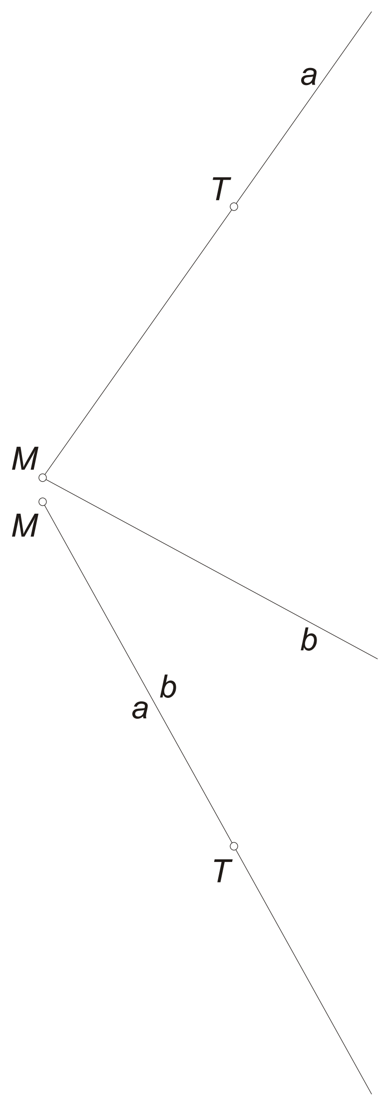
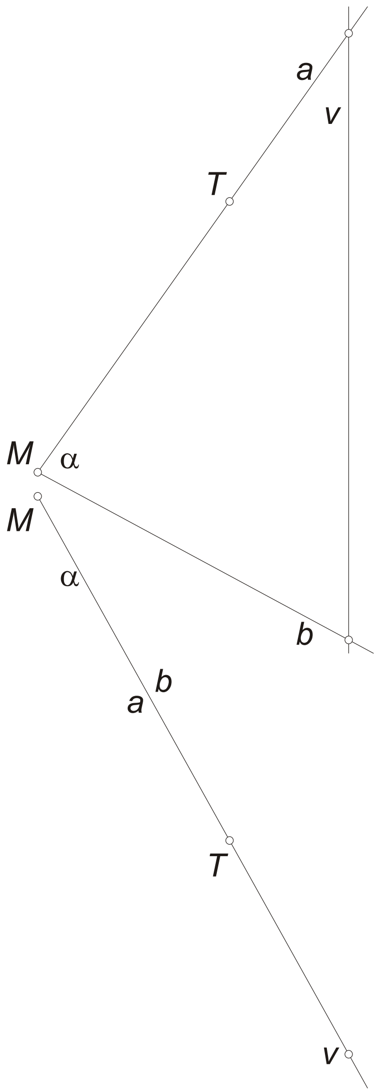
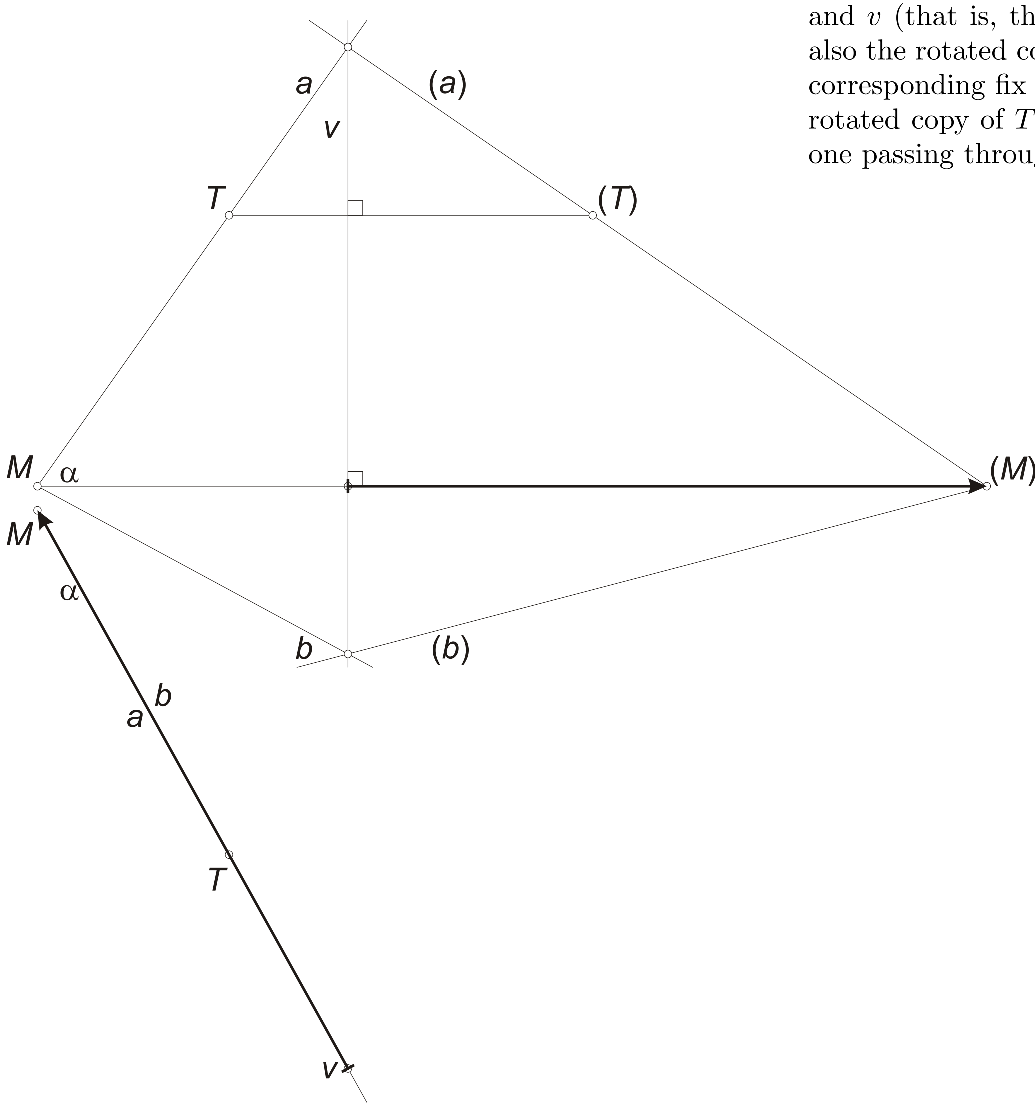
Projections of a circle in a projecting plane

