Mathematical Models For Malaria Disease

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Malaria is an epedemic and vector borne disease that has been plaguing mankind since before recorded history. The disease is carried by female *Anopheles* mosquitoes.

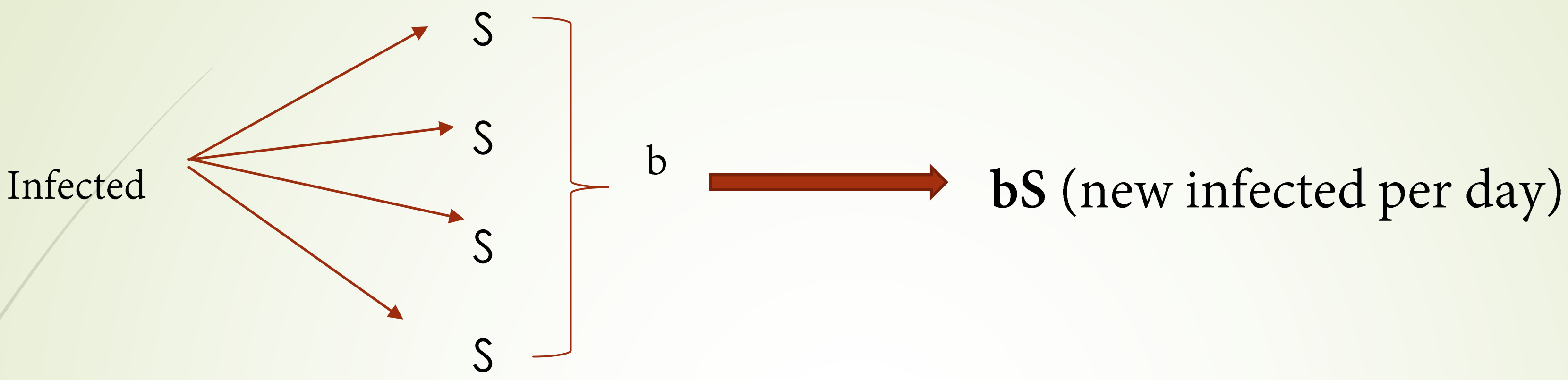
Modelling

SIR model

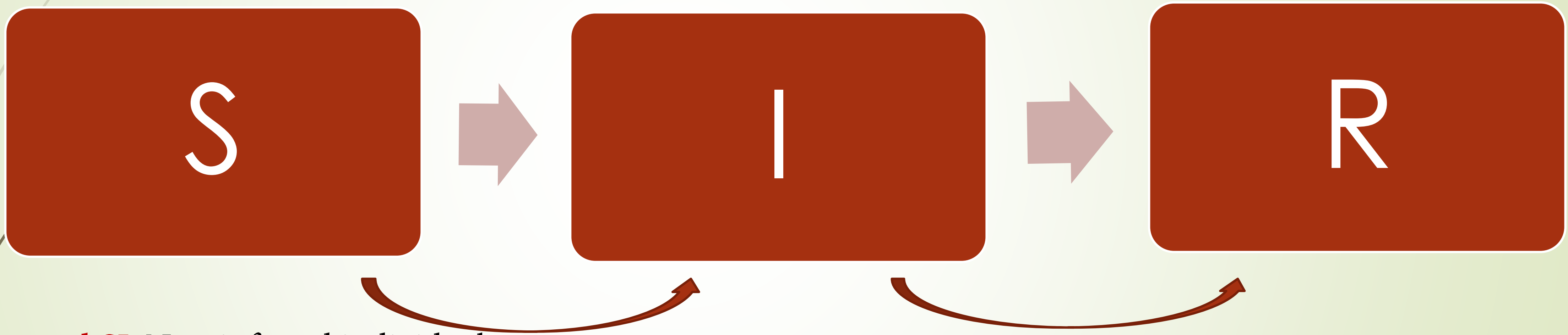
One of the bisc models for epidemic deases is SIR model.For convenience this model is considered in a closed host population of total size N .

