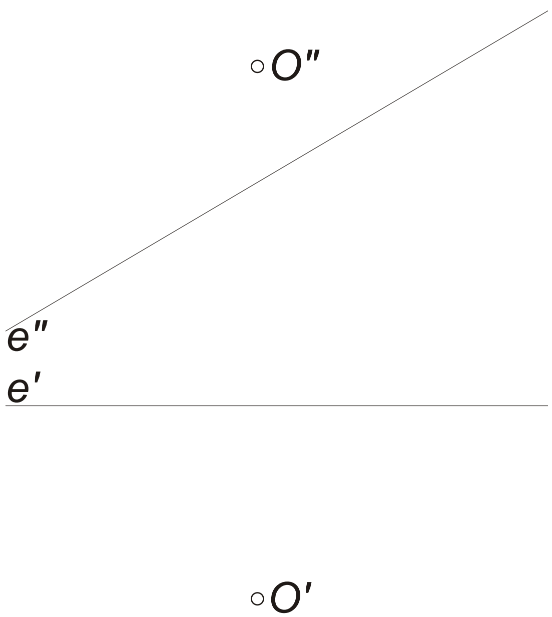
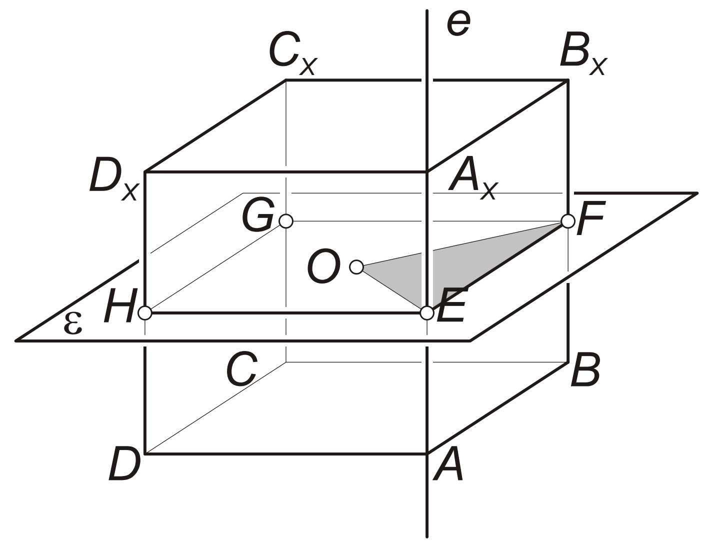
Transformation of a plane of projection

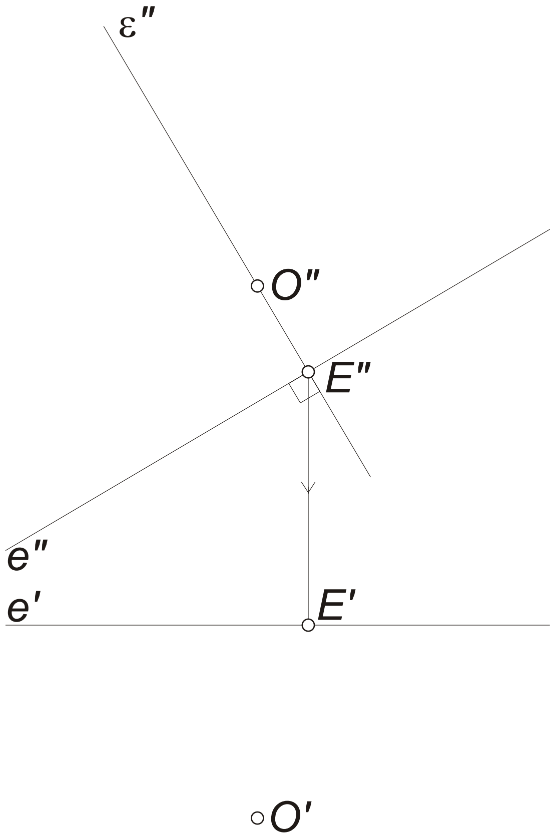
Construction of a cube given with its centre and a sideline