Exercise. *Given two half lines a and b, emanating from the point M, and a point T contained in a, draw the circle which is tangent to a at T, and is also tangent to b.*

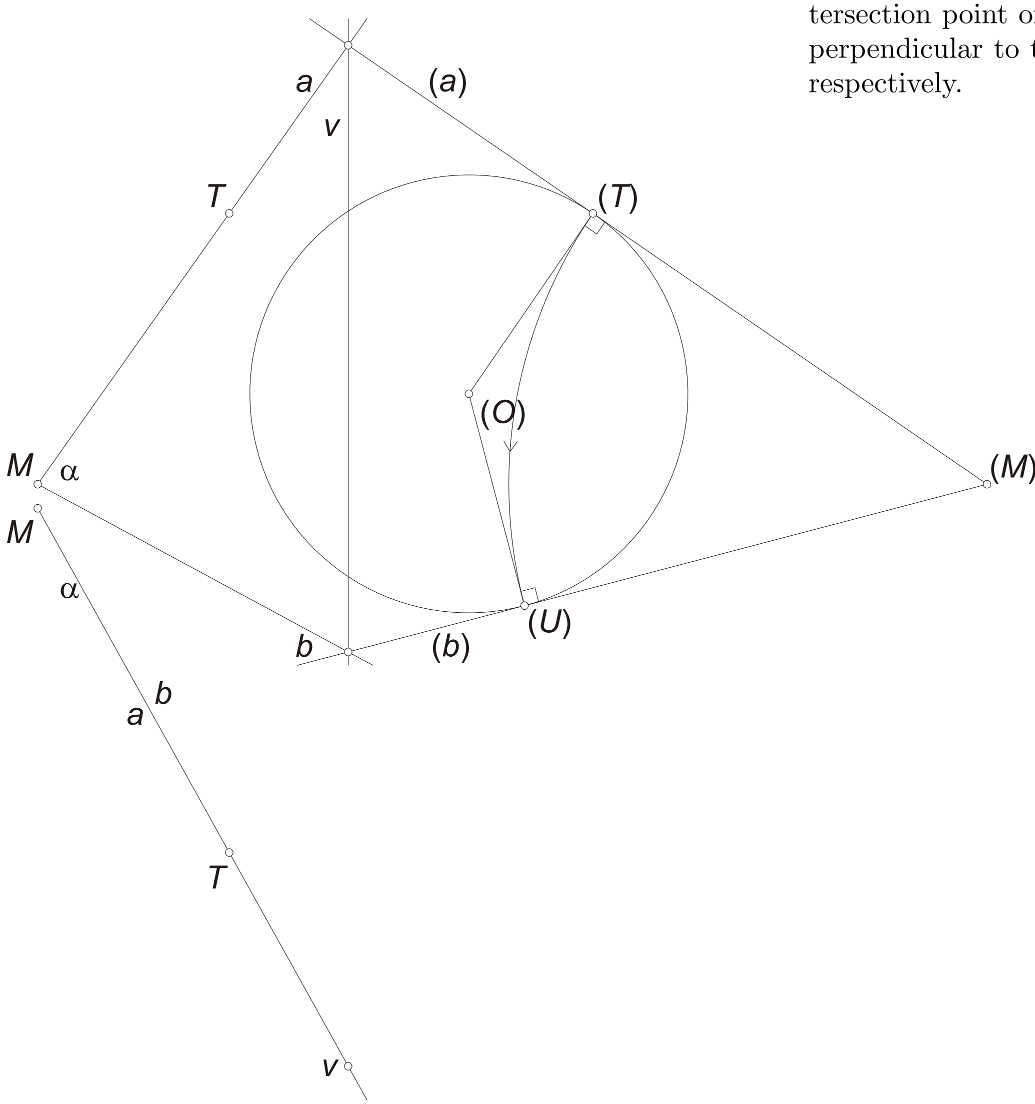




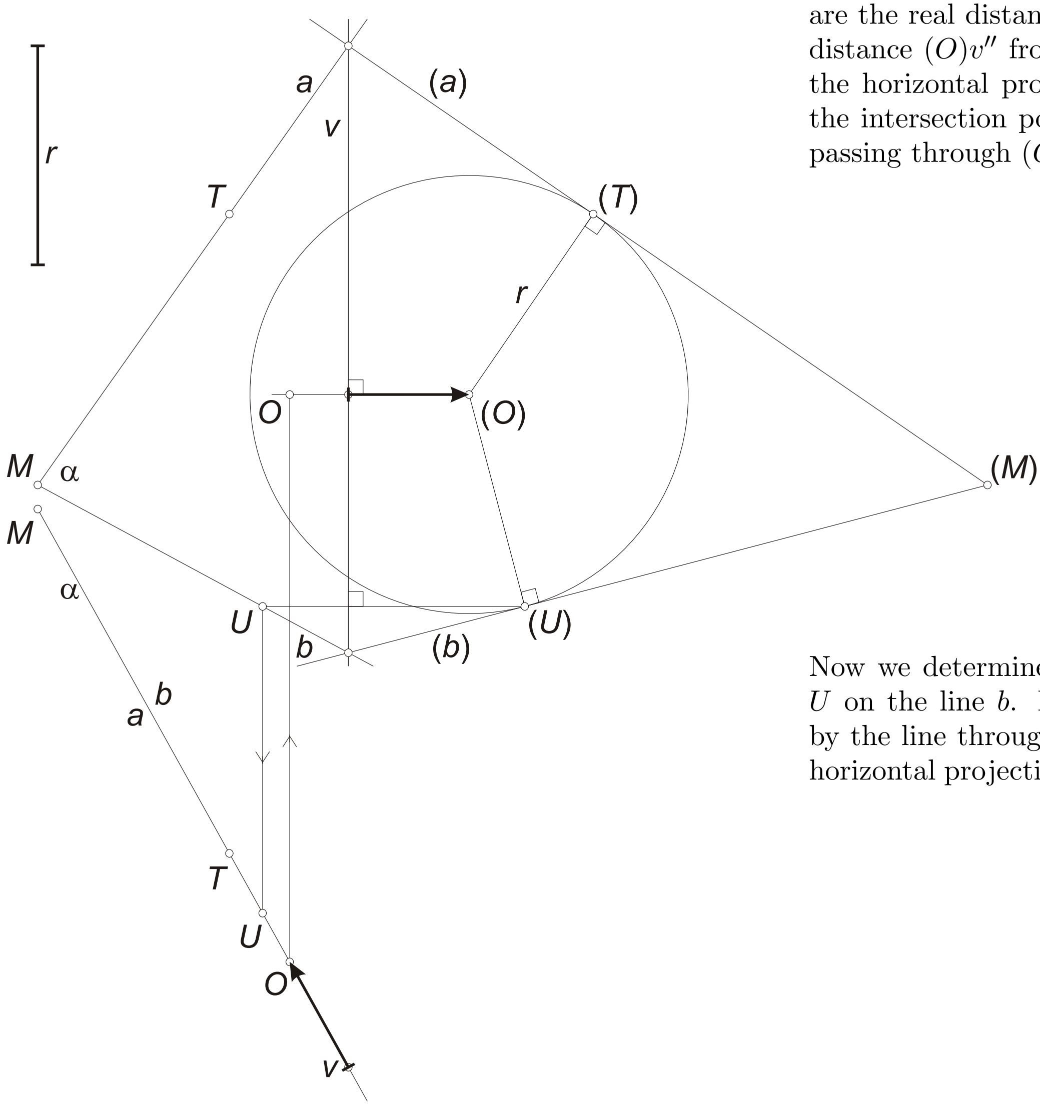
Since the horizontal projections of the half lines a and b are on the same line, the plane of the circle is a horizontal projecting plane α. Thus, the plane of the circle is given, and the first step is to determine its center and radius. To do this we need to rotate the plane α into a position parallel to the vertical plane of projection. For the axis of the rotation, we need a line in the plane that is parallel to the vertical plane of projection. These lines of α are the its horizontal projecting lines. For the axis, we can choose an arbitrary line v from amongst these. During the rotation we use the fact that the distance between a point of α and the axis v is shown directly in the horizontal projection.



