Name: ..... Neptun code: ..... Tutor: .....

Probability Theory 1 midterm, 13th October 2025. 17:15–18:00

Working time: 45 minutes. Only simple scientific, non-programmable calculators are allowed.
Maximum score (with the bonus exercise): 24 points, but we consider 20 points already as 100%.

Group A

1. In an urn, there are 20 balls of 4 colours, red, blue, yellow and white. (So, 5 balls of each colour.) We pick 5 balls randomly without replacement.

   (a) (2 points) What is the probability that we pick exactly 2 yellow balls?

   (b) (3 points) What is the probability that there will at least two different colours within the selected balls?

   (c) (5 points) What is the probability that we pick at least one ball of each colour?

2. I go fishing every weekend. I have two favourite lakes I like to go, Huron and Erie. I observed that I didn't catch any fish in roughly half of the cases when I went to Huron. Meanwhile I didn't catch any fish in roughly $1/3$ of the cases when I chose Erie. Last weekend I went to Erie. Today I choose the lake by tossing an unbiased coin.

   (a) (2 points) What is the distribution of the number of fish that I caught last weekend? Why?

   (b) (3 points) What is the probability that I will catch at least two fish today?

   (c) (3 points) I caught exactly two fish today. What is the probability that I went to Erie?

   (d) (2 points) My friend accompanied me today. I caught exactly two fish today. What is the probability that he catches at least one? (He is as good of a fisherman as me, and number of fish that we catch are independent (once the lake is decided).)

Bonus: (4 points) Let $X \sim \mathrm{BIN}(n,p)$. Give a closed explicit closed formula for $\mathbb{E}\left(\frac{1}{X+1}\right)$.

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Group B

1. In an urn, there are 20 balls of 4 colours, red, blue, yellow and white. (So, 5 balls of each colour.) We pick 5 balls randomly with replacement.

   (a) (2 points) What is the probability that we pick exactly 2 yellow balls?

   (b) (3 points) What is the probability that within the selected balls there will be at least two different colours?

   (c) (5 points) What is the probability that we pick at least one ball of each colour?

2. In the students canteen, two chefs are cooking the beef stew for the students, Gordon and Jamie. Jamie cooks on 3 days of a week, while Gordon cooks on the remaining two days. (No canteen at weekends.) Jamie is mingy, he puts so few meat in the stew that the average number of pieces of meat on a plate is 1 when he cooks, while Gordon is more generous and the average number of meat pieces per plate is 3 when he cooks. Last week I went to the canteen on a day when Gordon was the chef. This week I go to the canteen on a random day together with a friend of mine.

   (a) (2 points) What was the distribution of the number of meat pieces on my plate last week? Why?

   (b) (3 points) What is the probability that there will be at least one piece of meat on my plate this week?

   (c) (3 points) I found three pieces of meat in my plate this week. What is the probability that Gordon was the chef on that day?

   (d) (2 points) This week I found three pieces of meat in my plate. What is the probability that my friend will get at least two pieces of meat on his plate? (The number of pieces in the plates are independent, once the chef is revealed.)

Bonus: (4 points) Let $X \sim \mathrm{POI}(\lambda)$. Give a closed explicit formula for $\mathbb{E}\left(\frac{e^X}{X+1}\right)$.