PhD research topic proposal BME, Doctoral School of Mathematics and Computer Science

Name of the supervisor: Dr. Róbert Horváth

Degree: PhD

<u>**Title of the topic:**</u> Qualitative investigation of the numerical solutions of partial differential equations

Short description:

Partial differential equations frequently serve as mathematical models of real-life phenomena (e.g. in the case of physical, chemical, biological and economic processes). It is important that the approximate solutions obtained with numerical techniques should possess the characteristic qualitative properties of the original phenomenon. This requirement generally gives some restrictions for the adequate discretization of the problem.

The goal of the research is to investigate the qualitative properties of linear and nonlinear parabolic partial differential equations. Sufficient conditions are to be deduced for the model parameters, for the discrete mesh structures and for the time step that guarantee the validity of the qualitative properties. The research can be extended to the mathematical models of processes that are closer to the interest of the applicant.

The theoretical results are to be verified on numerical test problems.

<u>Requirements</u>: Standard courses on partial differential equations and on their numerical solutions, programming skills in Matlab.

Contact:

Phone: +36 1 463-5129, E-mail: <u>rhorvath@math.bme.hu</u> (preferred)

Place of work:

Department of Analysis, Institute of Mathematics, BME

Statement:

PhD research topic BME, Doctoral School of Mathematics and Computer Science

Name of the supervisor: Gábor Pete, PhD

<u>Title of the topic:</u> Noise sensitivity of Boolean functions, with applications to Markov chains in statistical physics and group theory

<u>Short description</u>: Noise-sensitivity of a Boolean function with iid random input bits means that resampling a tiny proportion of the input makes the output unpredictable. This notion arises naturally in computer science, but perhaps the most striking example comes from statistical physics, in large part due to the work of the supervisor: the macroscopic geometry of planar percolation is very sensitive to noise. This can be recast in terms of Fourier analysis on the hypercube: a function is noise sensitive iff most of its Fourier weight is on "high energy" eigenfunctions of the random walk operator.

The focus of the current research of the supervisor is on applying such ideas in a variety of settings, and the PhD student would take part in these efforts. Here are some key sample problems:

Problem 1: Prove noise sensitivity results for interesting Boolean functions with iid input bits coming from computer science or statistical physics, such as First Passage Percolation. What is the exact relationship between noise sensitivity and different computational complexity measures?

Problem 2: For Glauber dynamics of the Ising model on any finite transitive graph at its critical temperature, is it true that total magnetization is the observable that mixes the most slowly? What are the observables that get mixed much faster? This is interesting even on the complete graph, called the Curie-Weiss model. Another key example is the 2-dimensional critical Ising model, where conformal invariant techniques may be applied.

Problem 3: The Interchange Process on a graph with *n* vertices is the following random walk on the permutation group Sym(n): we start with a different particle at each vertex, and at each step of the walk, we interchange the two particles at a uniformly chosen random edge of the graph. Understand the mixing time of this random walk. How much time is needed to get macroscopic cycles in the resulting random permutation? Relate these questions to the representation theory of Sym(n).

Problem 4: Study the behavior of statistical physics models along Benjamini-Schramm convergent graph sequences. When is the critical temperature determined by the local structure?

<u>Requirements:</u> An MSc degree in mathematics, with a very strong background in probability and stochastic processes is a must. Strong background in combinatorics or group theory or complex analysis is welcome.

Contact: http://www.math.bme.hu/~gabor

BME, Doctoral School of Mathematics and Computer Science

Name of the supervisor: Balázs Ráth

Degree: PhD

<u>Title of the topic:</u> Self-organized critical random graph models

Short description:

Random graphs (e.g. the Erdős-Rényi model) are the mathematical models of the connectivity phenomena observed in large complex networks (e.g., social networks, wold wide web, spreading of infections). The PhD candidate is invited to investigate a the dynamic random graph models defined in [Tóth, Ráth, 2009, EJP] and [Ráth, 2009, JSP] and further investigated in [Martin, Ráth, 2017, EJP]. One of these random graph models is the so-called "forest fire model", the time evolution of which is driven by two opposite effects: the creation of edges and the destruction of large connected components by "fires". The balance of these two effects drive the model to a "critical" state which is on the boundary of "safe" (subcritical) and "very inflammable" (supercritical) regimes. This phenomenon is called self-organized criticality (S.O.C.) by physicists. The mathematically rigorous theory of these S.O.C. random graph models still has plenty of interesting open questions suitable for a PhD student in mathematics.

