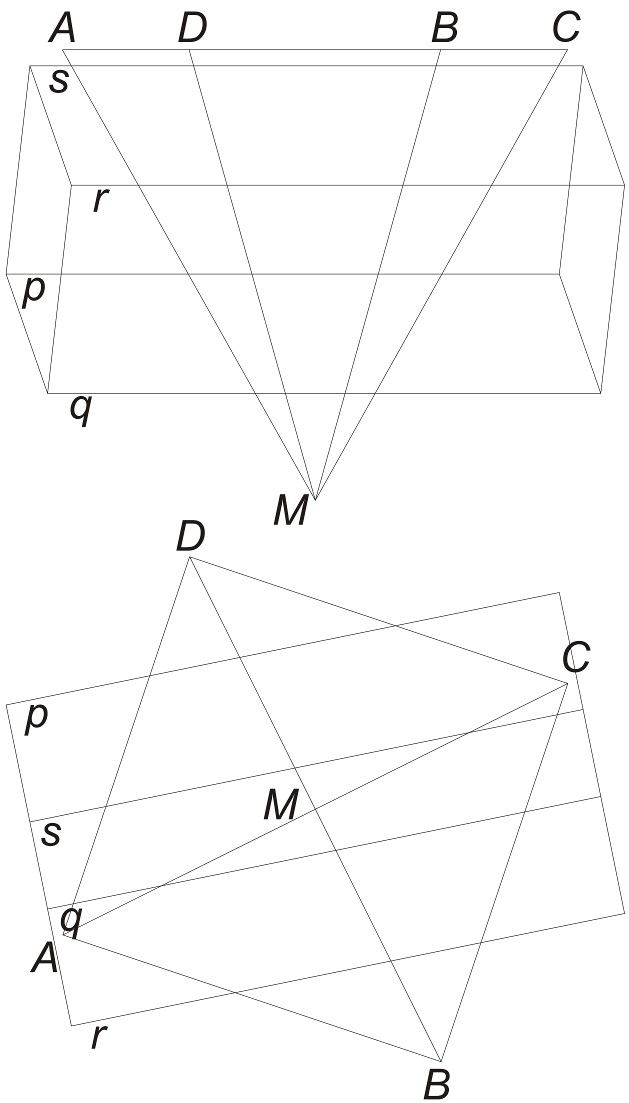
# Intersection of polyhedra

Intersection of a regular square-based pyramid and a prism with parallel lines as its sidelines

