Info1, 1st and 2nd midterm retakes

December 13, 2022

Download `https://math.bme.hu/~asimon/info1/retake.tex` (or `retake.tex` from Teams) and write in it your solutions to *both* the Ti*k*Z and the Sage problems (the latter after `\end{document}`). You have 60 minutes for each of the retakes. When ready, send the `.tex` file to `sa42bme@gmail.com`, and put your neptun code in the subject.

1 First midterm retake

1.1 LaTeX

1. There aren't many who know that $\arctan x = \sum_0^\infty (-1)^n \frac{x^{2n+1}}{2n+1}$ on the interval $(-1, 1]$.

(10 points)

Solution.

```
There aren't many who know that $\arctan x =
  \sum_0^\infty(-1)^n\frac{x^{2n+1}}{2n+1}$ on the interval $[-1,1]$.
```

inline math, `\sum` (with limits), exponent, `\frac`, `\arctan`

2. $$\begin{pmatrix} 1 & e^x & e^{-x} \\ 0 & e^x & -e^{-x} \\ 0 & e^x & e^{-x} \end{pmatrix} \neq \begin{vmatrix} 1 & e^x & e^{-x} \\ 0 & e^x & -e^{-x} \\ 0 & e^x & e^{-x} \end{vmatrix}$$

(10 points)

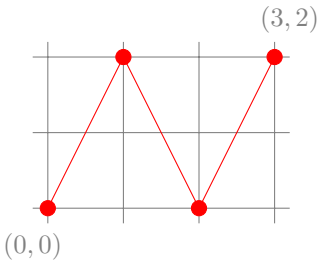
Solution.

```
\begin{pmatrix}1 & e^x & e^{-x} \\ 0 & e^x & -e^{-x} \\ 0 & e^x & e^{-x}
\end{pmatrix}\neq
\begin{vmatrix}1 & e^x & e^{-x} \\ 0 & e^x & -e^{-x} \\ 0 & e^x & e^{-x}
\end{vmatrix}\]
```

displayed math, `pmatrix`, `vmatrix`, `&`s and `\\`s, exponent

1.2 Ti*k*Z

1. Draw the red part with one command (that is, as one *path*), if you can. Do the same with the gray part, too.



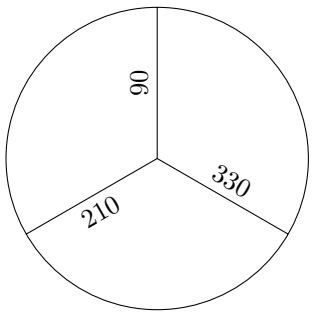
(10 points)

Solution.

```
\draw[thin,gray] (-0.2,-0.2) node[below] {$(0,0)$}
    grid (3.2,2.2) node[above] {$(3,2)$};
\draw[red,fill] (0,0) circle (0.1) -- (1,2) circle (0.1)
    -- (2,0) circle (0.1)  -- (3,2) circle (0.1);
```

draw[fill] or `fill`, circle (4, if used as a path ext. op.), grid, node

2. The radius of the circle below is 2.



(10 points)

Solution.

```
\draw (0,0) -- (90:2) node[midway,above,sloped]{$90$} (0,0)
-- (210:2) node[midway,below,sloped]{$210$} (0,0) -- (330:2)
node[midway,above,sloped]{$330$} (0,0) circle (2) ;
```

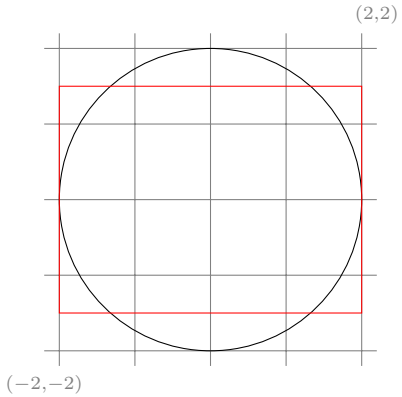
`circle`, polar coordinates, `midway`, `above/below`, `sloped`

2 Second midterm retake

2.1 Ti*k*Z

1. Write Ti*k*Z code that produces the following drawing.

Hint:



(6 points)

