SageLab

November 15, 2022

1. Define in Sage various functions, such as $f(x) = \sin^2(x)\cos(x)$, $g(x, y) = (x - y)^3$,... and some functions that appeared in one of your Algebra or Calculus homework. Evaluate them in different places, make sure you got a reasonable result. For example, evaluate them at places where you know what result to expect: in the above examples, $f(\pi/2)$ and $g(42, 42)$ should evaluate to 0.

2. Compute the first, second, etc. derivatives of your functions. Find primitive functions of these functions. Check that they *really* are their primitive function.

3. Give all complex solutions of the equation $z^5 + 4z = 0$. Check that these are indeed solutions! Do this without writing the equation again (give a name to the equation and use substitution).

4. $\lim_{x\to\infty} \frac{\operatorname{arctg}(x^2)}{\sqrt{x}}$

5. $\int (x + 2)e^{2x+1}\,dx =$? Check the answer by derivation!

6. $\int \frac{x^2+4x}{x^3+6x^2+5}\,dx =$? Check the answer by derivation!

7. On which interval is the function $f(x) = (x \cdot \ln x)^3$ monotonous? Where and what are its local minima and maxima (if any)? (Beware of false roots!)

8. Solve the linear systems of equations

   $2x + y + z = -1 \qquad x + y - 2z = 1 \qquad 3x + 2y - z = 1 \qquad x + 3z = -2$

   and

   $2x + y + z = -1 \qquad x + y - 2z = 1 \qquad 3x + 2y - z = 0 \qquad x + 3z = -2$

   Check the ranks of their (augmented) matrices, to see that the number of solutions are what the ranks say they should be.

9. For what values of a will the matrix $\begin{pmatrix} 2 & 4 & 0 \\ 0 & -1 & 1 \\ 1 & 1 & a \end{pmatrix}$ be invertibe? What is the inverse of the matrix when it is invertible?

10. For what values of a and b will the linear system of equations

$$2x + 4y = -2 \qquad -y + z = 1 \qquad x + y + az = b$$

have 0, 1 or infinitely many solutions?

11. $2^{67} \pmod{71} = ?$

12. What are the last two digits of $3^{3^{3^2}}$ written in base 8?

13. $\varphi(10!) = ?$

14. Solve the congruence $3x^7 \equiv 1 \pmod{26}$ and check that what you got is indeed a solution!

15. Solve the following system of congruences and check that what you got is indeed a solution!

$$x \equiv 2 \pmod{3} \qquad x \equiv 8 \pmod{9} \qquad x \equiv -4 \pmod{1}1$$

16. Is the polynomial $x^4 + x^3 + x + 2$ divisible by $x^2 + 1$ in $\mathbb{Z}_3[x]$?

17. Are the polynomials $x^3 - 2x^2 + x - 1$ and $x^2 + 1$ relative primes in $\mathbb{Q}[x]$?

18. Plot the function arctg on the interval $[-5, 5]$. Then plot its first few derivatives. Finally, put all these plots in one figure (make sure that they're drawn with different colours.)