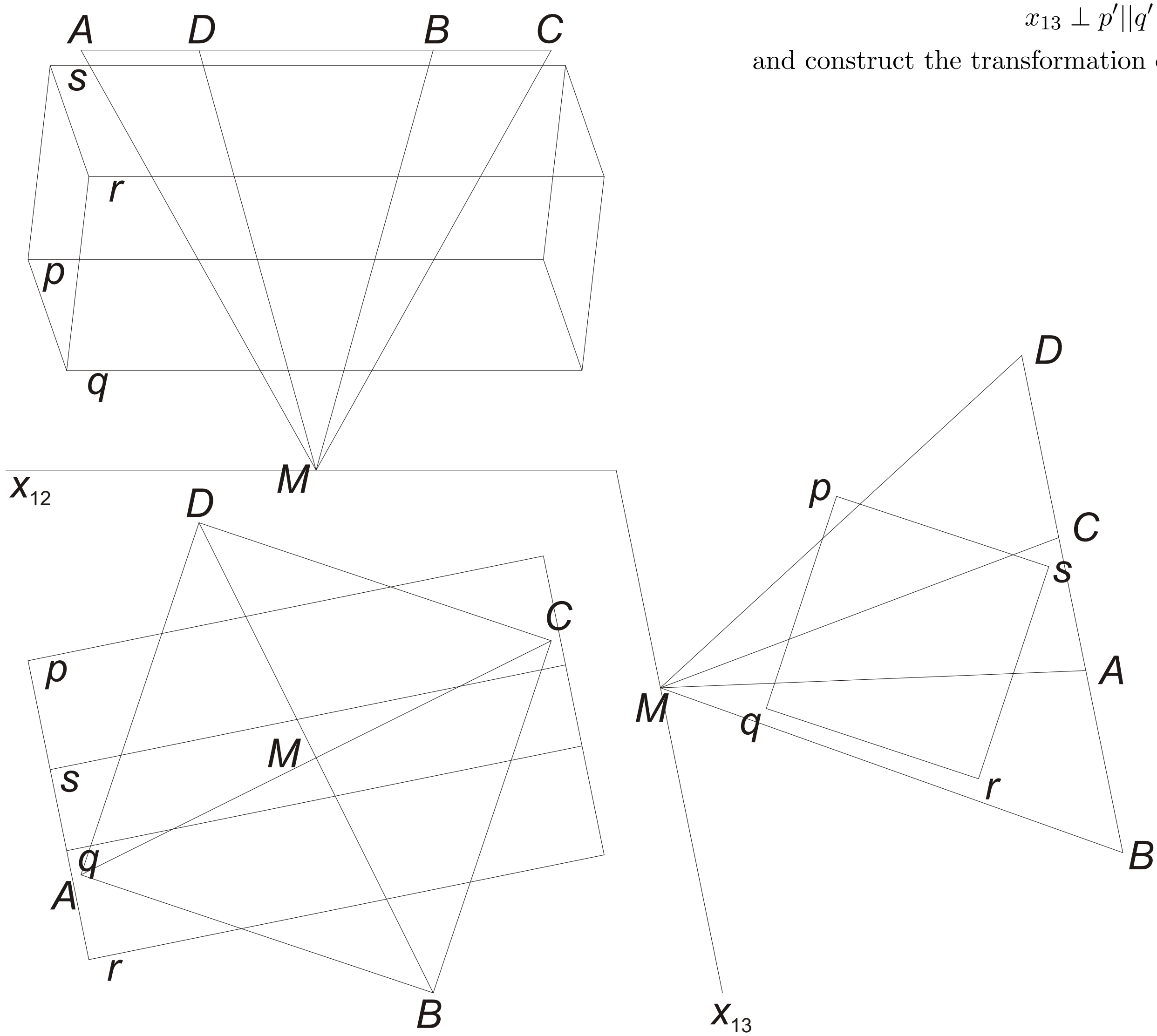
**Exercise.** *Given the $ABCDM$ pyramid with its base $ABCD$ lying in a horizontal plane. Furthermore, we are also given a prism such that its sidelines $p, q, r$ and $s$ are horizontal lines. Construct the intersection of the two bodies. For visibility, we