The rotation of the point M can be done, for instance, by drawing a line passing through M and perpendicular to the axis v'', and then measuring on it from v'' the distance of M and v (that is, the distance of M' and v'). Then we obtain also the rotated copies of the lines a and b, by connecting the corresponding fix points on the axis v with (M). Finally, the rotated copy of T is the intersection of the line (a) with the one passing through T'' and perpendicular to v''.

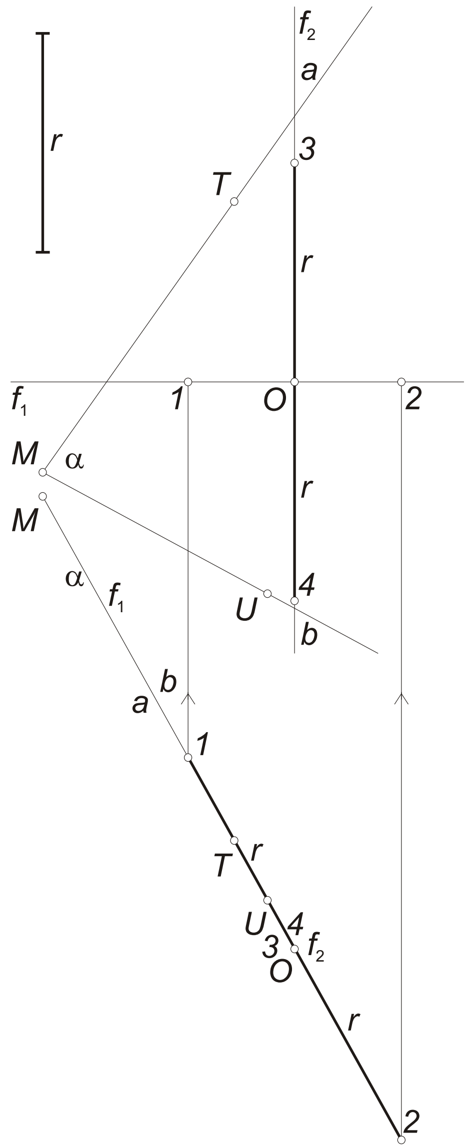


Now we construct the center of the circle in the rotated plane. To construct the tangent point (U) on b we can use the equality $(U)(M) = (T)(M)$. Then the center (O) will be the intersection point of the lines passing through (T) and U, and perpendicular to the corresponding tangent lines (a) and (b) respectively.