Requirements: deep knowledge of probability theory

Contact:

Phone: 36 1 463 1111 / 5904 ext.

e-mail: rathb@math.bme.hu

Place of work: Department of Stochastics, Institute of Mathematics, BME

Statement

PhD research theme BUTE Doctoral School of Mathematics and Computer Science

Name of the supervisor: János Tóth

Degree: CSc

Title of the topic: Algebraic Methods in the Theory and Applications of Differential Equations

Short description:

The candidate can join to research in the topics described by the title which mainly means that systems of polynomial equations are investigated applied in the fields of chemistry, biochemistry, chemical engineering, biology and other parts of science. One can investigate special models or classes of models. The questions of interest are the qualitative behaviour of the solutions (inverse problems, nonnegativity, blow up, periodicity, monotonicity, controllability, numerical approximations etc.) The main tools of investigations are---beyond the classical methods of the qualitative theory of differential equations---chapters of algebra, including linear algebra, polynomial equations, Lie algebra, etc. but graph theory may also play a special role. The work is done in strong cooperation with researchers from other countries. It is required an possible to present the results at international conferences. Students having previous work in related fields are especially welcome.

Requirements: MSC degree in mathematics, physics, chemistry or informatics, solid background in the theory of differential equations, interest in applications in the sciences, basic knowledge in programmig (Wolfram language preferred).

Contact:

Phone: 36 1 463 2314, cellular: 36 70 509 1398

E-mail: jtoth@math.bme.hu

Place of work: Department of Analysis, Institute of Mathematics, BUTE

Statement: The conditions of research are provided by the Department of Analysis. The announcement of the topic was approved by the head of the Department.

BME, Doctoral School of Mathematics and Computer Science

Name of the supervisor: Bálint Vető

Degree: PhD

<u>Title of the topic:</u> Asymptotics of exactly solvable models in the Kardar-Parisi-Zhang universality class

Short description:

n the physics literature, a wide class of surface growth phenomena is investigated since the 1980s which appear naturally, e.g. crystal and facet growth, boundaries, solidification fronts, paper wetting or burning fronts. In their seminal paper Kardar, Parisi and Zhang (Phys. Rev. Lett. 56, 1986) gave a stochastic differential equation which is believed and since then partially proved to described these phenomena. To access the solution of the KPZ equation, various mathematical models for surface growth are studied which mimic this behaviour. These models show the same universal scaling and asymptotic properties and hence said to belong to the KPZ universality class. The PhD candidate is assumed to study the limiting fluctuations of certain models in the KPZ universality class which include interacting particle

system models, directed polymers, non-intersecting trajectories and random tiling models.

<u>Requirements</u>: strong background in probability and analysis

Contact:

Phone: +36 1 463 1111 / 5904 ext. or +36 21 380 6526

e-mail: vetob@math.bme.hu

Place of work: Department of Stochastics, Institute of Mathematics, BME

Statement

BME, Doctoral School of Mathematics and Computer Science

Name of the supervisor: Marianna Bolla

Degree: CSc

<u>Title of the topic:</u> Mixed Graphical Models

Short description:

Graphical models provide a framework for describing statistical dependencies in (possibly large) collections of random variables. At their core lie various correspondences between the conditional independence properties of a random vector and the structural properties of the graph used to represent interactions (directed or undirected) between the vertices assigned to the random variables. These so-called causality models have been investigated since the 1980s, the first steps were made by J. Pearl. However, it was S. L. Lauritzen who showed how loglinear models can be used to estimate joint, marginal, and conditional probabilities taking into consideration the graph structure. The candidate is assumed to master some routine in hierarchical and decomposable loglinear models, based on the book of S. L. Lauritzen (Graphical Models, Oxford Univ. Press, 1995). Then the task of the candidate would be to develop the models and algorithms in the following. The underlying variables are usually categorical (e.g., symptoms, medical diagnoses), but so-called mixed models, incorporating continuously distributed random variables (mainly Gaussian, conditioned on the deiscrete ones) are also proposed in the above book. The estimation methods could be extended to these mixed types of models, via standard methods of multivariate statistics working with covariances. The models are applicable in machine learning for building artificial intelligence (e.g., in medical diagnostic systems), so testing the models on real-life data is also welcome.

