Establishing and maintaining databases of self-affine tiles

Christoph Bandt (Greifswald, Germany), Dmitry Mekhontsev (Novosibirsk, Russia)

Self-affine graph-directed constructions (graph-IFS for short) include Rauzy-type tiles obtained from substitutions as well as Penrose-type tiles obtained from cut-and-project schemes. A projection approach based on mappings was implemented on computer by the second author to find large lists of new examples. Sometimes, especially in the fractal case, such lists become so extensive that careful visual inspection is impossible. Thus the computer must eliminate equivalent datasets and determine properties of the examples which can be used to select the most interesting specimen. We describe algorithms for the search as well as for the management of the database. The package is available at `https://ifstile.com`.

Our setup is a graph-IFS given by an expanding integer matrix M and integer translations in high-dimensional space. Tiles, or fractal attractors, are studied in a two-dimensional invariant subspace of M. Moreover, there can be a discrete symmetry group S which commutes with M, the simplest symmetry being $s^{-}(x) = -x$. Our figure shows a few modifications which are obtained from the Rauzy substitution matrix M with appropriate choice of translations and adding s^{-} to some of the contraction maps.

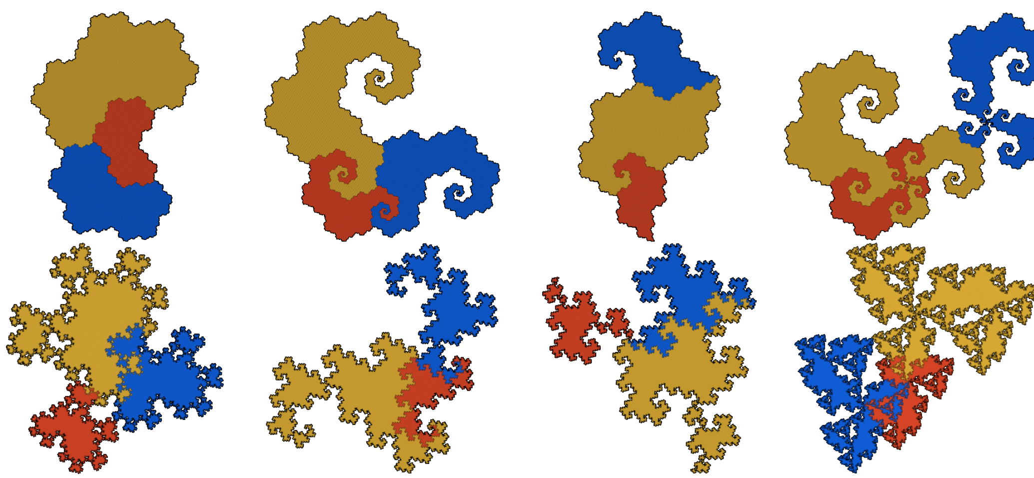

Figure 1: Simple modifications of the Rauzy tile.

The search for new examples is performed by a random walk on the parameter space of integer translations and discrete symmetries, while the given M and graph structure of the IFS remain fixed. The main algorithmic ingredient is a check for the open set condition of the graph-IFS which corresponds to the Arnoux-Ito coincidence property in the substitution approach. In contrast to the algorithms described in the monograph of Siegel and Thuswaldner (2010), we use only one automaton which we call the neighbor graph of the IFS. It describes in a canonical way the dynamical boundary of the given graph-IFS as a new graph-IFS.

To eliminate equivalent datasets from our list, we calculate various properties for each new dataset. Parameters of the neighbor graph, like number of vertices and edges, can be taken as invariants. The boundary dimension(s) and certain moments of the equidistribution on the tiles form other invariants. Topological properties can be calculated from the neighbor graph. The list of data sets can be sorted with respect to any property, and examples with desired properties can be selected.

In the talk, mathematical background will be given, and experiments with Rauzy-type fractals will be demonstrated.