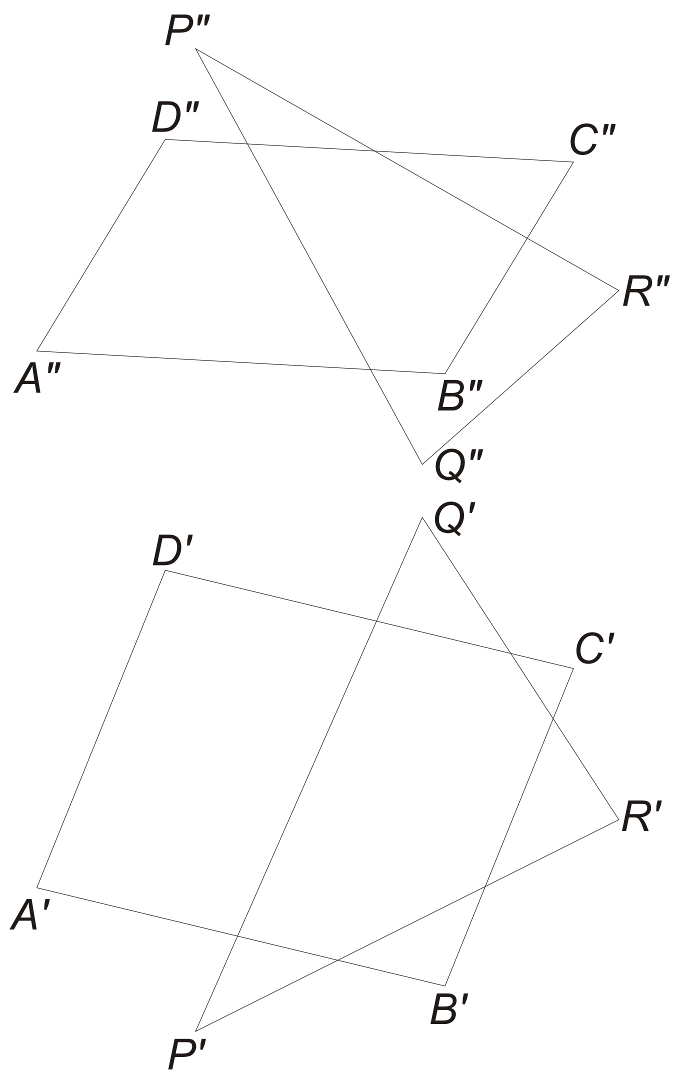
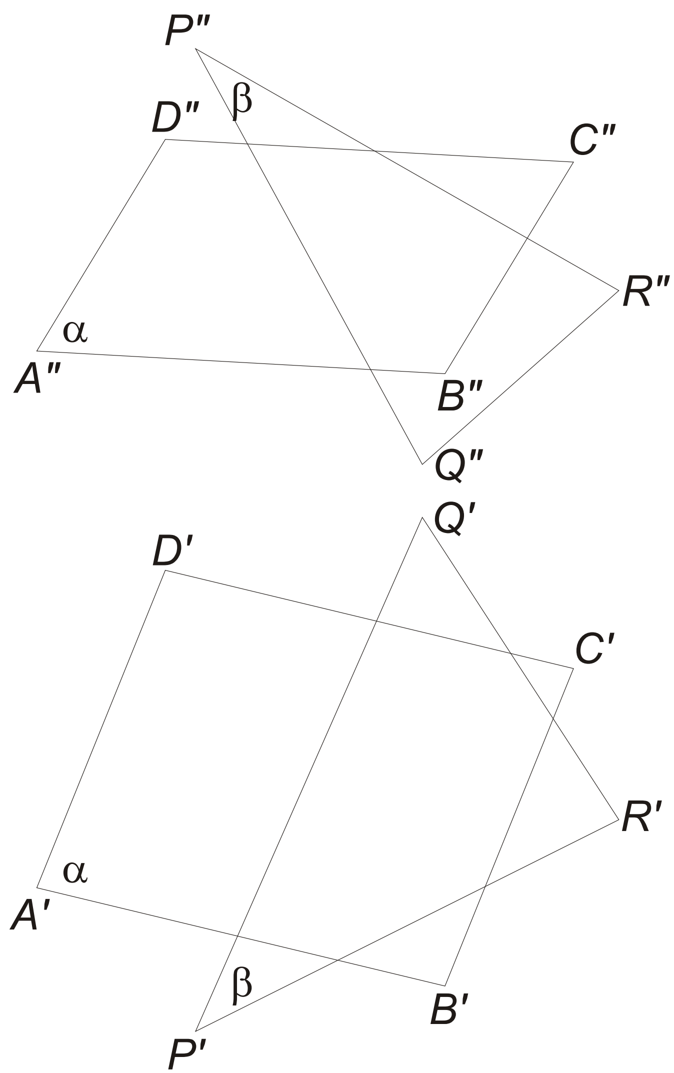
Intersection of planar figures

