1) $\begin{cases} u''_{tt}(0,t) = 4u''_{xx}(0,t) \\ u(x,0) = 0 \\ u'_t(x,0) = \begin{cases} 3 & 0 \le x \le 10 \\ 0 & \text{otherwise} \end{cases} \end{cases}$

$u(-1,2) = ?$

$\Rightarrow c = 2$

$$\Rightarrow u(x,t) = \frac{f(x+ct)+f(x-ct)}{2} + \frac{1}{2c}\int_{x-ct}^{x+ct} g(\tau)\,d\tau$$

$\Rightarrow f \equiv 0$

$$\Rightarrow u(-1,2) = \frac{1}{4}\int_{-5}^{3} g(\tau)\,d\tau = \frac{1}{4}\int_{0}^{3} 3\,d\tau = \frac{9}{4}.$$

2/ a/ $\vec{F}(x,y) = \left(-5(x^2+y^2)y,\ 5(x^2+y^2)x\right)$

$$\operatorname{curl}(\vec{F}) = \left(5(x^2+y^2)x\right)'_x - \left(-5(x^2+y^2)y\right)'_y$$

$$= 10x^2 + 5(x^2+y^2) - \left(-10y^2 - 5(x^2+y^2)\right)$$

$$= 15x^2 + 5y^2 + 5x^2 + 15y^2 = 20x^2 + 20y^2 \neq 0$$

$\Rightarrow$ not conservative!

b) $\vec{F}(\underline{r}(t)) =$

$\left(-5(4\cos^2(t)+4\sin^2(t))\,2\sin(t),\ 5(4\cos^2(t)+4\sin^2(t))\,2\cos(t)\right)$

$= (-40\sin(t),\ 40\cos(t))$

$\dot{\underline{r}}(t) = (-2\sin(t),\ 2\cos(t))$

$\vec{F}(\underline{r}(t)) \cdot \dot{\underline{r}}(t) = 80\sin^2(t) + 80\cos^2(t) = 80$

$$\int_\gamma \vec{F}(\underline{r})\, d\underline{r} = \int_0^{2\pi} 80\, dt = \underline{\underline{160\pi}}$$

3) $\operatorname{div} \vec{G} = 3x^2 + 3y^2 + 3z^2$ $\quad \mathcal{F} :=$ surface of the unit sphere

! $B :=$ unit ball

$$\iint_{\mathcal{F}} \vec{G}\, d\vec{A} = \iiint_B \operatorname{div}(\vec{G})\, dx\, dy\, dz =$$

$$= \int_0^1 \int_0^{2\pi} \int_0^{\pi} 3(r\sin(u)\cos(v))^2 + 3(r\sin(u)\sin(v))^2 + 3(r\cos(u))^2\, du\, dv\, dr$$

$$= \int_0^1 \int_0^{2\pi} \int_0^{\pi} 3r^2 \, du \, dv \, dr = 6\pi^2 \int_0^1 r^2 dr =$$

$$= 6\pi^2 \left[\frac{r^3}{3} \right]_0^1 = \underline{\underline{2\pi^2}}$$