Name:..... Neptun code:.....

2nd Midterm test, 12th December 2025, 16:10-16:55

1. *(25 points)* Find the solution $u(x,t)$ of the following vibrating string problem:

$$\begin{cases} u''_{tt}(x,t) = 4u''_{xx}(x,t) & x \in [0,3], t \geq 0 \\ u(x,0) = 0 & x \in [0,3] \\ u'_t(x,0) = \sin(\frac{2\pi}{3}x)\cos(\frac{\pi}{3}x) + \sin(2\pi x) & x \in [0,3] \\ u(0,t) = u(3,t) = 0 & t \geq 0 \end{cases}$$

2. *(25 points)* Consider the vector field

$$\overrightarrow{F}(\mathbf{r}) = \overrightarrow{F}(x,y,z) = (y^3z + 3, 3y^2xz, xy^3 - 2z).$$

(a) Apply the Curl-test to decide if $\overrightarrow{F}(\mathbf{r})$ is a potential vector field. If it is, then determine the potential function.
(b) Let γ denote the straight line segment starting at $C = (0,1,1)$ and terminating at $D = (1,0,2)$. Compute the line integral $\int\limits_{\gamma} \overrightarrow{F}\, d\mathbf{r}$.

3. *(25 pont)* Let $\mathcal{F}$ denote the triangular domain ABC, with upward pointing normal vector, where $A(2,1,-1)$; $B(5,1,3)$; and $C(2,1,3)$. Consider the constant vector field $\overrightarrow{F} = (-1,5,2)$. Compute the surface integral $\iint\limits_{\mathcal{F}} \overrightarrow{F}\, d\overrightarrow{A}$.