Intersection of a parallelogram and a triangle



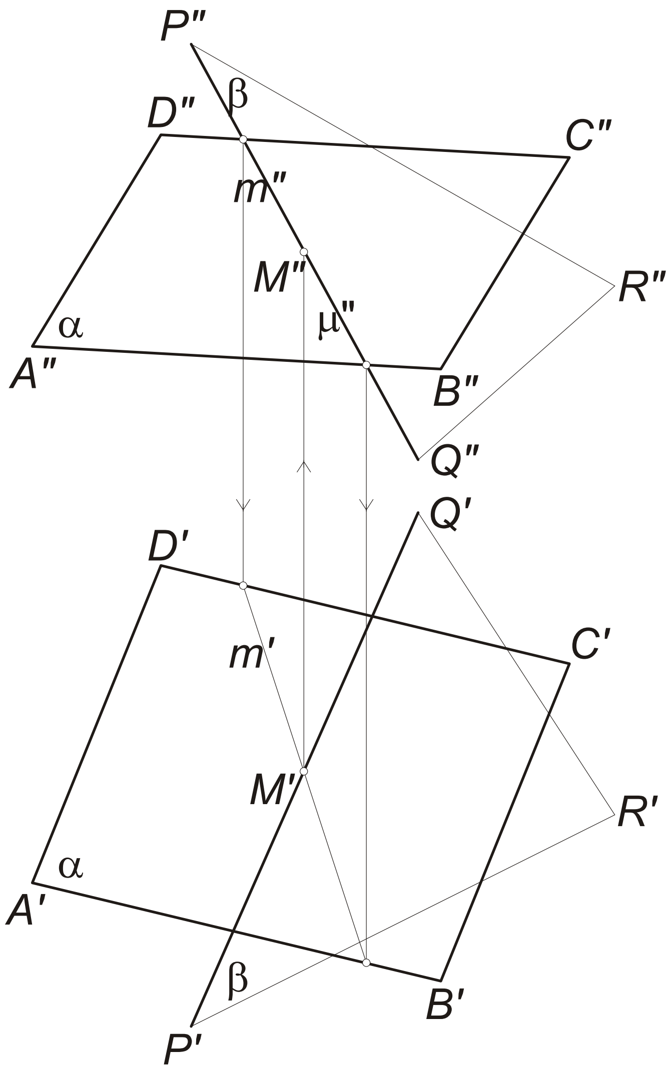
Exercise. *Given a parallelogram $ABCD$ and a triangle PQR. Construct the intersection of the two figures.*



First, we consider the *complete* planes containing the figures:

$$\alpha = [A, B, C, D] \quad \text{and} \quad \beta = [P, Q, R]$$

We construct the intersection line $x = \alpha \cap \beta$ of the two planes. To do this, we construct two points of this line; the intersection line will be the line connecting these two points. We can obtain points of this line by constructing the intersection point of any line of one of the planes with the other plane.