assume that the two bodies are made from plate, and their bases, top faces, and the parts of their faces that are in the interior of the other body are removed.*



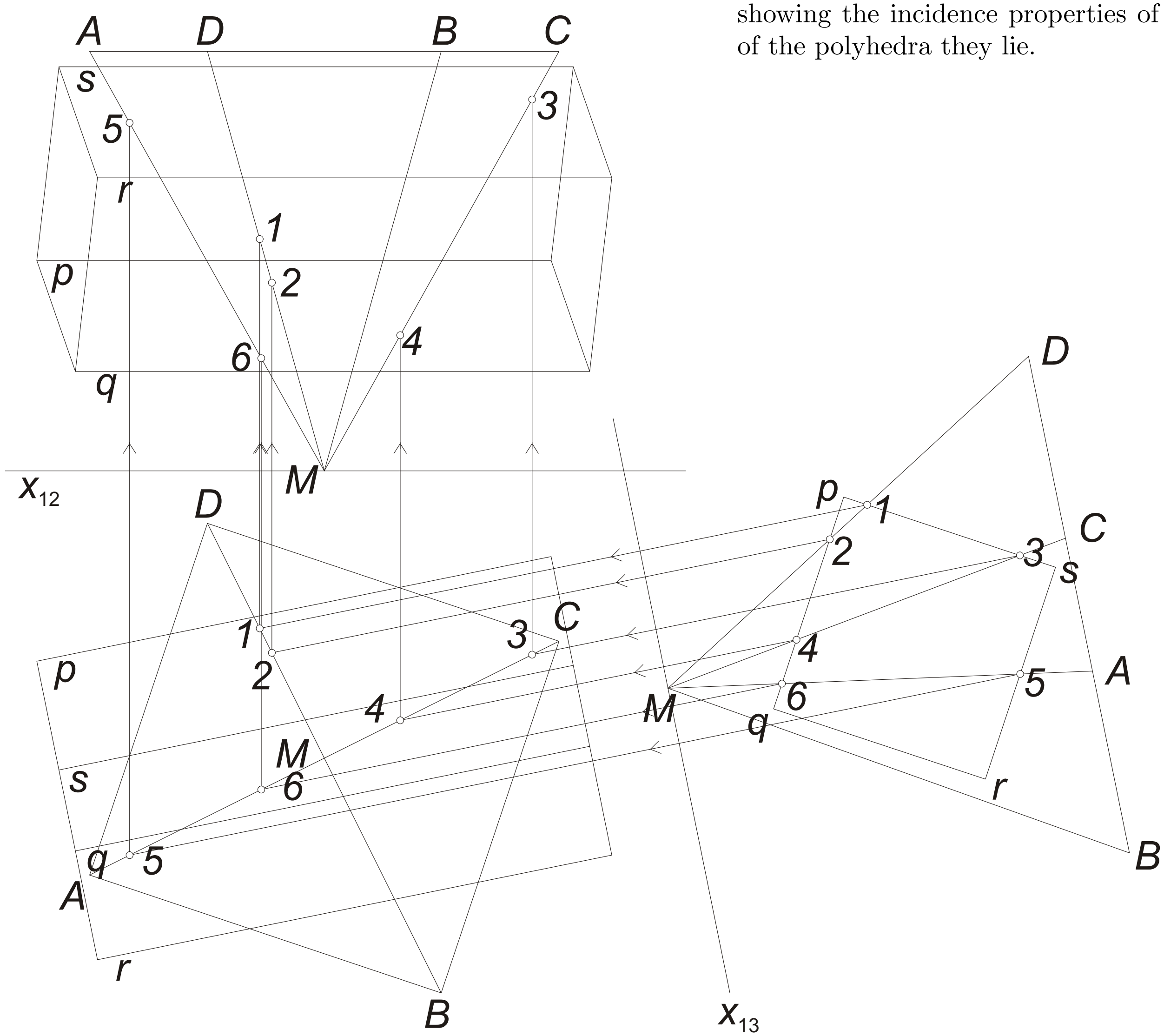
We transform the prism in a way that its sidelines $p, q, r$ and $s$ become projecting lines:

