

Name:.....

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Sample test for 2nd Midterm test,

1. (25 points) Let us consider the following infinite vibrating string equation:

$$\begin{cases} u''_{tt}(x, t) = 4u''_{xx}(x, t) & x \in \mathbb{R}, t \geq 0; \\ u(x, 0) = 0 & x \in \mathbb{R} \\ u'_t(x, 0) = \begin{cases} 3 & 0 \leq x \leq 10, \\ 0 & \text{otherwise.} \end{cases} \end{cases}$$

Determine  $u(-1, 2) = ?$

2. (25 points) Consider the following planar vector field  $\vec{F}(x, y) = (-5(x^2 + y^2)y, 5(x^2 + y^2)x)$ .

(a) Using the planar Curl-test, determine whether the vector field  $\vec{F}$  is conservative or not!  
If it is then find its potential function!

(b) Let  $\gamma$  be the curve parametrized by  $\mathbf{r}(t) = (2 \cos(t), 2 \sin(t))$ ,  $0 \leq t \leq 2\pi$ . What is  $\int_{\gamma} \vec{F}(\mathbf{r}) d\mathbf{r} = ?$

3. (25 points) Let  $\mathcal{F}$  be surface of the ball with radius one and centered at the origin with orientation point outwards. Furthermore, let  $\vec{G}(x, y, z) = (x^3 + e^{z^2+y^2}, e^x z + y^3, \cos(x) \sin(y) + z^3 + xy)$ . Using Gauss' theorem, find  $\iint_{\mathcal{F}} \vec{G} d\vec{A} = ?$