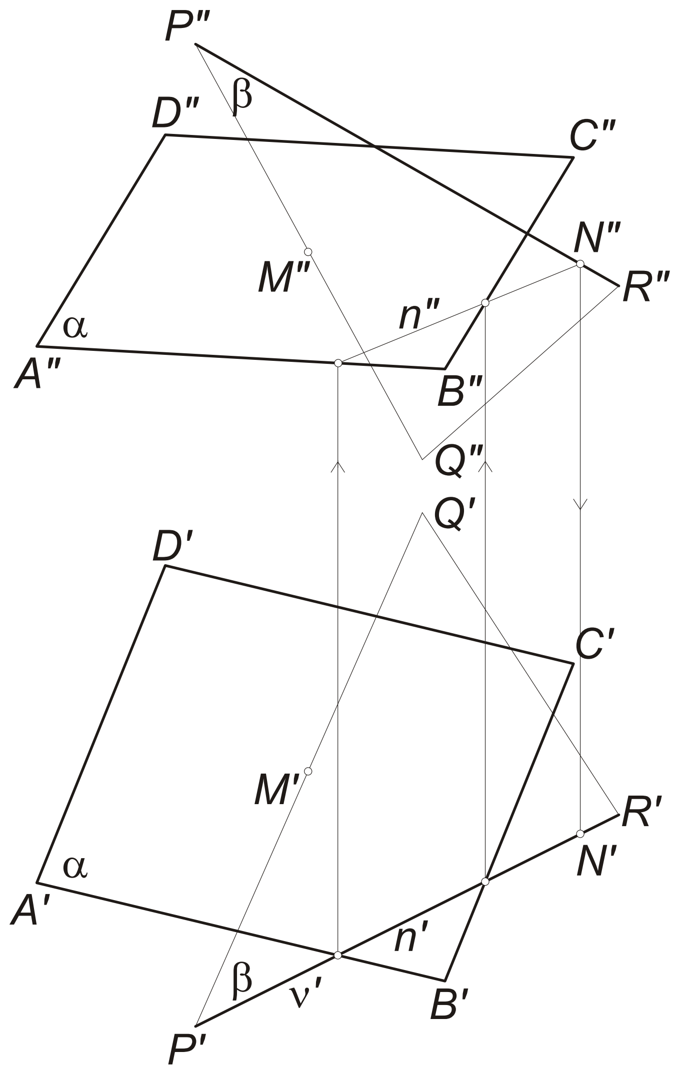


First, we find, for instance, the intersection point M of the the plane α with the line PQ of β. As auxiliary plane, we choose the vertical projecting plane μ of PQ, which intersects α in the line m. We can find M as the intersection point of m with PQ.