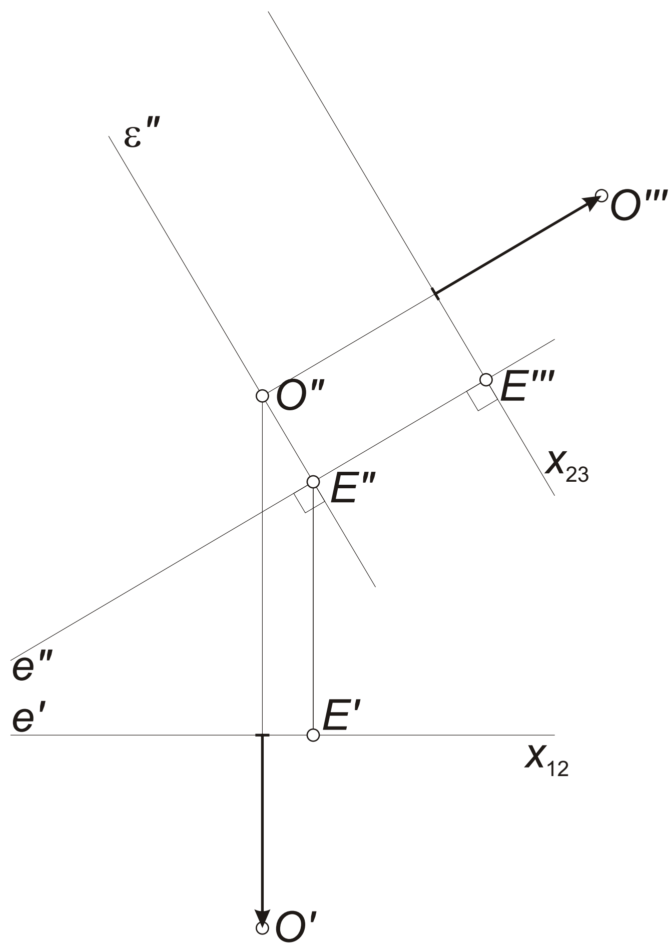
Exercise. *Given the center O and a sideline e of a cube, where e is a vertical line. Construct the projections of the cube, and show its visibility under the condition that the cube is a solid.*



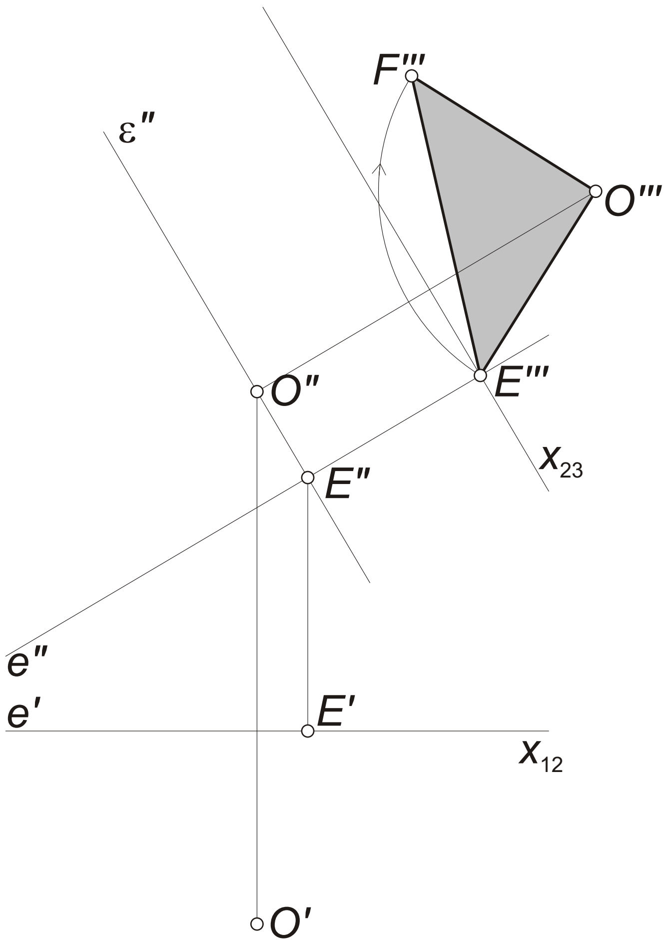
The plane ε, containing O and perpendicular to the line e, intersects the cube in the square $EFGH$, the edge-length of which is equal to that of the cube. Using O and e, we construct this square by transforming the plane ε into a parallel plane. To obtain the vertices of the cube, we draw lines through E, F, G and H, parallel to e, and measure half the edge-length of the cube on each in both directions.