The total population is divided into three classes:

- S: the class of suceptibles
- I:the class of infectives
- R:the class of removed individuals



b: the number of contacts that each infected individual has with sesuceptibles per day



-bSI :New infected individuals per day

+kI :Infected group will recover during any given day

K:the rate of recovery

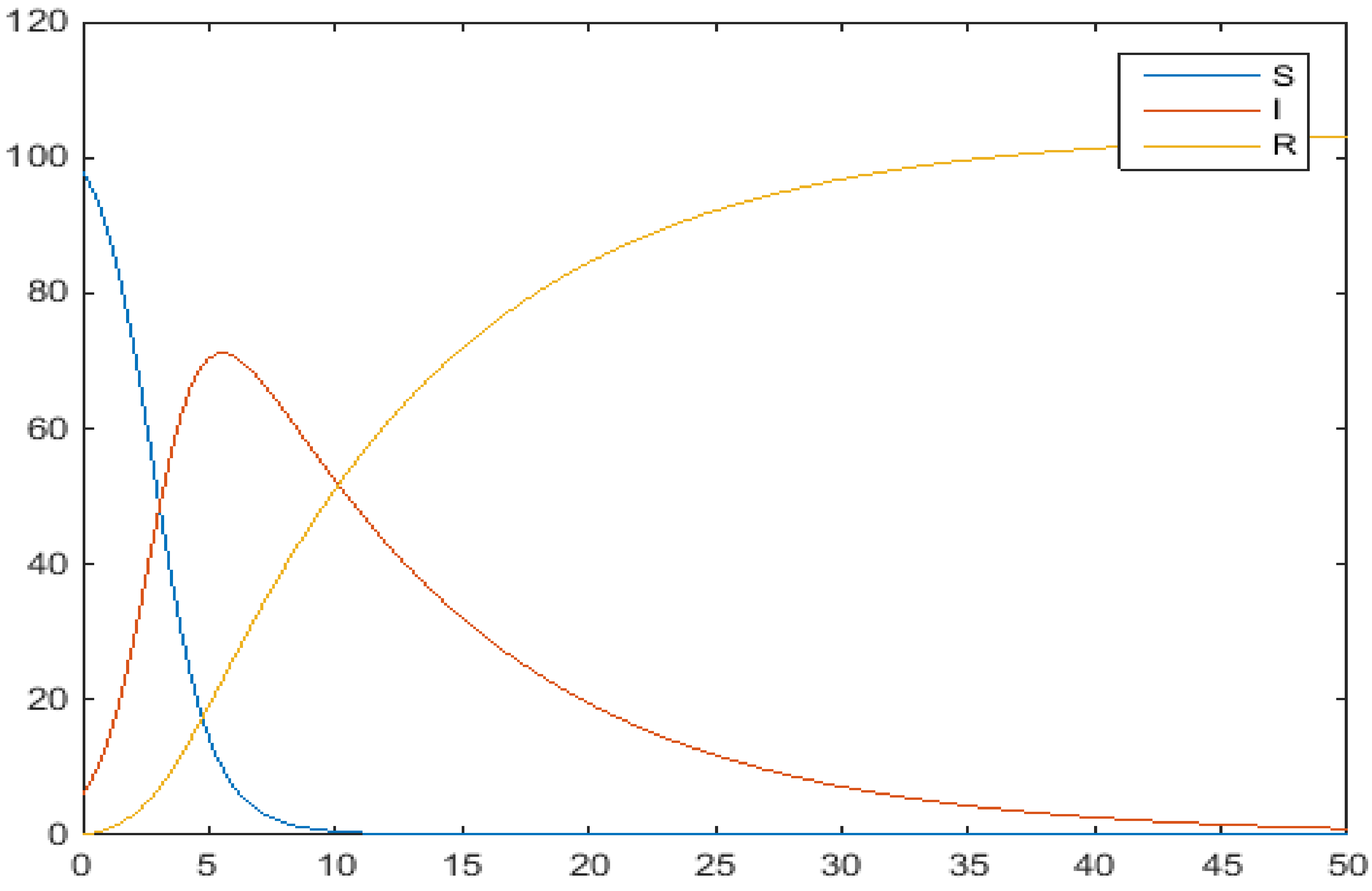
Mathematical models

SIR MODEL

$$\frac{dS(t)}{dt} = -bS(t)I(t)$$

$$\frac{dI(t)}{dt} = bS(t)I(t) - kI(t)$$

$$\frac{dR}{dt} = kI(t)$$



b=0.1,k=.01

Interpretation of SIR model