Requirements: to be graduated in introductory Probability, Statistics, and Graph Theory

Contact:

Phone: 36 1 463 1101.....

e-mail: marib@math.bme.hu

Place of work: Department of Stochastics, Institute of Mathematics, BME

Statement

BME, Doctoral School of Mathematics and Computer Science

Name of the supervisor: Miklós Ferenczi

Degree: DSc

<u>Title of the topic:</u> Investigations in Algebraic Logic

Short description:

Algebraic Logic is at the borderline of Mathematical Logic and Abstract Algebra. George Boole developed the connection between these two topics already in the 19th century. In Algebraic Logic, it is investigated how logical problems can be translated to algebraic problems and conversely, how algebraic problems can be translated into Logic and how these problems can be solved in the new area. Inside Algebra, Universal Algebra is the topic which has priority from the viewpoint of Algebraic Logic. The candidate is supposed to have some routine both in Logic and in Algebra, of course. Furthermore, one of the tasks of the candidate to deepen this routine in the future. Today, Algebraic Logic is a classical area of the mathematical researches. Some famous researchers of the area are: George Boole, Alfred Tarski, Paul Halmos, Leon Henkin, William Craig, Roman Sikorski, Donald Monk.

<u>Requirements</u>: To graduate introductory courses from Mathematical Logic, Set Theory and Abstract Algebra

Contact:

Phone: 36 1 4632094 e-mail: ferenczi@math.bme.hu

Place of work: Department of Algebra, Institute of Mathematics, BME

Statement

PhD theme

Doctoral School of Mathematics and Computer Science

Name of the supervisor: Erzsébet Horváth

Degree: PhD

Title of the topic: Representaion theory with computers

Short description:

The PhD student gets acquainted with the computer algebra system GAP and

with the basics of the ordinary and modular representation theory of finite groups.

To this the following special courses give help:

Represenation theory with computers,

Modular represenation theory,

Representaion theory seminar.

The student makes researches on up to date problems of representation theory, coordinated and supervised by the advisor. The student uses the GAP program system in these researches.

Requirements:

MSC degree in mathematics, physiscs or informatics

Contact:

Telephone: 5670

email: <u>he@math.bme.hu</u>

Place of the work: BME Faculty of Natural Sciences. Math. Inst. Dept. of Algebra H-1111 Budapest, Egry J. u.1.

Statement

PhD research theme BME, Doctoral School of Mathematics and Computer Science

Name of the supervisor: Gábor Ivanyos

Degree: DSc

Title of the topic: Algebraic methods in quantum information processing

Short description:

Physicists proposed the study of the question of applicability of quantum phenomena in information processing. In this regard considerable progress has been achieved during the past few years. On the one hand, there already exist devices for transmitting information that are encrypted using quantum mechanics. On the other hand, in a certain model of quantum computers in principle it is possible to factor integers and computing discrete logarithms. It is not yet clear how realistic this model is. Anyway, there are some promising results with implementing simple quantum machines consisting of a very limited number of gates.

Here are some topics in which application of algebraic methods may result in further progress:

- comparing complexity classes defined by various models of quantum computers with classical complexity classes
- looking for novel computational problems from algebra and arithmetics that can be solved efficiently on quantum computers
- designing and testing quantum gates
- algebraic methods for quantification and classification of basic quantum mechanical phenomena (e.g., entanglement)
- study of quantum communication complexity using algebraic tools
- algebraic constructions for quantum error correcting codes

Requirements:

Msc/diploma in mathematics, physics, computer science, electrical engineering or related disciplines; solid background in algebra, skills in reading specialized literature in English.

Contact:

Email: Gabor.Ivanyos@sztaki.mta.hu

Tel: +36-1-27961764

<u>Place of work:</u> Mathematical Institute of Budapest University of Technology and Economics, Department of Algebra.