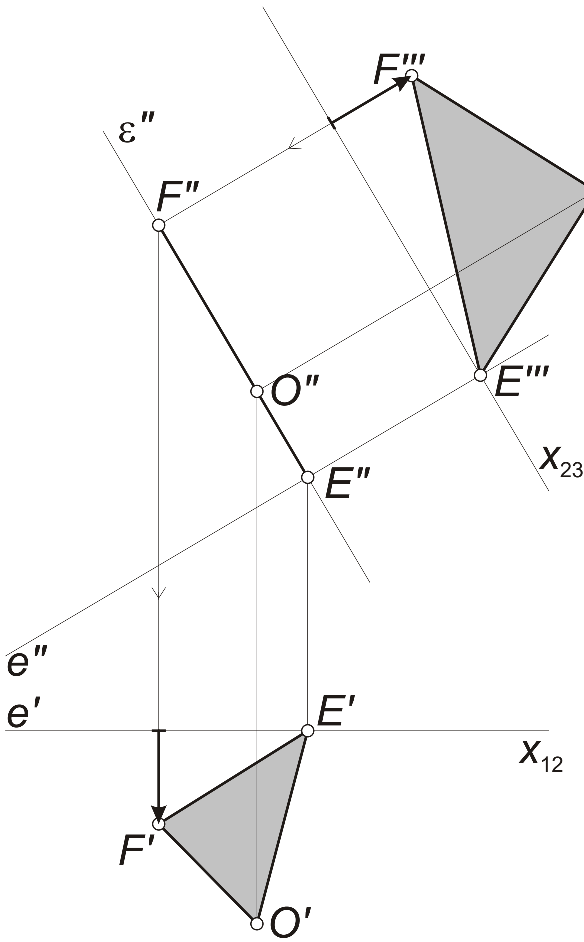
We draw the plane ε, containing the square $EFGH$ and perpendicular to the line e. Since e is a vertical line, ε is a vertical projecting plane: $e'' \perp \varepsilon''$. There common point is the vertex E of the square $EFGH$, the horizontal projection of which can be obtained as the intersection of its line of recall with e'.



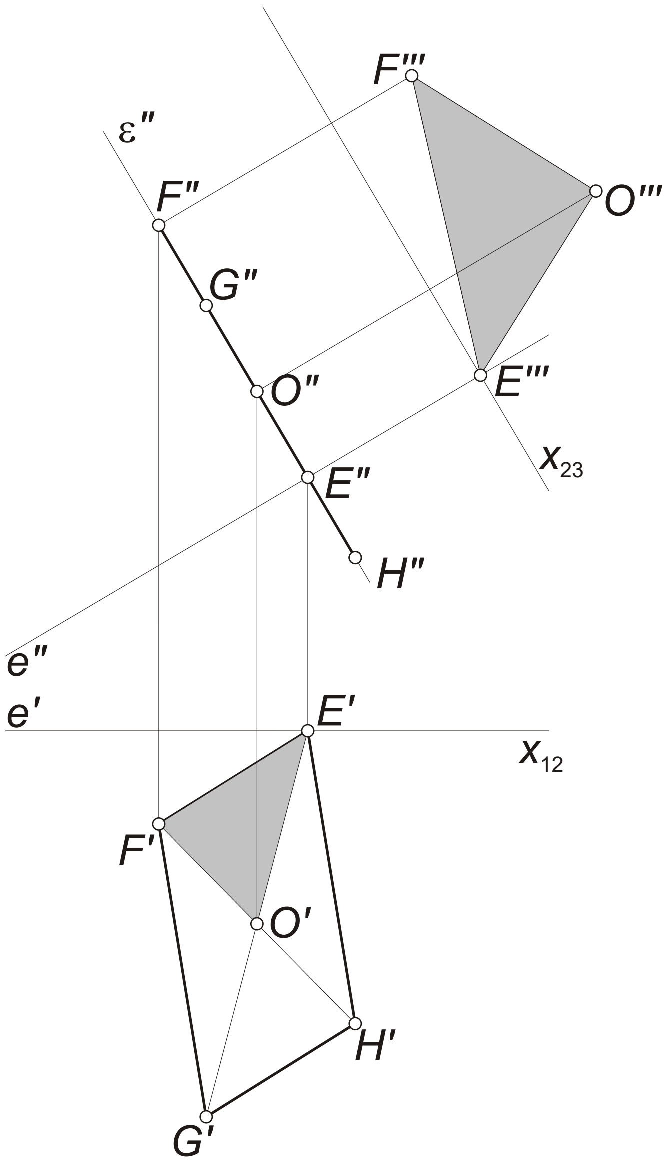
By introducing a third plane of projection, perpendicular to the vertical projection plane, we transform the plane ε into a parallel plane. To do this, we choose the x_{23} axis parallel to ε''. We fix the axis x_{12} of the original system as well, for example in a way that it coincides with e'. Then the (signed) distance of the point E from this axis is zero, and thus, its (new) third projection is on x_{23}. To transform the point O, we measure off the distance of O' from x_{12}, and construct the point O''' at the same distance from x_{13}.