according to SIR model in epidemic:

$$\frac{dS}{dt} \leq 0;$$
$$\frac{dI}{dt} \geq 0;$$
$$\frac{dR}{dt} \geq 0$$

Consequently:

If S(0)=$\boldsymbol{S_0 \geq \frac{k}{b} = \rho}$

$$\boldsymbol{I = -S + \frac{k}{b}\, lnS + c}$$

If S=$\frac{k}{b}$=$\boldsymbol{\rho}$

$$\boldsymbol{I_{max} = -\frac{k}{b} + \frac{k}{b} ln\frac{k}{b} + S_0 + I_0 - \frac{k}{b} lnS_0}$$

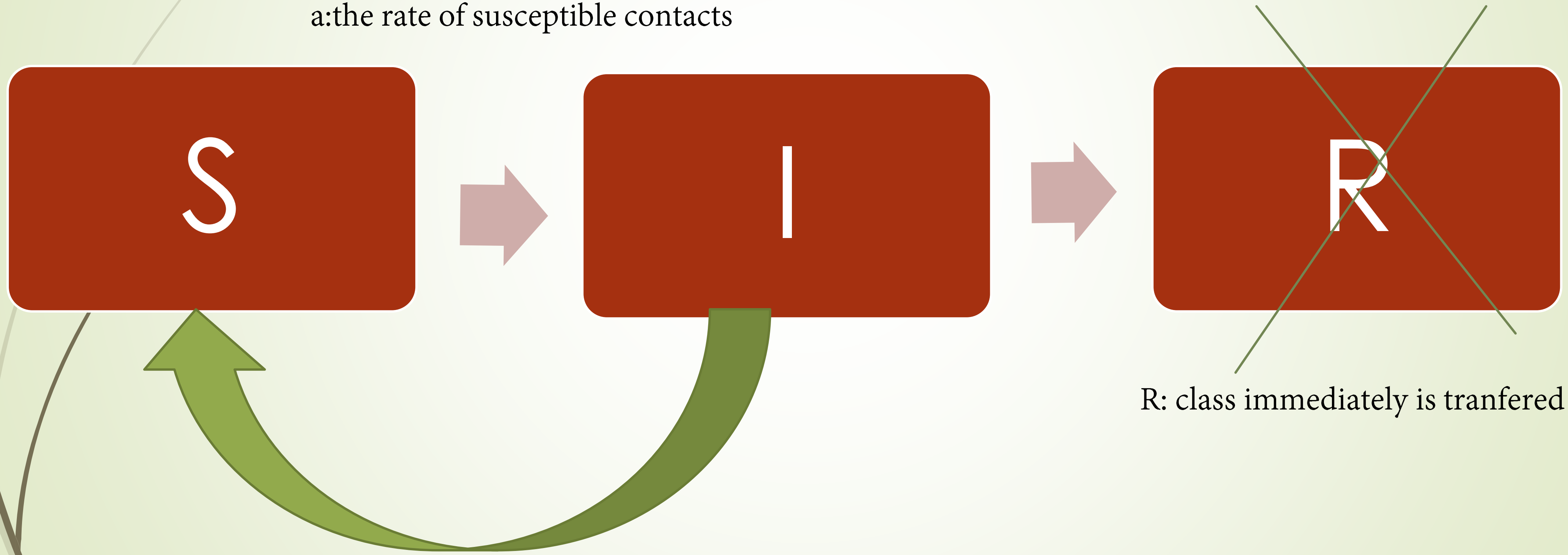
if $S_0 < \frac{k}{b} = \boldsymbol{\rho}$

$$\frac{dI}{dt} \leq 0 \;\; \forall\, t \geq 0, \lim_{t\to\infty} I(t) = 0$$

In this case no epidemic can occur.