<u>Statement</u>

BME, Doctoral School of Mathematics and Computer Science

Name of the supervisor: Károly Simon

Degree: DSc

<u>Title of the topic:</u> Fractal geometry

Short <u>description</u>: There is a very successful research group at our department, which investigates the chaotic systems. Part of our group <u>is specialized</u> in researching the geometry of fractals sets. These are sets which appear in all parts of natural sciences, and their geometry is significantly different from the geometry of the objects we are used to. In many <u>cases</u> there is a pattern which <u>is repeated</u> on every scale. The Ph.D. student will work on problems related to both deterministic of random fractals.

<u>Requirements:</u> Deep knowledge in Measure Theory and Probability Theory

Contact:

Phone: 36 1 4631607

e-mail: simonk@math.bme.hu

Place of work: Department of Stochastics, Institute of Mathematics, BME

<u>Statement</u>

PhD theme Doctoral School of Mathematics and Computer Science

Name of the supervisor: Sándor Kiss

Degree: PhD

<u>Title of the topic:</u> Sumsets and difference sets

Short description:

The investigation of the sumsets and difference sets is a very important topic in Additive Number Theory. Among the plenty of beautiful and interesting results in this topic one of the oldest is the famous theorem of Cauchy and Davenport. There is some unsolved problems in this field as well. An immediate question arises when the size of the sumset and difference set is small: what can one say about the size of the original set? Another classical problem is to estimate the cardinality of a subset of the finite field of p elements with the property that the difference of any two elements from the subset is a quadratic residue modulo p. To handle these problems and similar questions there are new and exciting methods developed by outstanding research mathematicians recently. The task of the PhD student is to learn and improve the known methods and try to solve some problems in this field. The student have to publish his results in high quality journals.

<u>Requirements:</u> Basic knowledge of Algebra, Analysis and Combinatorics are needed. The PhD student should like to read and learn new tools from several fields of mathematics.

Contact:

Email: ksandor@math.bme.hu Tel: +3614631785

<u>Place of work:</u> Mathematical Institute of Budapest University of Technology and Economics, Department of Algebra.

Statement

PhD research theme BME, Doctoral School of Mathematics and Computer Science

Name of the supervisor: András Kornai

Degree: DSc

<u>Title of the topic:</u> Mathematical linguistics

Short description:

The research proposed here addresses several subjects within the study of semigroups and monoids. The main task of mathematical linguistics is to investigate the mathematical theories behind language technology algorithms such as spellchecking, speech- and optical character recognition (ASR, OCR) as well as speech synthesis, machine translation, semantic information retrieval and information extraction, etc. Because of their importance in applied work, we single out the methods based on finite automata, finite transducers, and their generalizations; machine learning of algorithms based on these; and vector-based semantic methods (semantic web, weak inferencing, glue semantics). Within formal language theory the central areas are the subregular families on the one hand, and the mildly context-sensitive classes of languages on the other. Their weighted (probabilistic) generalizations are of particular interest.

<u>Requirements:</u> Basic knowledge of Algebra, the PhD student should like to read and learn new tools from several fields of mathematics.

Contact:

Email: <u>kornai@math.bme.hu</u> Tel: +3614632094

<u>Place of work:</u> Mathematical Institute of Budapest University of Technology and Economics, Department of Algebra.

Statement

BME, Doctoral School of Mathematics and Computer Science

Name of the supervisor: Edith Alice Kovacs

Degree: PhD

<u>Title of the topic:</u> Approximation of regular vine copulas by using truncated vine copulas and cherry-tree copulas

Short description:

Copulas are a useful tool in modeling multivariate probability distribution which play an important role in generating scenarios for different fields: Finance: estimate the credit risk and the market risk, Insurance, Hydrology. As Fisher (1997) notes in the Encyclopedia of Statistical Sciences : "Copulas are of interest to statisticians for two main reasons: First, as a way of studying scale-free measures of dependence; and secondly, as a starting point for constructing families of bivariate distributions, [...]" For the fitting of two dimensional copulas to sample data there are a lot a very good algorithms and programs.