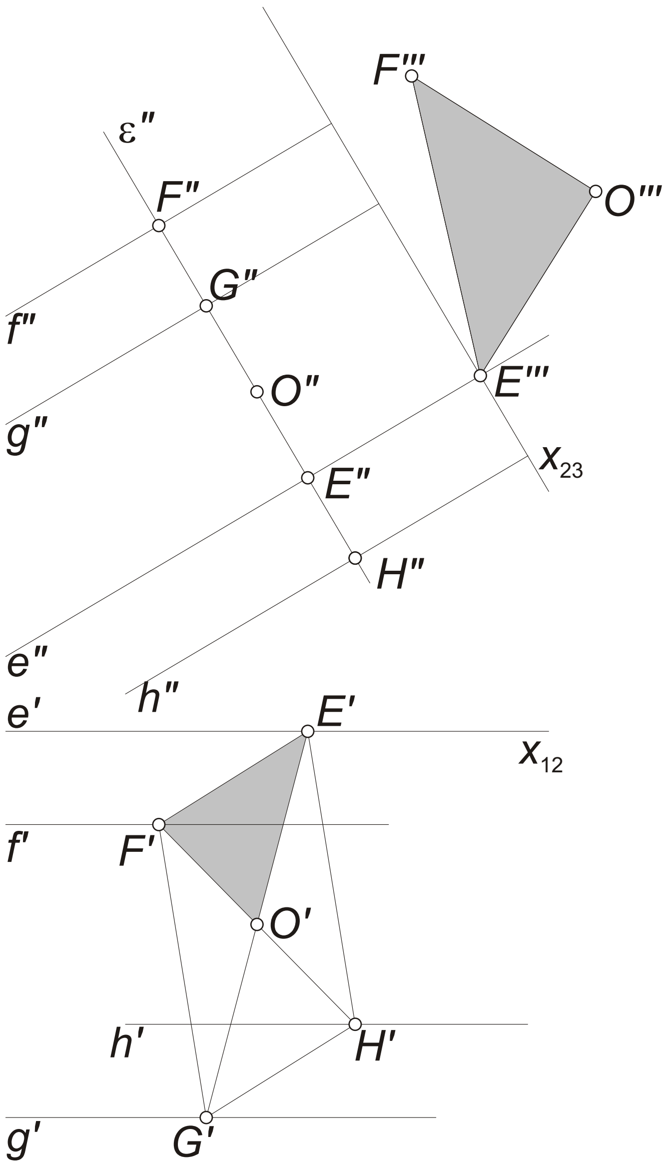
Now the third plane of projection is parallel to the plane ε containing the section $EFGH$, and thus, its third projection can be constructed directly. By rotating the half diagonal OE of the square by 90° about O, we obtain the hald diagonal OF. which yields a quarter of the square: the isosceles right triangle OEF. We are looking for the horizontal and vertical projections of this triangle; from these, using its central symmetry to O, we can obtain the first two projections of the square as well.