SIS MODEL

a:the rate of susceptible contacts



R: class immediately is tranfered

γ:The rate of transfer of infectives to S class

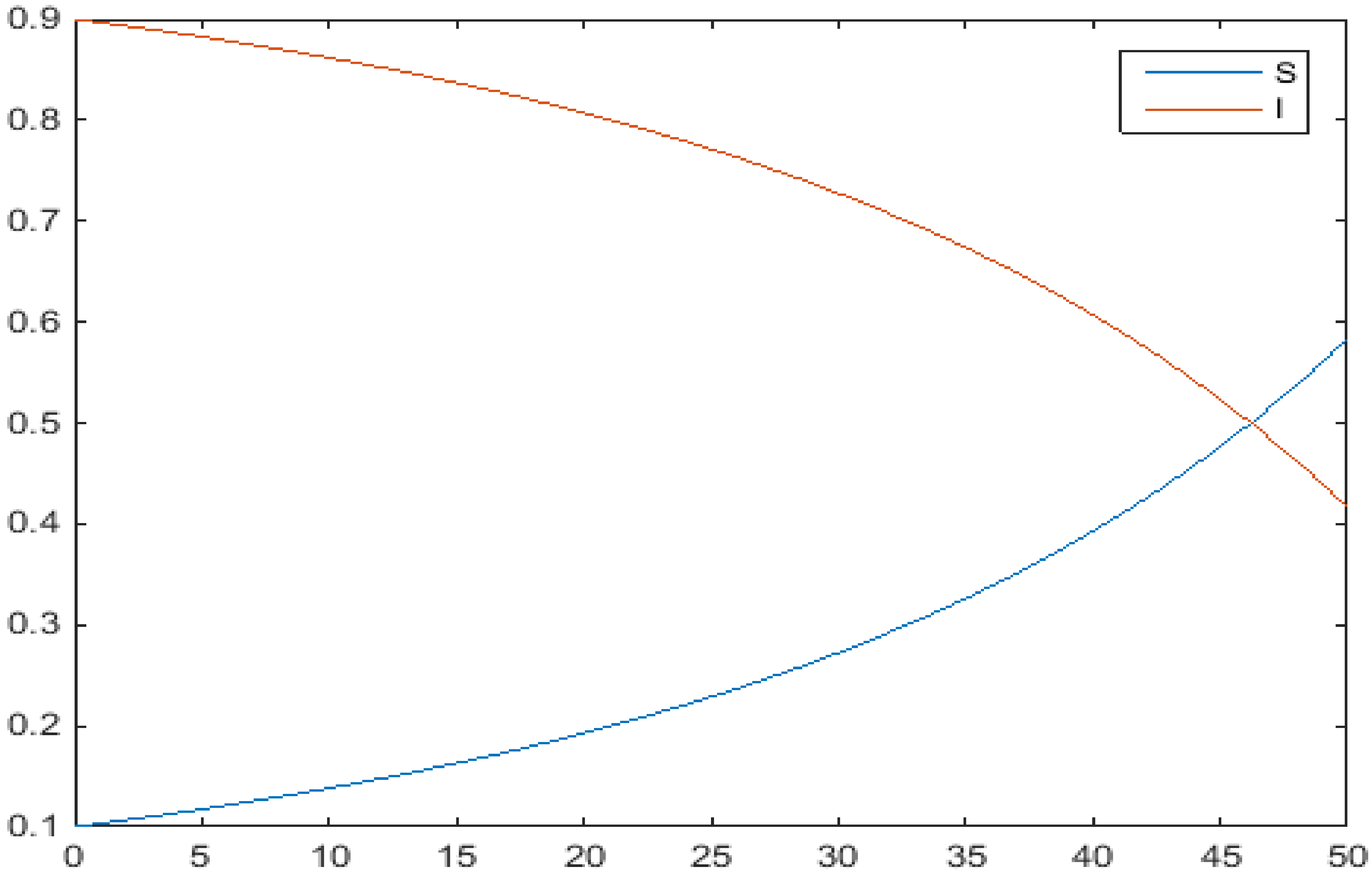
Mathematical Model

SIS MODEL

$$\begin{cases} \frac{dS}{dt} = \gamma I - aSI \\ \frac{dI}{dt} = aSI - \gamma I \end{cases}$$

Since S+I=1 then,

$$\begin{cases} \frac{dS}{dt} = \gamma I - a(1 - I)I \\ \frac{dI}{dt} = a(1 - I)I - \gamma I \end{cases}$$



$\gamma = .5\ , a = .2$

Interpretation of SIS MODEL

Based on logistic equation :

$\frac{dI}{dt} = rI(1 - \frac{I}{K})$, r=$\boldsymbol{\alpha - \gamma}$, $\boldsymbol{K = \frac{r}{\alpha}}$

r<0:

$$\mathrm{I(t)} = \boldsymbol{I(0)e^{rt}}, \ \boldsymbol{\lim_{t\to\infty} I(t) = 0}$$

r>0:

$$\boldsymbol{I = \frac{KBe^{rt}}{1 + Be^{rt}}}$$

Where B=$\frac{I(0)}{K-I(0)}$,

$$\boldsymbol{\lim_{t\to\infty} I(t) = k}$$

MALARIA MODELS

The models presented here are based on SIS model and the life-cycle of malaria parasites.