$$x_{13} \perp p'||q'||r'||s',$$

and construct the transformation of the pyramid as well.

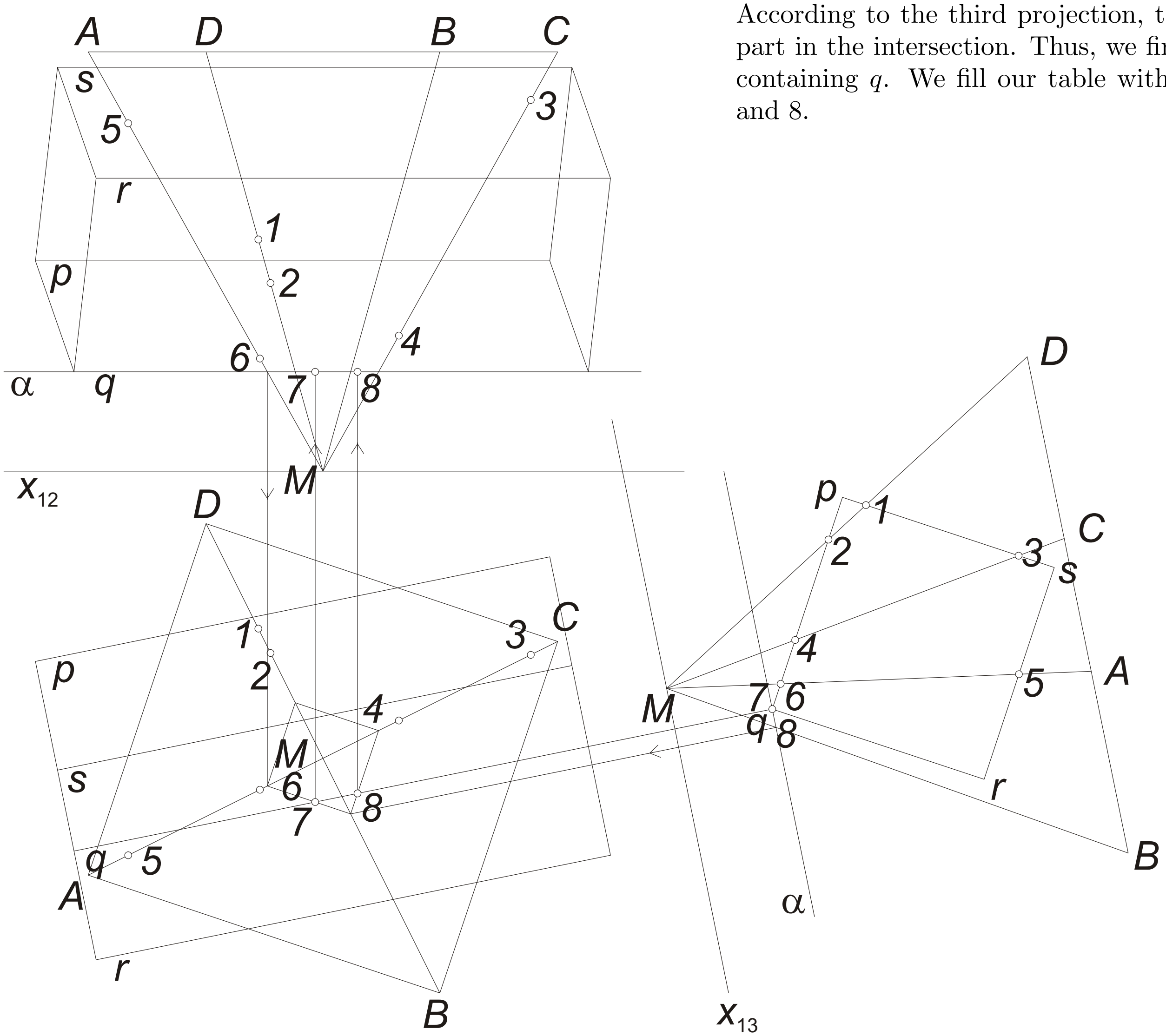


We construct the intersection points of the edges of the pyramid and the fthe prism. These can be found in the third projection, and then, using their lines of recall, their horizontal and vertical projections can be constructed as well. Finally, we make a table showing the incidence properties of the points: on which faces of the polyhedra they lie.




| | Pyramid | Prism |
|---|---|---|
| 1: | $CDM, DAM;$ | $sp$ |
| 2: | $CDM, DAM;$ | $pq$ |
| 3: | $BCM, CDM;$ | $sp$ |
| 4: | $BCM, CDM;$ | $pq$ |
| 5: | $DAM, ABM;$ | $rs$ |
| 6: | $DAM, ABM;$ | $pq$ |