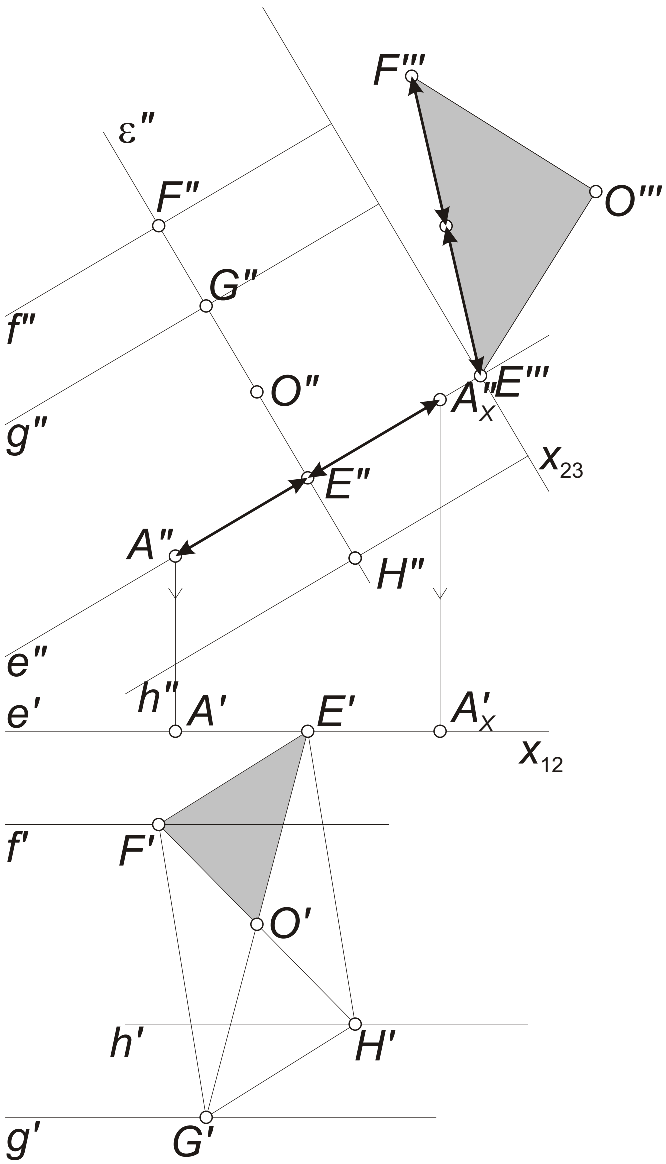
The vertical projection of the vertex F of OEF is obtained as the intersection of ε'' and its line of recall in the $(2,3)$ system. Then, we can construct its horizontal projection by transforming it back; that is, by measuring the distance of F''' and x_{23} on the line of recall of F'' in the $(1,2)$ system, from $x_{1,2}$.
After constructing the projections of the vertices of the triangle, we can construct the triangle itself, we obtain a general triangle in the horizontal projection, and a segment, $F''E''$, in the vertical one.



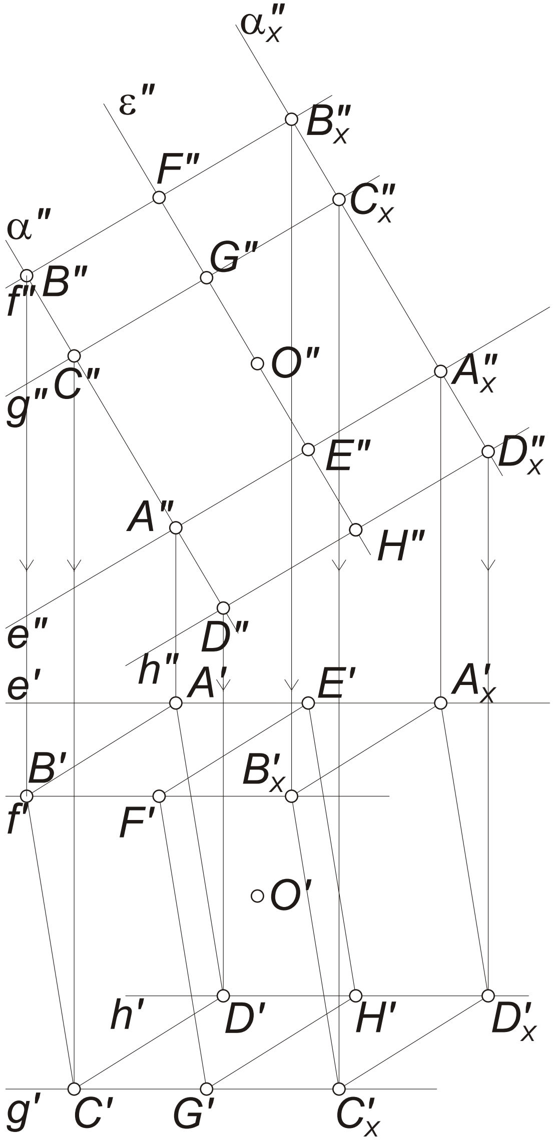
In both projections (even independently of each other), we can carry out the reflections of E and F about O, and thus constructing the missing vertices of the section of the cube in ε. Finally, we can draw the horizontal projection of the square, which will be a parallelogram, and the vertical one, which will lie on a straight line segment, $E''F''$.