ROSS model

c=the rate of bites which one susceptible mosqito becomes infected by human

a

b=the rate of bites that produce infection in human

b

a=the average of bites per night

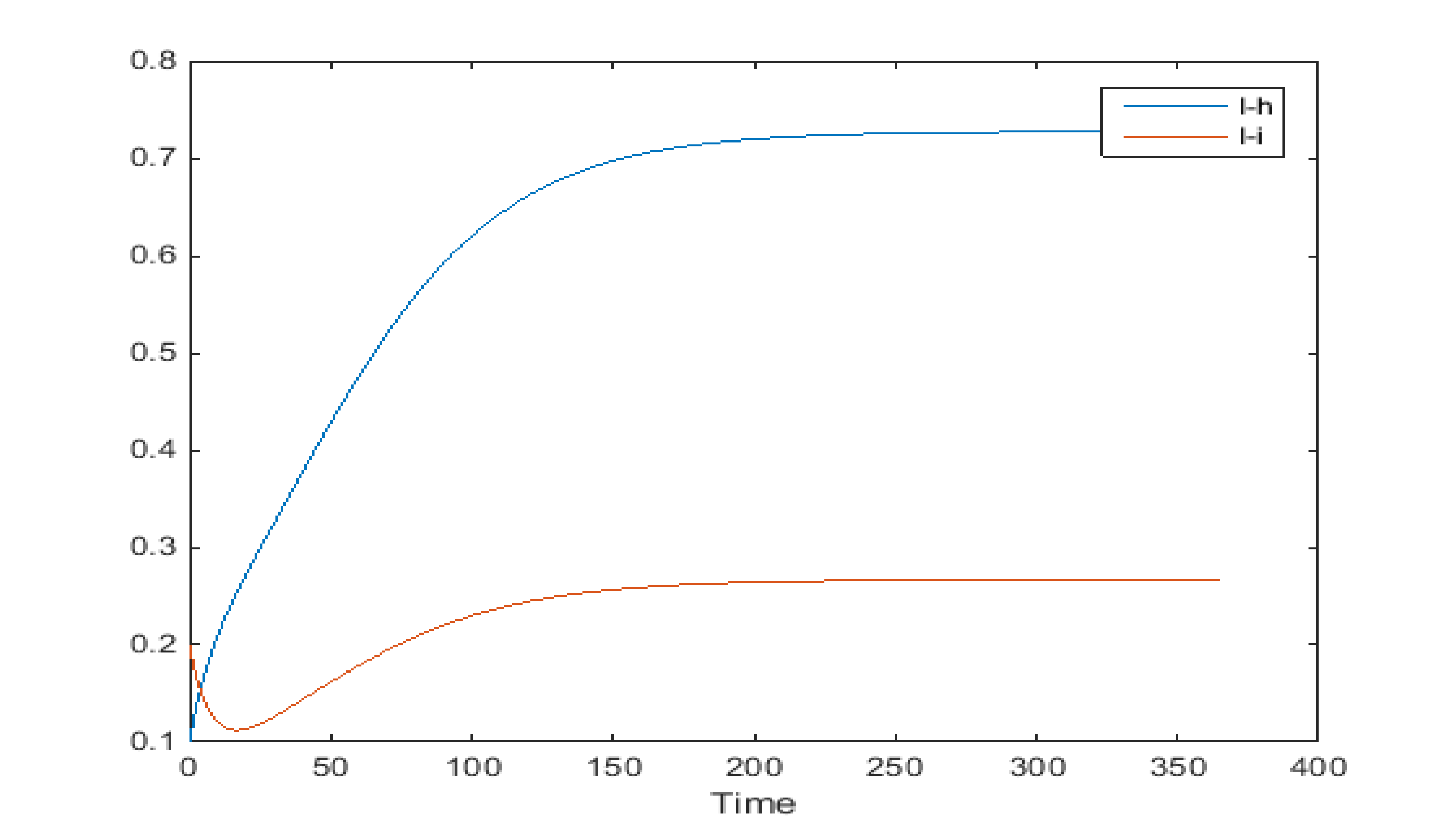
Female mosquito

m=the density of female mosquitos

Ross mathematical Model

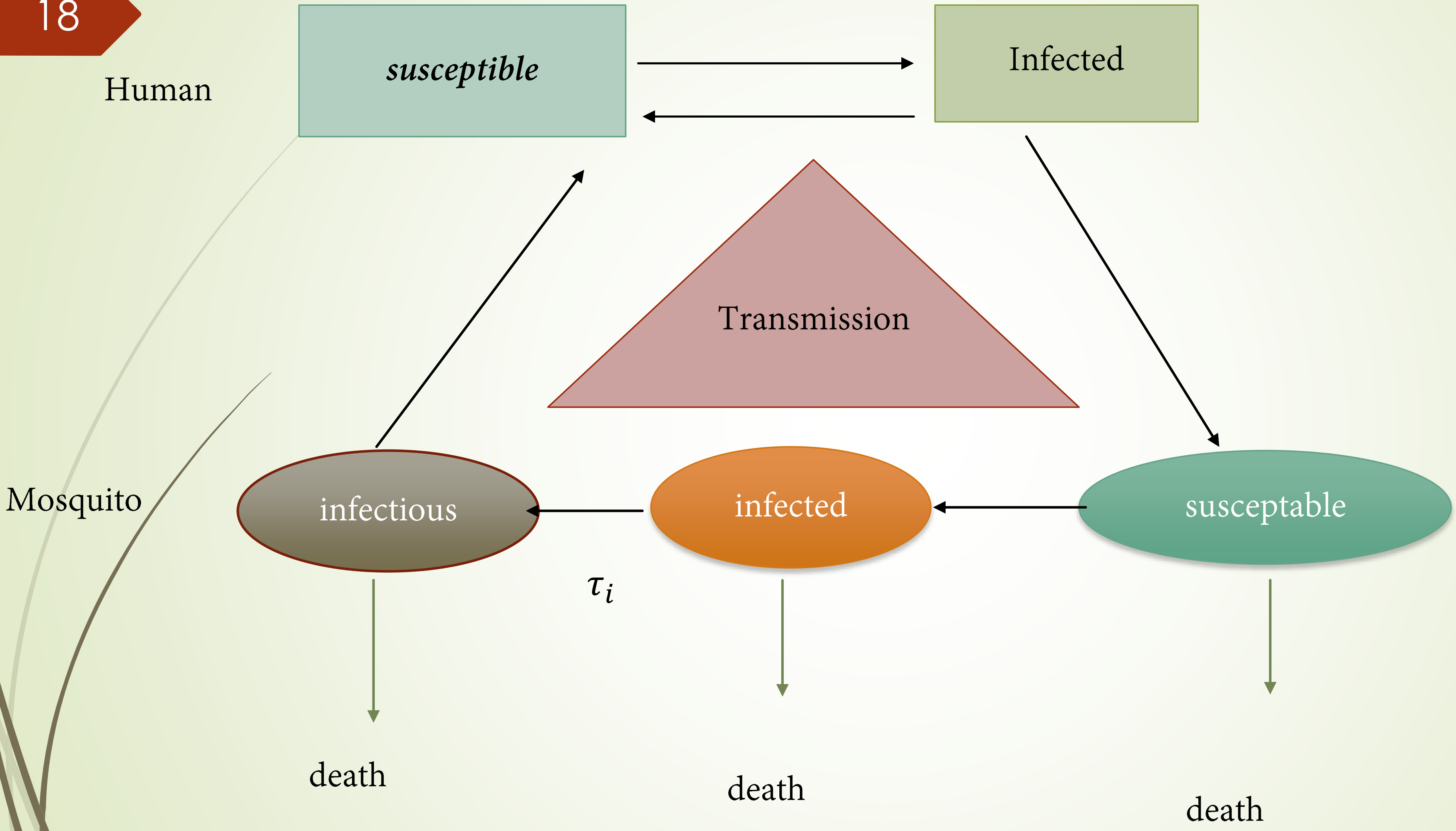
$$\frac{dI_h}{dt} = abmI_i(1 - I_h) - rI_h$$