In higher dimensions often appear different types of dependences between the random variables involved. For this aim there were introduced the regular vine copulas, which can model many types of dependences between different pairs of random variables. However this is also a drawback since the number of parameters becomes very large, as the dimension of the multivariate random variables grows. To reduce this large number of parameters, the Truncated vine copulas and the Cherry-tree copulas were introduced. The candidate is supposed to do research in developing the modeling of cherry tree copulas and truncated vines.

The candidate is supposed to have some routine in Probability Theory. Furthermore, one of the tasks of the candidate will be deepening this knowledge by adding Copula Theory and some parts of Information Theory.

<u>Requirements</u>: To graduate introductory courses from Algebra, Probability Theory.

Contact:

Phone: (36-1)-4631397

e-mail: kovacsea@math.bme.hu

edith kovacs@yahoo.com

Place of work: Department of Differential Equations, Institute of Mathematics, BME

Statement

BME, Doctoral School of Mathematics and Computer Science

Name of the supervisor: Edith Alice Kovacs

Degree: PhD

<u>Title of the topic:</u> Algorithms for fitting cherry-tree copulas to sample data and their applications

Short description:

Copulas became a popular tool in modelling multivariate probability distributions. Copulas make possible the modelling separately the one dimensional marginal probability distributions and the dependency between the random variables.

In higher dimensions often appear different types of dependences between the random variables involved. To model these there were introduced the regular vine copulas, however this regular vine copulas use a large number of parameters, as the dimension of the multivariate random variables grows. To reduce this large number of parameters, the Truncated vine copulas and the Cherry-tree copulas were introduced. The candidate is supposed to do research on developing algorithms for finding good fitting cherry tree copulas and truncated vines, and to implement them.

The candidate is supposed to have some routine in Probability Theory and Algorithms furthermore, one of the tasks of the candidate to deepen this knowledge by adding Copula Theory and some parts of Information Theory.

<u>Requirements</u>: To graduate introductory courses from Algebra, Probability Theory, Algorithms.

Contact:

Phone: (36-1)-4631397

e-mail: kovacsea@math.bme.hu

Place of work: Department of Differential Equations, Institute of Mathematics, BME

<u>Statement</u>

BME, Doctoral School of Mathematics and Computer Science

Name of the supervisor: András Kroó

Degree: DSc

<u>Title of the topic:</u> Modern Aspects of Constructive Function Theory

Short description:

The main goal of this PhD Programme is to introduce the students to the main topics and methods of the Constructive Function Theory and Approximation Theory. By the completion of the course the students are enabled to conduct independent study and research in fields touching on the topics of the course. They also learn how to use these methods to solve specific problems. In addition, the students develop some special expertise in the Constructive Function Theory, which they can use efficiently in other mathematical fields, and in applications, as well.

The main topics covered by this PhD Programme are as follows:

1. Classical polynomial inequalities (Bernstein, Markov, Remez, Schur, Nikolskii inequalities).

2. Approximation by linear operators. Fourier and Fej'er operators, Bernstein Polynomials, positive linear operators. Korovkin theorem.

3. Lacunary polynomial approximation, incomplete polynomials, M[°]untz type theorems.

4. Bernstein-Markov type inequalities for multivariate polynomials on convex and star like domains in uniform and integral norms.

5. Markov type inequalities for homogeneous polynomials on convex bodies and Tangential Bernstein-Markov type inequalities.

6. Remez type inequalities for multivariate polynomials on star like domains and convex bodies and their application.

7. Admissible and optimal meshes for multivariate polynomials.

8. Approximation by ridge functions and incomplete polynomials in several variables.

9. Weierstrass type theorems for approximation by homogeneous polynomials on the boundary of convex domains.

10. Approximation of convex bodies by convex algebraic level surfaces.

Requirements:

Contact:

Phone: 36 1 463 1111 /5705 ext.

e-mail: kroo@math.bme.hu

Place of work: Department of Analysis, Institute of Mathematics, BME

Statement

BME, Doctoral School of Mathematics and Computer Science

Name of the supervisor: Molnár Lajos

Degree: DSc

<u>Title of the topic:</u> Transformations on structures of matrices, operators, and functions

Short description:

In Hermann Weyl's fundamental book "Symmetry" one can read the following famous sentences: "Whenever you have to do with a structure-endowed entity \$\Sigma\$, try to determine its group of automorphisms, the group of those element-wise transformations which leave all structural relations undisturbed. You can expect to gain a deep insight into the constitution of \$\Sigma\$ in this way."

