Topics for the exam on Random Fractals

1. **Basics of iterated function systems:** contractions, IFS, attractor, Hutchinson's theorem on the existence and uniqueness,
2. **Basics of fractal geometry:** (upper and lower) box-counting dimension, basic properties, equivalent definitions, Hausdorff measure and Hausdorff dimension, basic properties, mass distribution principle, Frostman's lemma
3. **Self-similar sets**: open set condition, strong separation condition, Hutchinson's theorem, symbolic space, upper bound on the box-counting dimension,
4. **Fractal percolation I**: probabilistic tools: probability generating function, Galton-Watson process, probability of extinction; construction of the fractal percolation, probability of being empty
5. **Fractal percolation II**: probabilistic tools: Borel-Cantelli lemma; Falconer-Mauldin-Williams theorem on the dimension given non-extinction: almost sure upper bound on the box-counting dimension, "adapted Hausdorff measure", dimension of intersecting set
6. **Fractal percolation III**: probabilistic tools: subtrees of Galton-Watson processes, Dekking-Pakes Theorem; Chayes-Chayes-Durrett theorem,
7. **Geometric measure theory**: Martstand's projection theorem: dimension and Lebesgue measure, transversality lemma for orthogonal projection,
8. **Fractal percolation IV**: Probabilistic tool: Azuma-Höffding inequality; Falconer-Grimmett theorem, proof of the existence of interior point, Rams-Simon theorem (only stating),
9. **Statistically self-similar sets I**: definition of statistically self-similar set, Jordan-Pollicott-Simon theorem, transversality lemma for random translations, Dekking-Simon-Székely-Szekeres theorem (only stating),
10. **Statistically self-similar sets II:** Probabilistic tool: Doob's L^2-martingale convergence theorem (only stating); definition of random Cantor set, Falconer-Mauldin-Williams theorem on the dimension, almost sure upper bound, construction of the random mass distribution, Koivusalo theorem (only stating)
11. **Brownian motion:** definition of the Brownian motion, Hölder property of the Brownian motion, Taylor's theorem on the dimension of the graph and the image.