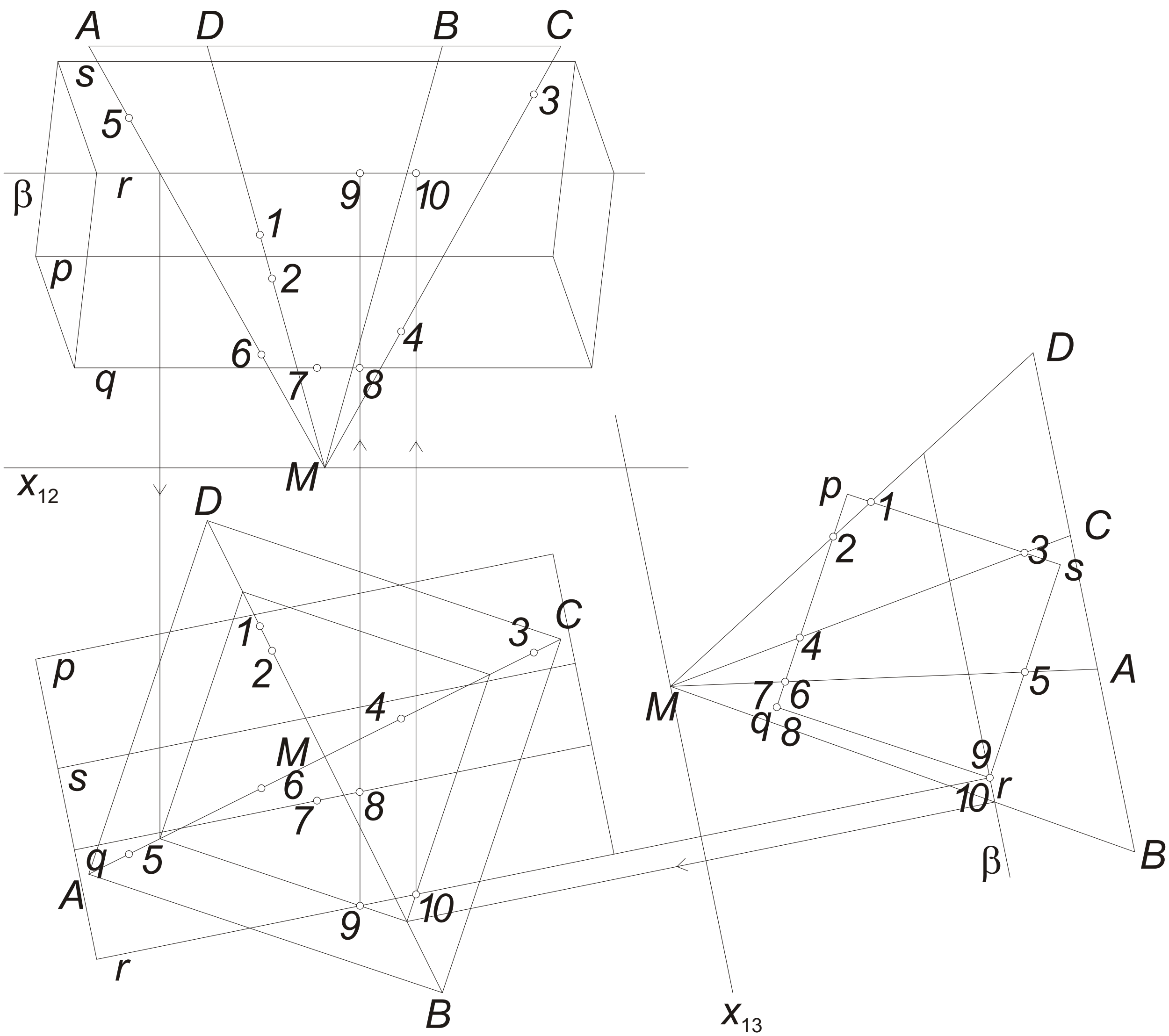
Now we construct the prism edge – pyramid face type intersection points. For slicing planes, we can choose, for example, horizontal planes. These planes intersect the pyramid in rectangles similar to its base, where the center of symmetry is $M$. According to the third projection, the sideline $p$ does not take part in the intersection. Thus, we first intersect with a plane $\alpha$ containing $q$. We fill our table with the new points, labeled 7 and 8.




| | Pyramid | Prism |
|---|---|---|
| 1: | $CDM, DAM;$ | $sp$ |
| 2: | $CDM, DAM;$ | $pq$ |
| 3: | $BCM, CDM;$ | $sp$ |
| 4: | $BCM, CDM;$ | $pq$ |
| 5: | $DAM, ABM;$ | $rs$ |
| 6: | $DAM, ABM;$ | $pq$ |

| | Pyramid | Prism |
|---|---|---|
| 7: | $ABM;$ | $pq, qr$ |
| 8: | $BCM;$ | $pq, qr$ |