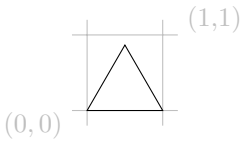
Solution.
```
\clip (0,0) circle (2);
\fill[red] (-2,-1.5) rectangle (2,1.5);
```
or

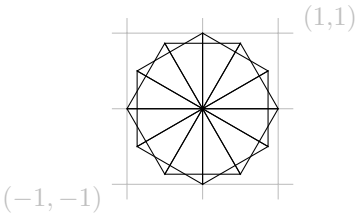
```
\clip (-2,-1.5) rectangle (2,1.5) ;
\fill[red] (0,0) circle (2);
```

`clip`, filled `rectangle` or `circle`, `circle`

2. Write Ti*k*Z code that uses a loop to produce the following picture!

Hint: the drawing contains rotated (by 30 degrees) versions of an equilateral triangle, around one of its vertices. You do *not* need any computation to draw the triangle.





(10 points)

Solution.
```
\foreach \x in {0,...,11} {
  \draw[red, fill,rotate={\x*30}] (0,0) -- (1,0) -- ++(120:1) -- cycle ;
}
```

relative coordinates, polar coordinates, loop, `rotate`,`fill`

2.2 Sage

Use whichever Sage client (the one running on your own laptop, `https://sagecell.sagemath.org/`, `https://cocalc.com/`, etc.) you're used to to solve the problems below. **The solution you need to submit is the Sage command or commands that you used**, not what Sage returns as its/their result(s).

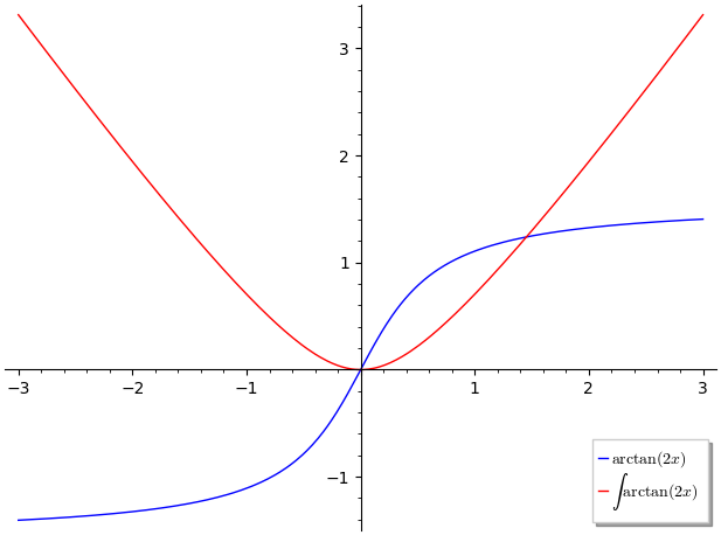
1. (a) $\lim_{x\to-0} \arctan(1/x) =$? (The name of the function arctan in Sage is `atan` or `arctan`.) (4 points)

`limit`, `dir`

(b) Define the function $f(x) = \arctan(2x)$. (2 points)

(c) Find a primitive function of $f(x)$. (2 points)

(d) Plot the graphs of f and its primitive function you've found in on the interval $[-3, 3]$ with different colors and so that it is shown which color belongs to which function, as in the figure below.



(12 points)

`plot`, `+`, interval, `legend_label`, `color`, raw string

Solution.
```
limit(atan(1/x), x=0, dir='-')
f(x) = atan(2*x)
g = integral(f(x),x)
plot(f, (x,-3,3), legend_label=r'$\arctan(2x)$')+
  plot(g, (x,-3,3),color='red', legend_label=r'$\int\, \arctan(2x)$')
```

or

```
plot((f,g), (x,-3,3), legend_label=(r'$\arctan(2x)$', r'$\int\arctan(2x)$'),
  color=('blue','red'))
```

2. Is the matrix $\begin{pmatrix} 1 & 2 & 1 \\ 0 & 1 & 0 \\ 2 & 3 & 2 \end{pmatrix}$ invertible?

Solution.
```
m = matrix([[1,2,1],[0,1,0],[2,3,2]]); rank(m) == 3
```

or

```
det(m) != 0
```

(6 points)

defining a matrix, `rank` or `det`, `==` or `!=`