$$PQ \subset \mu \,(\text{a vertical projecting plane})$$

$$m = \alpha \cap \mu \,(m'' = \mu'')$$

$$M = PQ \cap m = PQ \cap \alpha$$

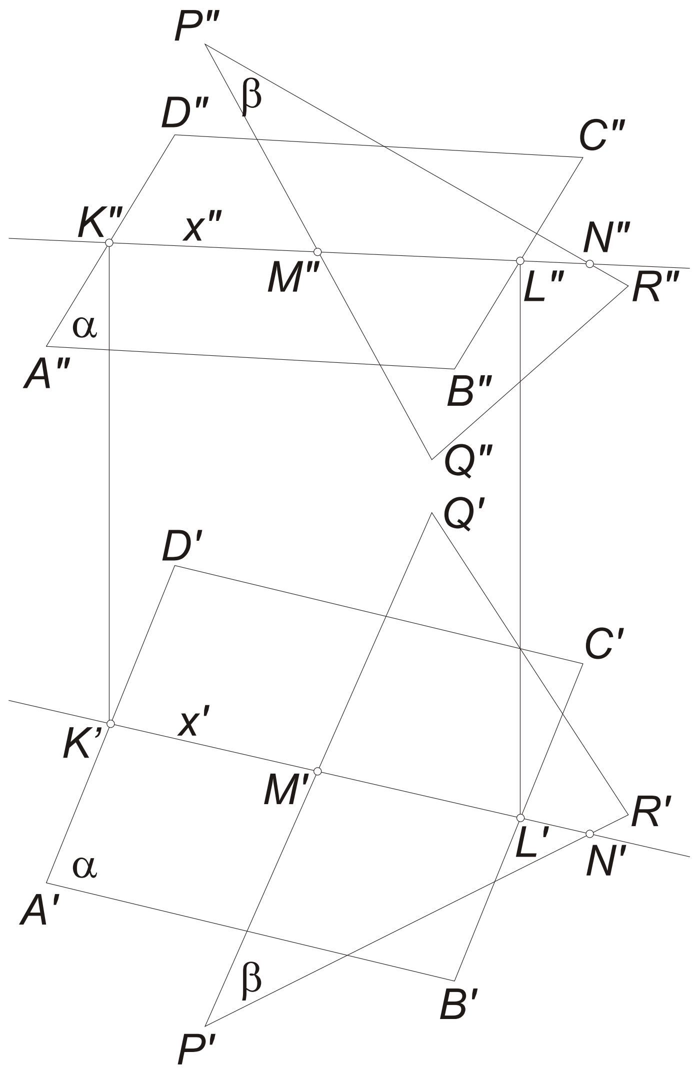


Now we find the intersection point N of the the plane α with the line PR. As auxiliary plane, we choose the horizontal projecting plane ν of PQ, which intersects α in the line n. We can find N as the intersection point of n with PR.

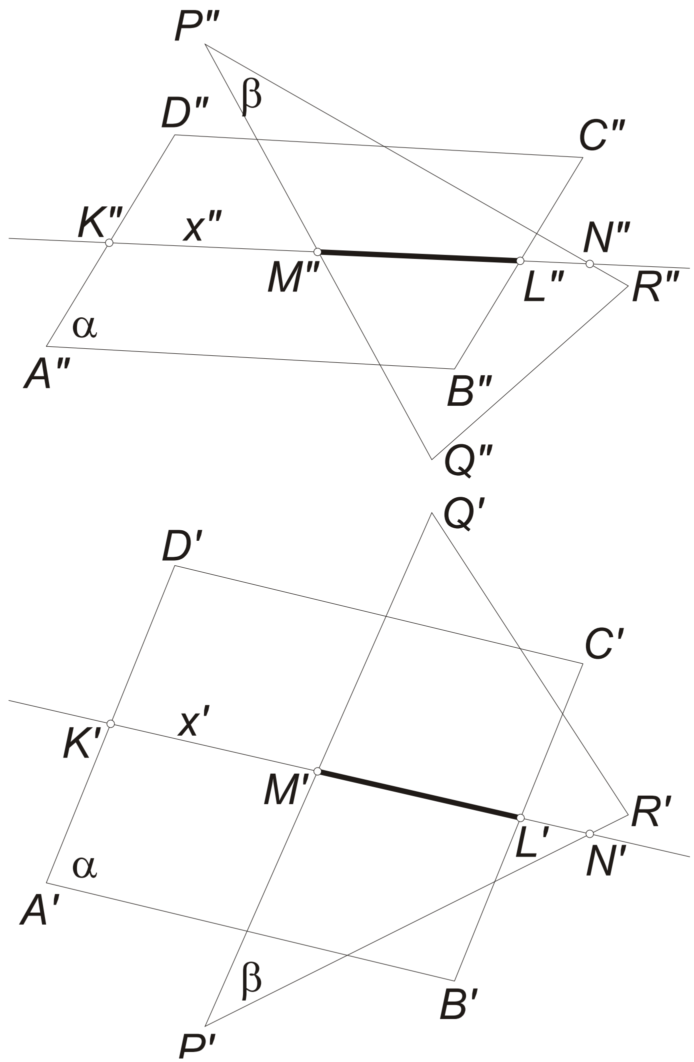
$$PR \subset \nu \,(\text{a horizontal projecting plane})$$