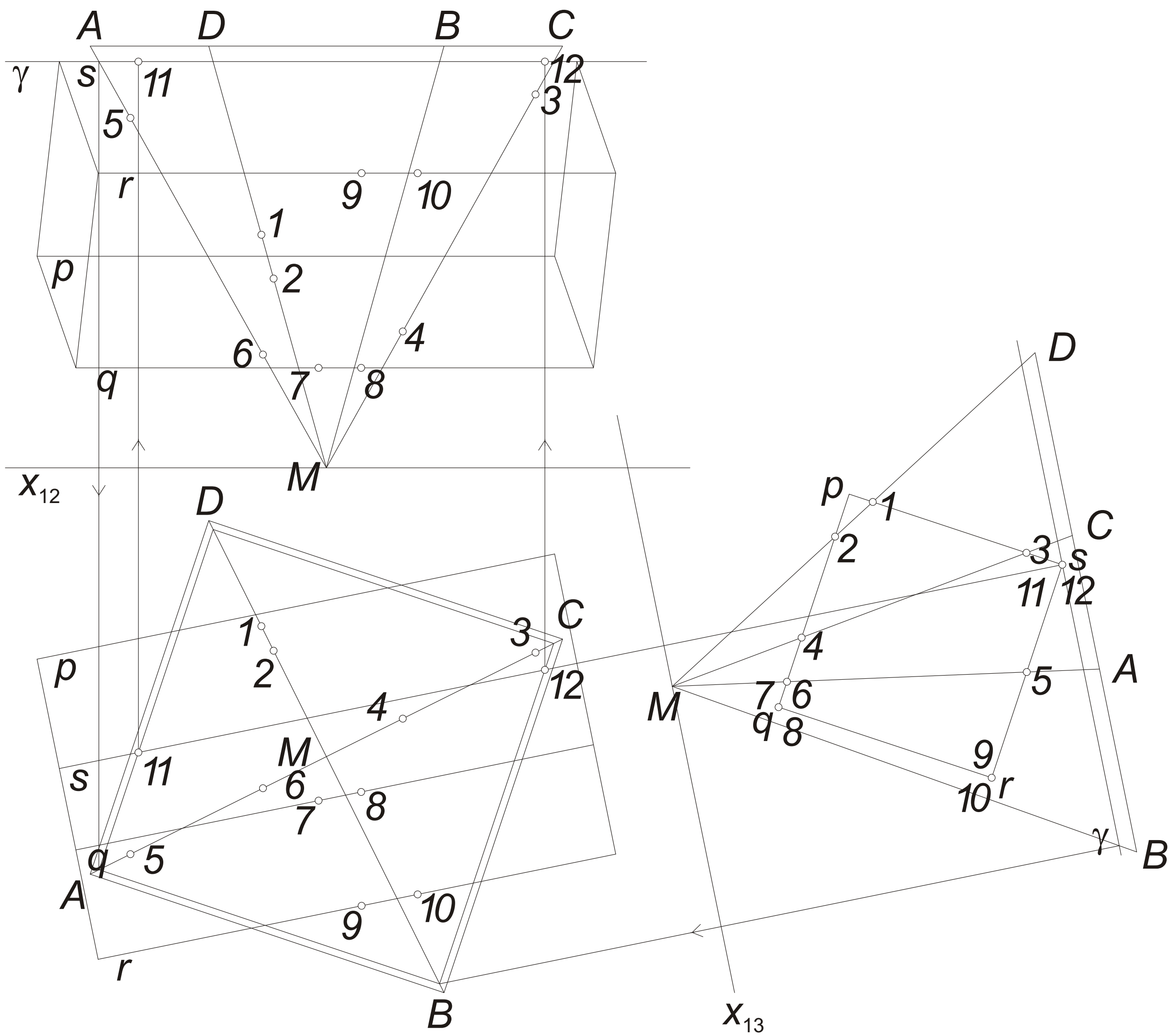
By slicing with the plane $\beta$ containing $r$, we find the intersection points, labeled 9 and 10, of the sideline $r$ and the pyramid.




| | Pyramid | Prism |
|---|---|---|
| 1: | $CDM, DAM;$ | $sp$ |
| 2: | $CDM, DAM;$ | $pq$ |
| 3: | $BCM, CDM;$ | $sp$ |
| 4: | $BCM, CDM;$ | $pq$ |
| 5: | $DAM, ABM;$ | $rs$ |
| 6: | $DAM, ABM;$ | $pq$ |

| | Pyramid | Prism |
|---|---|---|
| 7: | $ABM;$ | $pq, qr$ |
| 8: | $BCM;$ | $pq, qr$ |
| 9: | $ABM;$ | $qr, rs$ |
| 10: | $BCM;$ | $qr, rs$ |