The proposed PhD research topic concerns the determination of the automorphisms/symmetries of structures of rather wide range which consist of matrices, linear operators or functions. The particular area we are interested in is now labeled by the term "preserver problems" and it represents a rather vivid research field on the border of functional analysis and linear algebra. Generally speaking, the central problem here is to describe the maps (called preservers which are kinds of symmetries) that preserve certain important characteristics (numerical quantities, relations, operations, etc) of a given mathematical structure. Immediate and well-known examples for preservers are isometries (distance preserving maps) in various geometries, and algebraic automorphisms (operation preserving maps) in various parts of algebra.

In this doctoral topic we are concerned with structures which consists of matrices, linear operators, or scalar valued functions. Hence the research is connected to linear algebra, analysis, and functional analysis. Its aim is to contribute to and make developments in a chosen subfield of the area of preserver problems.

Requirements: ---

Contact:

Phone: 36 1 463 1111 /5704 ext.

e-mail: molnarl@math.bme.hu

Place of work: Department of Analysis, Institute of Mathematics, BME

Statement

BME, Doctoral School of Mathematics and Computer Science

Name of the supervisor: Mosonyi Milán

Degree: PhD

<u>Title of the topic:</u> PhD in Quantum Information Theory

Short description:

Quantum Information Theory is one of the most active research fields in present days, thanks partly to its great technological promises (quantum computer, unconditionally secure communication, ultra-precise metrology) and to the host of exciting problems it poses in Mathematics, Physics, and Computer Science. The candidate's task will be to work on problems at the mathematical/statistical foundations of Quantum Information Theory. The aim of the project is to identify the ultimate theoretical limits of the performance of coding strategies in various problems, like source compression, channel coding and hypothesis testing, by finding explicit descriptions of the exact trade-off curves between the competing quantities characterizing each problem, and linking them to measures of information, correlation, and dissimilarity of quantum states. An important part of the project is the study of various information quantities by the help of matrix analysis and other mathematical tools.

<u>Requirements</u>: The ideal candidate should have a solid background in mathematics. Prior experience (specialized courses, students projects, etc.) in any of the following fields is an advantage, though not necessary: Matrix analysis, functional analysis, operator algebras, information theory, quantum physics.

Contact:

Phone: 36 1 463 2767

e-mail: mosonyi@math.bme.hu

Place of work: Department of Analysis, Institute of Mathematics, BME

Statement

PhD research theme, BME, Doctoral School of Mathematics and Computer Science

Name of the advisor: Attila Nagy

Degree: PhD

<u>Title of the topic:</u> Algebraic examinations of semigroups

Short description:

The task of the PhD student is to study the algebraic theory of semigroups and try to solve problems in this fields. The following two topics are in the center:

Congruence permutable semigroups: A semigroup S is said to be congruence permutable if $\alpha \sigma \beta = \beta \sigma \alpha$ is satisfied for every congruences α and β of S, where σ denotes the usual composition of binary relations. The object is to determine congruence permutable semigroups in special classes of semigroups.

By the Birkhoff's theorem, every semigroup is a subdirect product of subdirectly irreducible semigroups. Thus it is an interesting problem to find subdirectly irreducible semigroups. The object is to determine subdirectly irreducible semigroups in special classes of semigroups.

The student have to publish his results in high quality journals.

<u>Requirements:</u> MSc degree from mathematics

Contact:

Email: <u>nagyat@math.bme.hu</u>

Tel: +3614632094

<u>Place of work:</u> Mathematical Institute of Budapest University of Technology and Economics, Department of Algebra.

<u>Statement</u>

PhD theme

Doctoral School of Mathematics and Computer Science

Advisor: prof. Lajos Rónyai

Title of the theme: Algebraic methods in computer science

Short description:

Algebraic tools and techniques have proved to be very efficient in the study of some problems of discrete mathematics and computer science. Particularly interesting are here the explicit constructions of algebraic nature. As examples, one can mention notable error correcting codes, such as Reed-Solomon codes. Some cryptographic techniques (such as ElGamal encryption, Diffie-Helmann key exchange, or ECC) also involve algebraic ideas. Algebraic methods have led to important constructions in combinatorics, such as the norm graphs. The main objective of the project would be the study and development of constructive applications in the spirit of the above examples. From this very wide area we could select specific topics according to the interest and background of the student. There are important theoretical problems as well as questions close to computational applications.