$$n = \alpha \cap \nu \,(n' = \nu')$$

$$N = PR \cap n = PR \cap \alpha$$



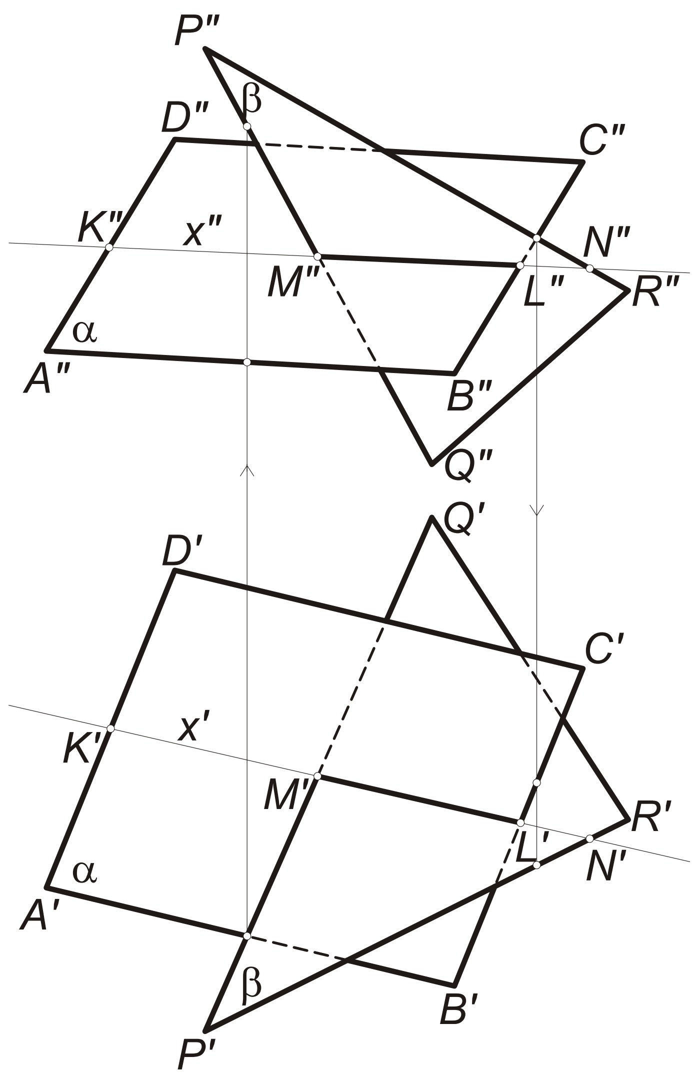
The points M and N are incidient to, and thus determine, the intersetion line x of α and β.
It is recommended to check the construction. Since x lies in both planes α and β, in any of the two projections, any intersection point on x with a line in one of the planes is the projection of a real (not virtual) intersection point. These points should appear in the other projection as well on the same line of recall.
For example, $A'D'$ and $A''D''$ intersect, respectively, x' at K' and x'' at K'', Then $K = AD \cap x$ is a real intersection point, and hence K' and K'' must lie on the same line of recall.
Similarly, $B'C'$ and $B''C''$ intersect, respectively, x' at L' and x'' at L'', Thus, $L = BC \cap x$ is a real intersection point, and L' and L'' lie on the same line of recall.



Now we find the **intersection segments**, the common part of the two given planar figures. That is, we find the part of x that lies in the interior of both figures. In our case, the points of KM are inside the parallelogram but outside the triangle. The points of LN are inside the parallelogram but inside the triangle. The points of x outside KN are outside both figures. Only the points of KN are inside both figures, and thus now the only intersection segment is LM.

In case of convex figures, their intersection is convex as well, which means that if they have common points, then they belong to only one intersection segment. One can get more than one segment only in case one of the figures is concave (e.g. has a hole).

When showing visibility, the intersection segments are always visible, since they cannot be hidden behind any of the planar figures.



To determine visibility, consider a covering pair of points not on x, one element of which lies in α and the other one is in β (in other words, consider a virtual intersection point not on x). We examine which point covers which one, which shows which plane covers which one near this point. In our case (when intersecting two planar figures), this neighborhood can be extended to the intersection line x of the two planes. On the other side of x, the visibility properties of the two figures switch.

In the vertical projection, for visibility, we can consider, for instance, the vertical covering pair determined by the line BC in α and PR in β (which are skew lines). We can find the horizontal projections of these two points using their line of recall. We can see that the horizontal projection of the point on PR is farther from the axis than that of the point on BC, which means that PR in front view covers BC. Thus, on the side of x'' containing this pair, the edges of PQR 'have priority', whereas on the other side of x'' the edges of $ABCD$ do.

In the horizontal projection we apply this principle to the horizontal covering pair determined by AB and PQ. We can check that the point of PQ is on a higher level in the vertical projection, which then covers the point of AB in top view. Hence, on the side of x' containing this pair, the triangle has 'priority', and on ther other side the parallelogram does.