Info1, 2nd midterm

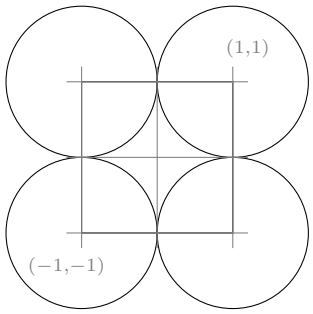
November 29, 2022

Download `https://math.bme.hu/~asimon/info1/midterm2.tex` and write in it your solutions to *both* the Ti*k*Z and the Sage problems (the latter after `\end{document}`). You have 75 minutes to finish the midterm. When ready, send the `.tex` file to `sa42bme@gmail.com`, and put your neptun code in the subject.

1. Ti*k*Z

1. Write Ti*k*Zcode that produces the following drawing.

Hint:



(6 points)

Solution.
```
\clip (-1,-1) rectangle (1,1);
\fill[red] (-1,-1) rectangle (1,1);
\fill[yellow] (-1, -1) circle(1) (-1, 1) circle(1) (1, -1) circle(1) (1, 1) circle(1);
```
or

```
\clip (-1,-1) rectangle (1,1);
\fill[red] (-1,-1) rectangle (1,1);
\fill[yellow] (-1,-1) circle (1);
\fill[yellow] (-1,1) circle (1);
\fill[yellow] (1,-1) circle (1);
\fill[yellow] (1,1) circle (1);
```

or

```
\fill[red] (-1,-1) rectangle (1,1);
\fill[yellow] (0,-1) arc (0:90:1) -- (-1,-1) -- cycle;
\fill[yellow] (-1,0) arc (270:360:1) -- (-1,1) -- cycle;
\fill[yellow] (0,-1) arc (180:90:1) -- (1,-1) -- cycle;
\fill[yellow] (1,0) arc (270:180:1) -- (1,1) -- cycle;
```

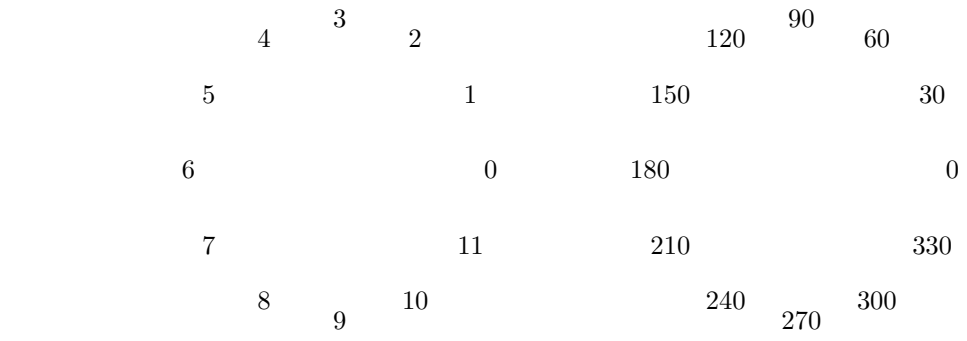
clip (or arcs), filled rectangle, filled circles

2. Write Ti*k*Z code that draws the triangle $(0,0)$, $(2,0)$, $(2,1)$ and its rotated version by 30 and 60 degrees around the point $(2,0)$. (6 points)

Solution.
```
\draw (0,0) -- (2,0) -- (2,1) -- cycle ;
\draw[rotate around={30:(2,0)}] (0,0) -- (2,0) -- (2,1) -- cycle ;
\draw[rotate around={60:(2,0)}] (0,0) -- (2,0) -- (2,1) -- cycle ;
```

triangle, rotate, around

3. Write Ti*k*Z code that uses a loop to produce the picture on the left (or, if that's too hard, then the one on the right, for a little fewer points).



(8 points)

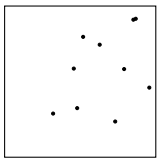
Solution.
```
\foreach \x in {0,...,11} {
  \draw (30*\x:2) node {$\x$} ;
}
```

and

```
\foreach \x in {0,30,...,330} {
  \draw (\x:2) node {$\x$} ;
}
```

loop, draw (polar coords), multiplication, node

4. Write Ti*k*Z code that draws 10 points with random coordinates (filled circles of radius 0.02) in the square whose lower left corner is at $(-1, -1)$ and upper right corner is at $(1, 1)$. The result should resemble this:



(8 points)

Solution.
```
\draw (-1,-1) rectangle (1,1);
\foreach \x in {1,...,10} {
  \draw[fill] (rand,rand) circle (0.02);
}
```

rectangle, loop, rand, filled circle

2. Sage

Use whichever Sage client (the one running on your own laptop, `https://sagecell.sagemath.org/`, `https://cocalc.com/`, etc.) you're used to to solve the problems below. **The solution you need to submit is the Sage command or commands that you used**, not what Sage returns as its/their result(s).

1. (a) Define the function $f(x) = \sin(9x^3)$. (2 points)
   (b) What is the first derivative of $f(x)$? (2 points)
   (c) What is the (exact) value of the first derivative of $f(x)$ at 5? (Do this by extending your solution to 1b, **don't type in or copy the function Sage returned there!**) (2 points)
   (d) Give a numerical approximation of the value of the first derivative of $f(x)$ at 5 with 25 digits precision. (Extend your solution to 1c, **don't type in the function that Sage returned there!**) (2 points)

(e) Plot the graph of the derivative of $f(x)$ on the interval $[-\pi, \pi]$ in such a way that only the values in the interval $[-1, 1]$ are drawn. (6 points)

plot, interval, yminmax

(f) Extend the plot in az 1e with the graph of f. Make sure the two graphs have different colours. (4 points)

+, color

Solution.
```
f(x) = sin(9*x^3)
diff(f(x),x) (or diff(f) or diff(f(x)) or...)
diff(f)(5) (or diff(f,x)(5) or diff(f(x),x).subs(x=5))
diff(f)(5).n(digits=25) (or n(diff(f)(5),digits=25) or...)
plot(diff(f(x),x),(x,-pi,pi), ymax=1,ymin=-1)
plot(f(x),(x,-pi,pi)) + plot(diff(f(x),x),(x,-pi,pi), ymax=1,ymin=-1, color='red')
```
or
```
plot((f(x),diff(f(x),x)),(x,-pi,pi), color=('blue', 'red'),ymax=1,ymin=-1)
```

2. (a) Let $p = x^3 + 2x^2 - 2 \in \mathbb{Z}_5[x]$ and $q = x + 3 \in \mathbb{Z}_5[x]$. Is p irreducible? What is the result of dividing p by q with remainder?

   (b) What's the answer to the same questions if the polynomials above are taken as elements of $\mathbb{Z}_{11}[x]$?

(10 points)

Solution.
```
x = polygen(GF(5),'x'); p=x^3 + 2*x^2 - 2 ; q = x+3; p.is_irreducible(), p//q, p%q
x = polygen(GF(11),'x'); p=x^3 + 2*x^2 - 2 ; q = x+3 ; p.is_irreducible(), p//q, p%q
```

$GF(5)$, $GF(11)$, `is_irreducibile`, //, %