1) a/
$$\begin{cases} u''_{tt}(x,t) = 4\,u''_{xx}(x,t) \\ u(0,t) = u(3,t) = 0 \\ u'_t(x,0) = \sin\left(\frac{2\pi}{3}x\right)\cos\left(\frac{\pi}{3}x\right) + \sin(2\pi x) \\ u(x,0) = 0 \end{cases}$$

$\Rightarrow$ $c = 2$

with Bernoulli; length $L = 3$

$\Rightarrow$ general solution

$$u(x,t) = \sum_{k=0}^{\infty} \left(A_k \cos\left(\frac{k\pi}{3} 2t\right) + B_k \sin\left(\frac{k\pi}{3} 2t\right)\right) \sin\left(\frac{k\pi}{3}x\right)$$

Since

$$0 = u(x,0) = \sum_{k=0}^{\infty} A_k \sin\left(\frac{k\pi}{3}x\right) \Rightarrow A_k = 0 \quad \forall k = 1,2,\dots$$

$$u'_t(x,0) = \sin\left(\frac{2\pi}{3}x\right)\cdot\cos\left(\frac{\pi}{3}x\right) + \sin(2\pi x) =$$

$$= \frac{1}{2}\sin\left(\frac{\pi}{3}x\right) + \frac{1}{2}\sin(\pi x) + \sin(2\pi x)$$

$$u'_t(x,0) = \sum_{k=0}^{\infty} B_k \frac{k\pi}{3} 2 \sin\left(\frac{k\pi}{3}x\right)$$

$$\Rightarrow \quad B_1 = \frac{1}{2}\cdot\frac{3}{2\pi} = \frac{3}{4\pi} \;\&\; B_3 = \frac{1}{2}\cdot\frac{3}{3\cdot\pi\cdot 2} = \frac{1}{4\pi}$$

$$B_6 = 1\cdot\frac{3}{6\cdot\pi 2} = \frac{1}{4\pi} \;\&\; B_k = 0 \quad \forall k \neq 1,3,6$$

$\Rightarrow$

$$u(x,t) = \frac{3}{4\pi}\sin\left(\frac{\pi}{3}2t\right)\sin\left(\frac{\pi}{3}x\right) + \frac{1}{4\pi}\sin(\pi 2t)\sin(\pi x) + \frac{1}{4\pi}\sin(4\pi t)\sin(2\pi x)$$

2) a) $\vec{F}(x,y,z) = (y^3z+3,\ 3y^2xz,\ xy^3-2z)$

$$\text{curl}(\vec{F}) = \det\begin{pmatrix} \underline{i} & \underline{j} & \underline{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y^3z+3 & 3y^2xz & xy^3-2z \end{pmatrix} =$$

$= 3y^2x\,\underline{i} + y^3\,\underline{j} + 3y^2z\,\underline{k} - (3y^2x\,\underline{i} + y^3\,\underline{j} + 3y^2z\,\underline{k})$

$= \underline{0}$. Since $\vec{F}$ & its partial derivatives are well defined & cont everywhere $\Rightarrow$ $\vec{F}$ is conservative

$$\left.\begin{aligned} f'_x &= y^3z+3 \\ f'_y &= 3y^2xz \\ f'_z &= xy^3-2z \end{aligned}\right\} \Rightarrow \begin{aligned} f &= \underline{xy^3z} + \underline{3x} + c_1(y,z) \\ f &= \cancel{y^3xz} + c_2(x,z) \\ f &= \cancel{xy^3z} - \underline{z^2} + c_3(x,y) \end{aligned}$$

$\Rightarrow f = \underline{xy^3z} + \underline{3x} - \underline{z^2} + \text{const}$

b) $\gamma: C = (0,1,1) \to D = (1,0,2)$

$$\int_\gamma \vec{F}\,d\underline{r} = f(D) - f(C) = 1\cdot 0^3\cdot 2 + 3\cdot 1 - 2^2 - (0\cdot 1^3\cdot 1 + 3\cdot 0 - 1^2)$$
$$= 0$$

3) $\mathcal{F}: \Delta\ A(2,1,-1),\ B(5,1,3),\ C(2,1,3)$

$\vec{CA} = (0,0,-4)$ & $\vec{CB} = (3,0,0)$

$$\vec{CA} \times \vec{CB} = \det\begin{pmatrix} \underline{i} & \underline{j} & \underline{k} \\ 0 & 0 & -4 \\ 3 & 0 & 0 \end{pmatrix} = -12\underline{j}$$

$\Rightarrow$ Area$(\mathcal{F}) = \frac{12}{2} = \underline{\underline{6}}$ & normal vector $(0,1,0)$

$\Rightarrow \langle \vec{F}, \underline{n} \rangle = -1 \cdot 0 + 5 \cdot 1 + 2 \cdot 0 = \underline{\underline{5}}$

$$\Rightarrow \iint_{\mathcal{F}} \vec{F}\, d\vec{A} = \langle \vec{F}, \underline{n} \rangle \cdot \text{Area}(\mathcal{F}) = 5 \cdot 6 = \underline{\underline{30}}$$