Requirements from applicants: MSc degree from mathematics, or computer science, or engineering.

Contact of the advisor:

Telephone:+36-1-4632094

e-mail: lajos@math.bme.hu

<u>Place of work:</u> Mathematical Institute of Budapest University of Technology and Economics, Department of Algebra.

Statement

PhD kutatási témajavaslat BME, Matematika és Számítástudományok Doktori Iskola

A témavezető neve, tud. fokozata (külső témavezető esetén tanszéki konzulens adatai is):

Dr. Szirmai Jenő, PhD

A PhD téma címe:

Ball packings, coverings and Dirichlet-Voronoi cells in Thurston geometries

A kidolgozandó feladat tömör leírása:

The classical sphere packing problems concern arrangements of non-overlapping equal spheres (rather balls) which fill a space. Space is the usual three-dimensional Euclidean space. However, ball (sphere) packing problems can be generalized to the other 3-dimensional Thurston geometries

 $E^3,\ S^3,\ H^3,\ S^2 \times R$, $\ H^2 \times R$, $\ S\widetilde{L}_2R$, Nil , Sol

and to higher dimensional various spaces.

In an n-dimensional space of constant curvature $d_n(r)$ be the density of n+1 spheres of radius r mutually touching one another with respect to the simplex spanned by the centres of the spheres. L. Fejes Tó'th and H.S.M. Coxeter conjectured that in an n-dimensional space of constant curvature the density of packing spheres of radius r can not exceed $d_n(r)$. This conjecture has been proved by C. Roger in the Euclidean space. The 2-dimensional case has been solved by L. Fejes Tóth. In an 3-dimensional space of constant curvature the problem has been investigated by Böröoczky and Florian and it has been studied by K. Böröczky for n-dimensional space of constant curvature (n> 3).

We have studied some new aspects of the horoball and hyperball packings in n-dimensional hyperbolic space and we have realized that the ball, horoball and hyperball packing problems are not settled yet in the n-dimensional n>2 hyperbolic space.

The goal of this PhD program to generalize the above problem of finding the densest geodesic and translation ball (or sphere) packing and covering to the other 3-dimensional homogeneous geometries (Thurston geometries) $S^2 \times R$, $H^2 \times R$, $S\tilde{L}_2R$, Nil, Sol. Moreover, we will study the structure of Dirichlet-Voronoi cells related to the packing configurations.

We note here that the greatest known packing density is realized in $S^2 \times R$ geometry with packing density is ~0.87499429.

We will use the unified interpretation of the Thurston geometries in the projective 3-sphere.

További információk: www.math.bme.hu/~szirmai

A jelentkezővel szemben támasztott elvárások (pl. idegen nyelv ismeret, matematika bizonyos irányainak alaposabb ismerete, stb.):

A témavezető elérhetősége (külső témavezető esetén tanszéki konzulens adatai is):

Telefon: 2645 E-mail: szirmai@math.bme.hu

<u>A doktori munka készítésének helye (tanszék megnevezése, külső témavezető esetén külső kutatóhely is):</u> BME, MI, Geometria Tanszék

<u>Nyilatkozat</u>

A javasolt témában kutatás feltételei a tanszéken biztosítottak, a téma meghirdetését a tanszékvezető jóváhagyta.

PhD thesis research topic Doctoral School of Mathematics and Computer Science, Budapest Univ. Techn.