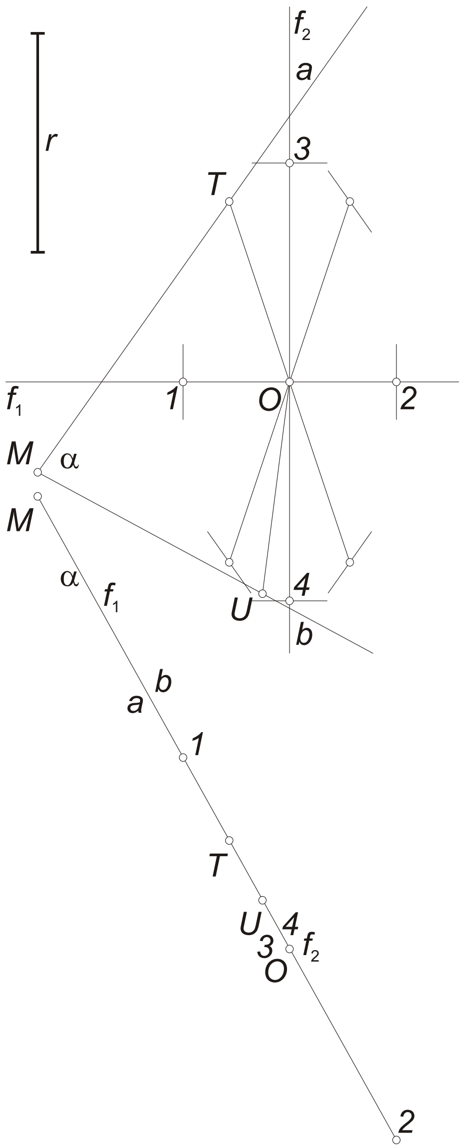


We can measure the radius r of the circle, and rotate back the center O. Since the distances measured from the axis v of the rotation in the horizontal plane of projection plane are the real distances, we can construct O' by measuring the distance $(O)v''$ from the vertical projection on the line α' in the horizontal projection, starting from the point v'. Then the intersection point of the line of recall of O' and the line passing through (O) and perpendicular to v'' is O''.

Now we determine the two projections of the tangent point U on the line b. Its vertical projection on b'' is determined by the line through (U) and perpendicular to v'', and for its horizontal projection we can use the line of recall through U''.

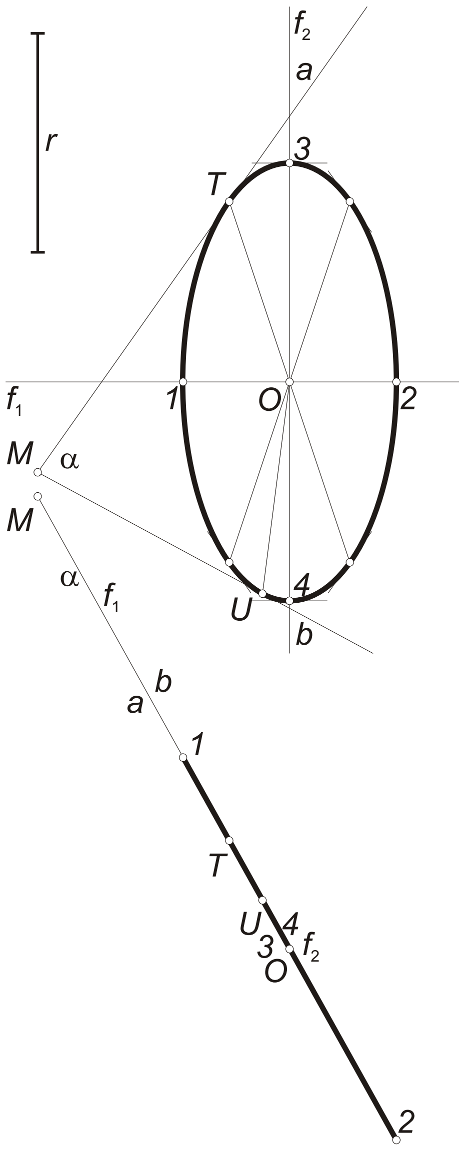


The major axes of the ellipses will lie on the parallel lines through the centre; and their lengths are equal to the diameter of circle (the real length of the diameter).
Hence we draw the horizontal line f_1 through the point O, and the vertical line f_2, which is, at the same time, a horizontal projecting line as well.
In the horizontal plane of projection, on f_1', we measure from O' in both directions the radius of the circle. We obtain the poins $1'$ and $2'$ in this way; their vertical projections can be found using their lines of recall. The points 1 and 2 are the endpoints of the major axis in the horizontal projection.
In the vertical projection, we measure on f_2'' the radius r to obtain $3''$ and $4''$, their horizontal projections coincide with the point f_2'. The points 3 and 4 are the endpoints of the major axis in the vertical projection of the circle.
The minor axes of the ellipses lie on the slope lines of the plane of the circle. Nevertheless, the horizontal and the vertical lines of a projecting plane are perpendicular, which means that a horizontal slope line is a vertical line, and a vertical slope line is a horizontal line as well. Thus, the minor axis in the horizontal plane of projection is $3'4'$ (of zero length), and that in the vertical plane of projection is $1''2''$.



At the endpoints of the axes of the vertical projection of the ellipse we draw the tangent lines.

We should construct more points of the ellipse, using, for example, the method of the two circles. Using symmetry and the known points and tangent lines, we can construct more points and tangents of the ellipse, by reflecting them about the center or the axes of symmetry. This is what we did about the point T.



Finally we connect the constructed points to get the circle.