$$\frac{dI_i}{dt} = acI_h(1 - I_i) - \mu_2 I_i$$



a=0.1,b=.2,c=.5.m=5,r=.01,$\mu_2 = 0.1$

Ross-Macdonald model

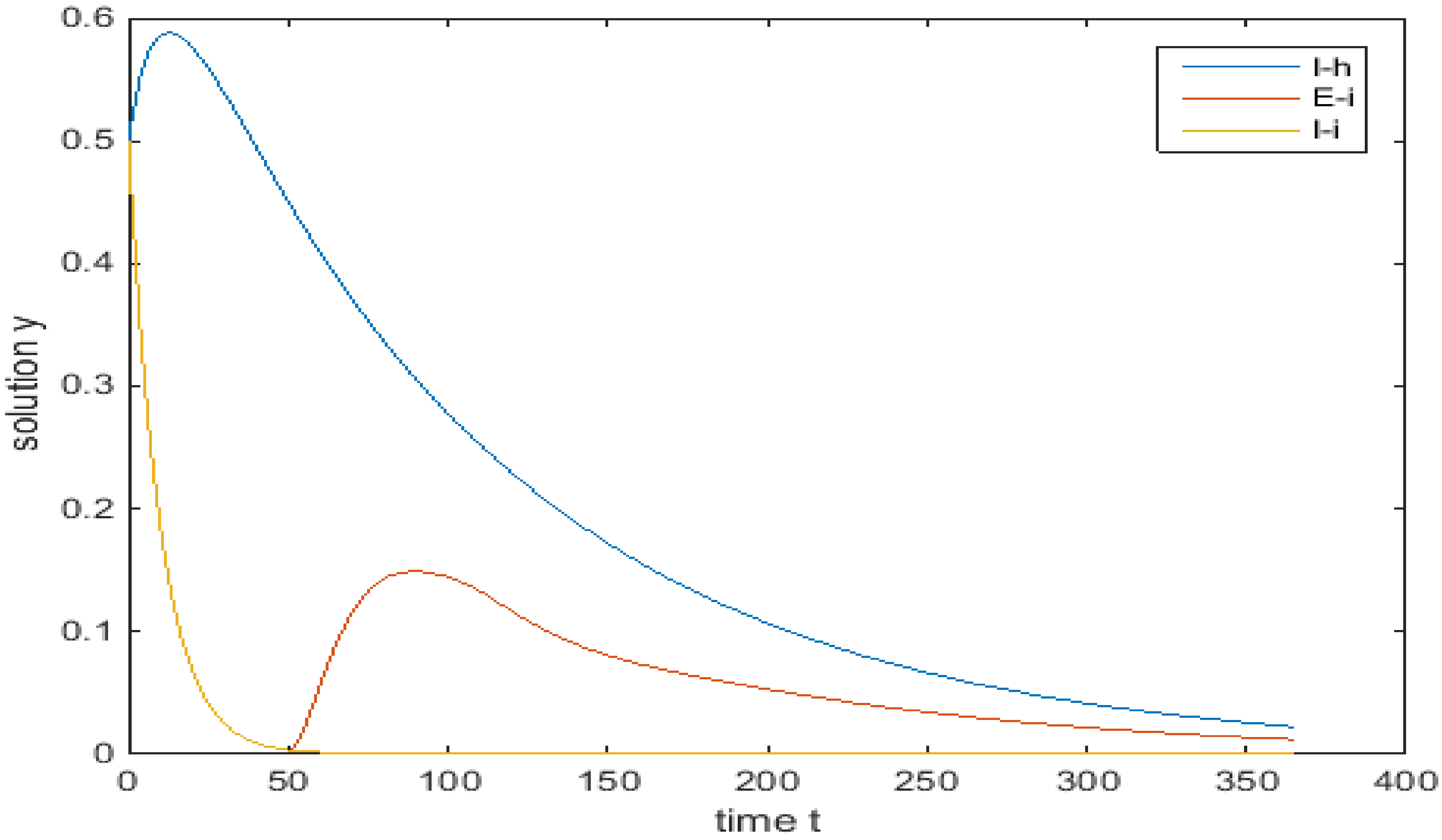


Ross-Macdonald mathematical Model

$$\frac{dI_h}{dt} = abmI_i(1 - I_h) - rI_h$$

$$\frac{dE_i}{dt} = acI_h(1 - E_i - I_i) - acI_h(t - \tau_i)\big(1 - E_i(t - \tau_i) - I_i(t - \tau_i)\big)e^{-\mu_2\tau_i} - \mu_2 E_i$$

$$\frac{dI_i}{dt} = acI_h(t - \tau_i)\big(1 - E_i(t - \tau_i) - I_i(t - \tau_i)\big)e^{-\mu_2\tau_i} - \mu_2 I_i$$



a=0.1, b=0.2,c=0.5,m=5,r=0.01,mu=0.1

Further research

Proposed Models

$$\frac{dI_h}{dt} = abmI_i(t-\tau_h)(1-I_h(t-\tau_h)) - rI_h$$

$$\frac{dI_i}{dt} = acI_h(t-\tau_i)(1-I_i(t-\tau_i)) - \mu_2 I_i$$

$$\frac{dI_h}{dt} = abmI_i(t-\tau_i)(1-I_h(t-\tau_h)) - rI_h$$

$$\frac{dI_i}{dt} = acI_h(t-\tau_h)(1-I_i(t-\tau_i)) - \mu_2 I_i$$

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THANK YOU FOR YOUR ATTENTION