Name of supervisor, research degree (in case of external supervisor also the data of the departmental supervisor):

Supervisor: Gábor Domokos, member of of Hungarian Academy of Sciences Co-supervisor: Zsolt Lángi, PhD

<u>The title of the PhD topic:</u> Morphodynamics of convex solids

Brief description of the task:

A physically extremely interesting area of convex geometry deals with the description of convex solids with the aim to identify, categorize and track the evolution of natural shapes. In addition to convex geometry, mathematical tools include geometric partial differential equations, in particular, curvature-driven flows which are closely related to the heat equation. An equilibrium point of a convex solid is a stationary point of the distance function measured from the center of gravity, placing the solid on a horizontal plane it can be statically balanced at these points. We can distinguish between maximum, minimum and saddle points, the numbers of which we denote by S, U and H, respectively. In case of convex solids, the Poincaré-Hopf Theorem implies the relationship

S+U–H=2,

and based on this any convex solid can be assigned to an $\{S,U\}$ equilibrium class. In addition to the number of equilibrium points, the topology of the integral curves in the gradient flow connecting these points also describes convex solids. Based on this aspect, within each equilibrium class we can distinguish topological subclasses. By the equilibrium class and subclasses, a very interesting and from geological point of view very useful classification system can be defined for shapes that can be found in nature (e.g. pebble shapes). Our former research verified that both the system of equilibrium and that of topological classes are complete in the sense there is neither empty class, nor empty subclass.

In the present PhD research we investigate some particularly interesting geometric properties of the above defined classification system, and also other possible classification systems. Our goal, among other things, is to find out how robust these classes and subclasses are; that is, by what probability a convex solid can move from one class or subclass into another one by abrasion. Our aim is a deeper understanding of the partial differential equations describing abrasion processes, and proving statements related to them. We can also track abrasion processes via computer models and by state of the art experimental equipment. Our goal is to compare these data to the mathematical models. We already have some initial results, but many questions are not yet answered which are essential from physical applications.

Expectations for the applicant (e.g. knowledge of foreign languages, deeper knowledge of certain areas of mathematics, etc.):

The topic esentially is geometrically motivated, within this knowledge of classical differential geometry is important. Expertise in low-dimensional dynamical systems is an asset, and also familiarity with numeric computations and programming is very useful. The topic has also statistical aspects, we welcome applicants with such interest as well. Primarily we expect the applications of students with a degree in mathematics or physics.

Contact information of the supervisor (in case of external supervisor also the data of the departmental supervisor):

Phone:463-1493E-mail:domokos@iit.bme.huContact information for the co-supervisor:Phone:463-1145E-mail:zlangi@math.bme.hu

<u>Research place (name of the department, in case of external supervisor also the name of the external research place</u>): Department of Geometry

Declaration

The conditions for research in the suggested topic are satisfactory at the department, the announcement of the topic has been approved by the department head.

PhD thesis research topic Doctoral School of Mathematics and Computer Science, Budapest Univ. Techn.

Name of supervisor, research degree (in case of external supervisor also the data of the departmental supervisor): Supervisor: Zsolt Lángi, PhD

The title of the PhD topic: Extremum problems in geometry

Brief description of the task:

Geometric extremum problems; that is, looking for minimal and maximal values of a geometric quantity and configurations for which they are attained, have been in the focus of mathematical research since ancient Greece. Examples of such problems are all packing and covering problems, the long list of variants of the isoperimetric inequality and many other related inequalities in convex geometry, or problems in geometric graph theory and combinatorial geometry. Usually these problems have many applications in science from physics and engineering to computer science to medical sciences, and sometimes are even motivated by a real world phenomenon.

The aim of the PhD research is to deal with geometric extremum problems, or problems related to extremum problems. This work may have many different aspects depending on the knowledge, expertise and interest of the applicant. Among other things, it may include mathematical research on convex, discrete or combinatorial geometry problems, on the algorithmic or numeric aspects of these problems, or their applications.

Expectations for the applicant (e.g. knowledge of foreign languages, deeper knowledge of certain areas of mathematics, etc.):

The topic is geometrically motivated, with special regard to convex, discrete and combinatorial geometry. In addition to geometry, knowledge of analysis, linear algebra or combinatorics is an asset. People interested in programming and numeric computations are also welcome. Mainly the applications of students with mathematics or physics degree are expected.

Contact information of the supervisor (in case of external supervisor also the data of the departmental supervisor):

Phone:463-1145E-mail:zlangi@math.bme.hu

Research place (name of the department, in case of external supervisor also the name of the external research place): Department of Geometry

Declaration

The conditions for research in the suggested topic are satisfactory at the department, the announcement of the topic has been approved by the department head.