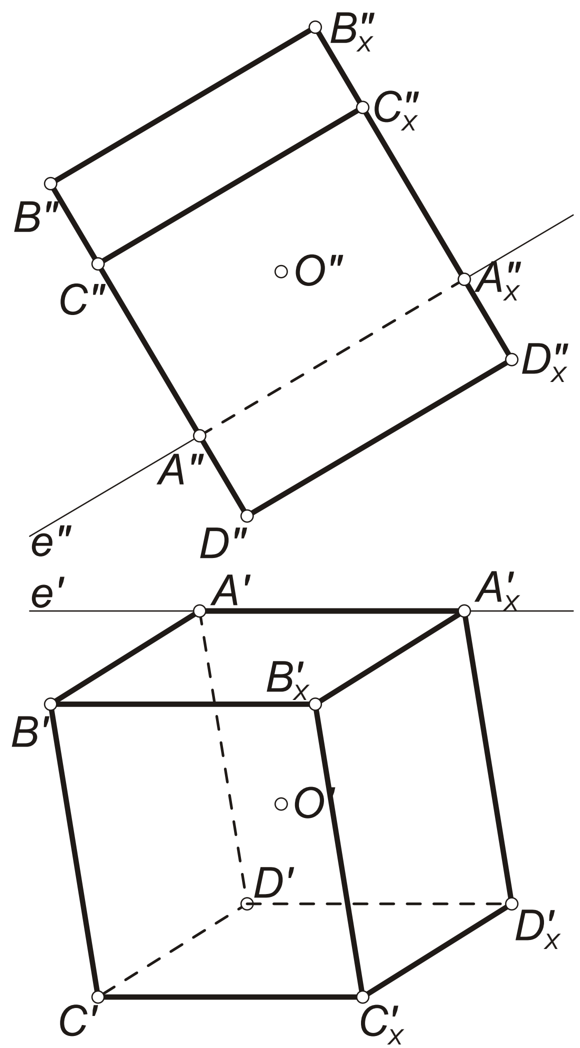
We draw the sidelines f, g and h of the cube, parallel to e.



In the third projection, we can see the segment EF in its real size, coinciding with the edge-length of the cube. On the othe hand, since e is a vertical line, in the vertical projection we can see the edge AA_x of the cube on the line e in its real size. Since the midpont of AA_x is E, in the vertical projection we need to measure half of the length of $E'''F'''$ from E'' in both directions. These points will be the vertical projections of A and A_x. The horizontal projections of these points can be obtained as the intersections of their lines of recall with e'.



The planes α and α_x, containing A and A_x, respectively, and parallel to ε, intersect the lines f, g and h at the remaining vertices of the cube. Drawing the vertical projection of α yields B'', C'' and D'' on f, g and h, whereas α''_x yields B''_x, C''_x and D''_x. The horizontal projections of these points are determined as the intersection points of their lines of recall, and f', g' and h'. Finally, connecting the vertices we obtain the horizontal projections of the faces $ABCD$ and $A_xB_xC_xD_x$.



When examining visibility, we can see that the points of CC_x are farthest from the axis in the horizontal projection, and thus, they will be visible on the vertical projection (front view). For AA_x, the converse is true, its points are closest to the axis, and hence we draw it with a dashed line to represent that it is invisible.

In the vertical projection, we can see that B_x'' is at the highest position, which means that this point, and the edges emanating from it, will be visible in the horizontal projection. On the other hand, D is on the lowest level, and thus this point and the edges starting here will not be visible in the horizontal projection (top view).