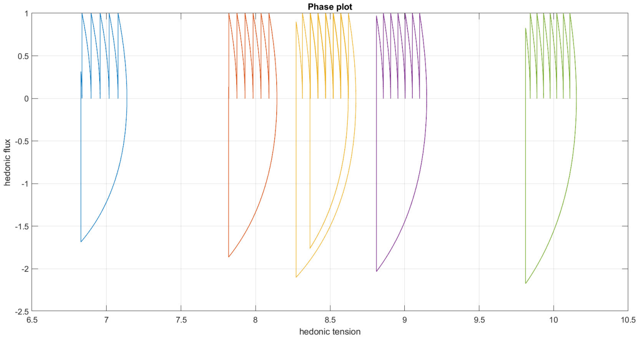


Figure: $\omega = 1,\ c = 1,\ T_1 = 0.65,\ T_2 = 0.35.$ Stable periodic orbits.

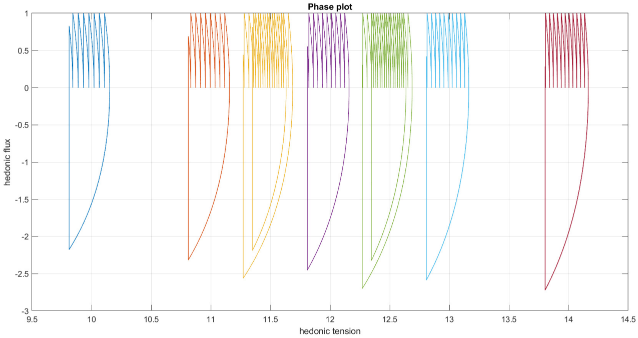


Figure: $\omega = 1,\ c = 1,\ T_1 = 0.7,\ T_2 = 0.3.$ Stable periodic orbits. Note that there are two periodic orbits with two loops.

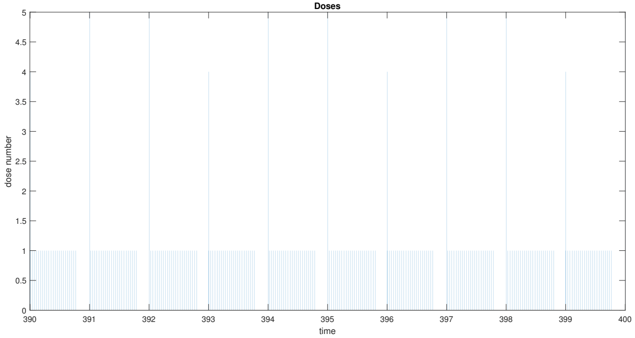


Figure: $\omega = 1,\ c = 1,\ T_1 = 0.8,\ T_2 = 0.2$. A stable periodic orbit with three loops. We have three different consecutive days which repeat.

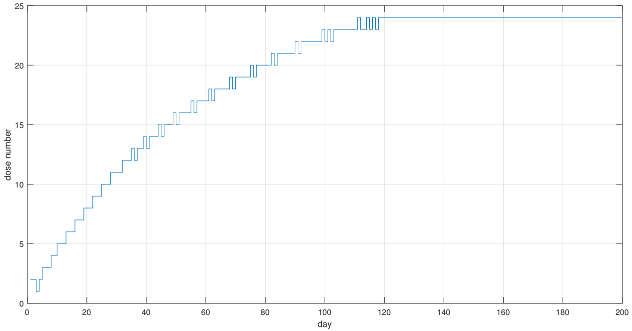


Figure: $\omega = 1$, $c = 1$, $T_1 = 0.8$, $T_2 = 0.2$. Dose numbers per day. The process started at 0 stress level.

Glider

- Héctor Mera Couto: Illustrating the dynamics of gliders using differential equations and flight simulators (talk);

Model (Zhukovski)

$$\begin{cases} \dot{v}(t) = -g \sin\theta - \mu_D v^2(t) \\ v(t)\dot{\theta}(t) = -g\cos\theta - \mu_L v^2(t) \end{cases}$$

- μ_D - drag coefficient
- μ_L - lift coefficient

Sources

- SIMIODE (low level mainly)
- SIAM Review (high level)
- Danaila I., Joly P., Kaber S.M., Postel M.: An Introduction to Scientific Computing, Fifteen Computational Projects Solved with MATLAB (book)
- Matlab materials
- Youtube (MIT, 3Blue1Brown, Michael Penn, Steve Mould ...)