

# Info 1 — Second midterm

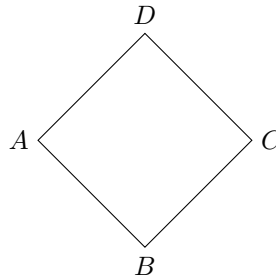
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## 1 TikZ

Download <https://math.bme.hu/~asimon/info1/preen.tex>, rename it to NNNNNN.tex, where NNNNNN is your neptun code, write your TikZ code in it, and email it to [asimon@math.bme.hu](mailto:asimon@math.bme.hu) with your neptun code in the subject.

- (a) Draw a square whose corners are at the points  $(0,0)$ ,  $(-\sqrt{2}, \sqrt{2})$ ,  $(0, 2\sqrt{2})$  and  $(\sqrt{2}, \sqrt{2})$ . If you can't, draw one with corners  $(0,0)$ ,  $(-1, 1)$ ,  $(0, 2)$  and  $(1, 1)$  (it will cost you 2 points). (4 points)  
(b) Draw the same square you drew in 1a, but label its corners like this:

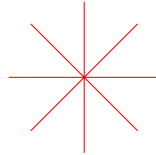


- (Just reuse the square you drew in 1a!) (2 points)
- (c) Draw the same square as in 1a, but with the restriction that you're only allowed to draw a **rectangle**. (Hint: the length of each side is 2.) (2 points)
  - (a) Draw the graph of  $\sqrt{x^3 + 1}$  on the interval  $[-1, 1]$ ! (4 points)  
(b) Draw (and fill with blue) the region enclosed by the segment whose endpoints are  $(0, 0)$  and  $(-1, 0)$ , the segment whose endpoints are  $(0, 0)$  and  $(1, \sqrt{2})$ , and the graph of  $\sqrt{x^3 + 1}$  on the interval  $[-1, 1]$ . It will look like this:



(4 points)

3. Draw the following picture with the help of a loop:



The endpoints are on the (invisible) circle of center  $(0, 0)$  and radius 1.

(6 points)

## 2 Sage

Use whichever interface (command line, <https://sagecell.sagemath.org/> or <https://leibniz.math.bme.hu:8888>) you prefer. (Use the one you're comfortable with.)

If you use <https://leibniz.math.bme.hu:8888>, you can download your work (File -> Download as -> Notebook (ipynb) — *make sure you know where it is saved*), rename it to `NNNNNN.ipynb` (or name it `NNNNNN.ipynb` when you're downloading it), where `NNNNNN` is your neptun code, and email it to [asimon@math.bme.hu](mailto:asimon@math.bme.hu) with your neptun code in the subject. If you want to do this, try it from home first. If you chose to submit your work this way, it's your responsibility that you find your file (notebook). So even if you use this client, you may want to copy your solutions to a separate text file and submit that instead (don't submit both!), as described below.

Otherwise, write all your solutions into a text file in the following format:

```
Question 1:  solution to question 1
```

```
Question 2:  solution to question 2
```

```
...
```

(Of course the solutions can take up more than one lines.) Name your file `NNNNNN.txt`, where `NNNNNN` is your neptun code, and email it to [asimon@math.bme.hu](mailto:asimon@math.bme.hu).

With the exception of 4b (where it is explicitly stated otherwise), **the solution you need to submit is the Sage command or commands that you used**, not what Sage returns as its/their result(s).

1. (a) Define the function  $f(x) = \cos^3(15x) \cdot \sin(x^2)$ . (2 points)

(b) What is the first derivative of  $f(x)$ ? (2 points)

(c) What is the 50th derivative of  $f(x)$ ? (Use the appropriate argument(s) to the appropriate command(s), **don't differentiate  $f$  50 times!**) (2 points)

(d) What is the (exact) value of the first derivative of  $f(x)$  at 10? (Do this by extending what you did in 1b, **don't type in or copy the function Sage returned there!**) (2 points)

(e) Give a numerical approximation of the value of the first derivative of  $f(x)$  at 10 with 25 digits precision. (Extend what you did in 1d, **don't type in the function that Sage returned there!**) (2 points)

2. Substitute  $z+1$  for the variable  $y$  in  $\sqrt{y^2(y+1)}$  with the help of the appropriate Sage command(s). (You will likely need more than one command for this to work!) (4 points)

3. (a) Find all the roots (the exact roots, not numerical approximations) of  $(x^3 - 3)\ln(x - 1) = 0$ . (2 points)

(b) Save one of the non-real roots in 3a in a (non-mathematical) variable (by extending what you did in 3a, **not by retyping or copy/pasting parts of what Sage returned there!**) (4 points)

(c) Check that what you saved in 3b is indeed a root of  $(x^3 - 3)\ln(x - 1) = 0$ . You can use the method `simplify_full()` to simplify an expression. (**Don't type the equation again with  $x$  replaced by the variable you used in 3b!**) (4 points)

4. (a) Find all solutions of the system

$$x + y + 4z = 8, \quad 3x + y + 10z = 18, \quad 5x + 3y + 18z = 34$$

of equations. (4 points)

(b) (Here we're interested in the result itself.) Give a solution of the system of equations above that you can read off from what Sage returned in 4a. Just write something like  $x = \dots, y = \dots, z = \dots$  (2 points)

5. Solve the system of congruences

$$x \equiv 1 \pmod{3} \quad x \equiv 2 \pmod{5} \quad x \equiv 5 \pmod{11}$$

(4 points)