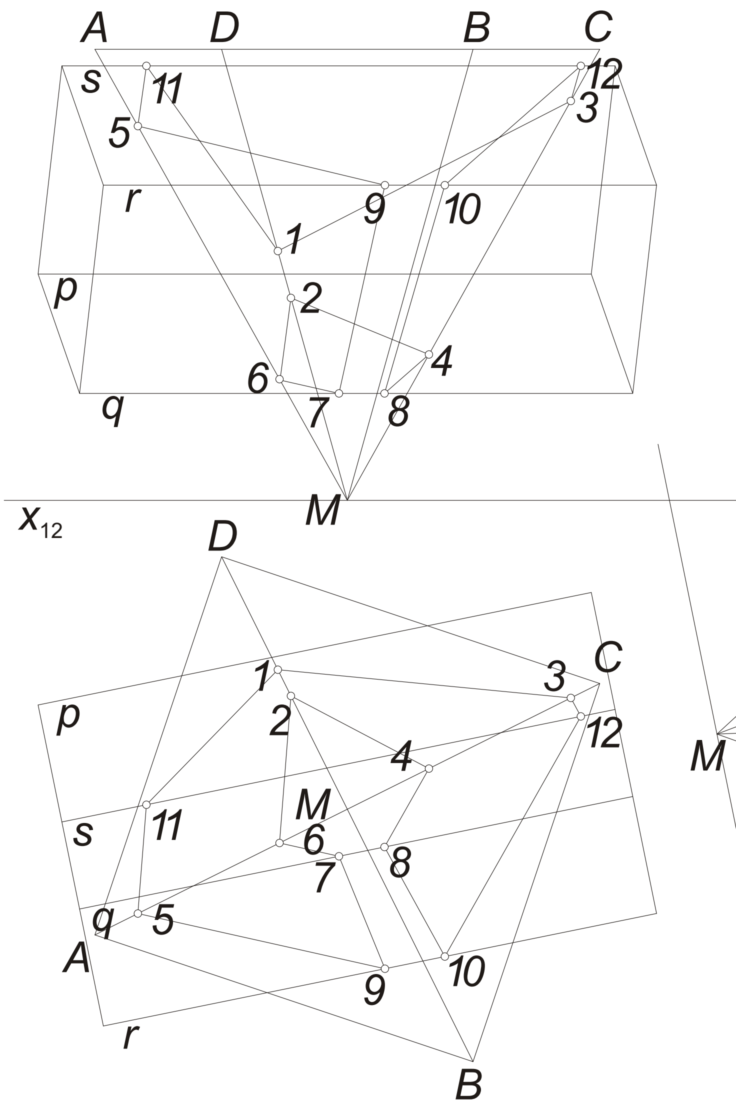
Finally, slicing with the plane $\beta$ containing $s$, we find the intersection points 11 and 12 of $s$ and the pyramid.



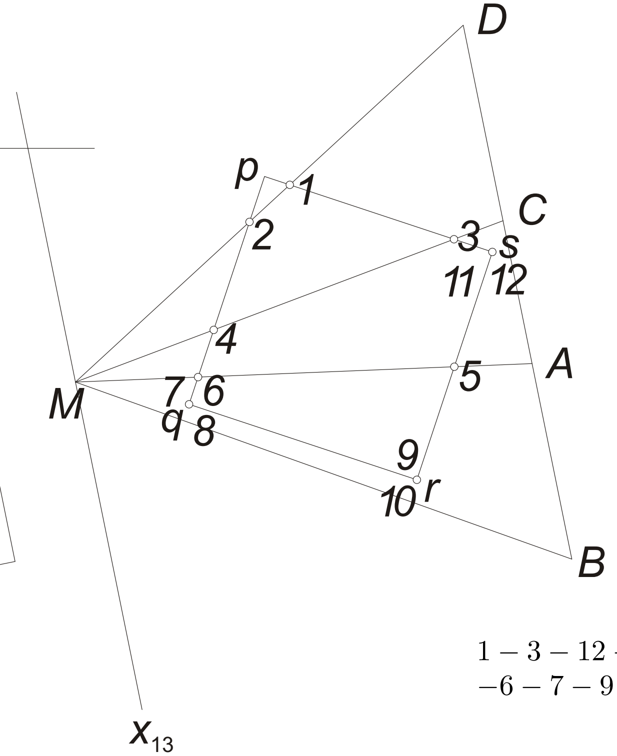

| | Pyramid | Prism |
|---|---|---|
| 1: | $CDM, DAM;$ | $sp$ |
| 2: | $CDM, DAM;$ | $pq$ |
| 3: | $BCM, CDM;$ | $sp$ |
| 4: | $BCM, CDM;$ | $pq$ |
| 5: | $DAM, ABM;$ | $rs$ |
| 6: | $DAM, ABM;$ | $pq$ |

| | Pyramid | Prism |
|---|---|---|
| 7: | $ABM;$ | $pq, qr$ |
| 8: | $BCM;$ | $pq, qr$ |
| 9: | $ABM;$ | $qr, rs$ |
| 10: | $BCM;$ | $qr, rs$ |
| 11: | $DAM;$ | $rs, sp$ |
| 12: | $BCM;$ | $rs, sp$ |

Based on our table we can find the order to connect the vertices. Our main observation is that two points are connected with an edge if there are faces of both the pyramid and the prism that contain them. For example, beginnig at the point 1, we can see that it lies on the face $CDM$ of the pyramid, and the face $sp$ of the prism. From the table, we can see that the point 3 also lies on these faces, and thus, there is an edge connecting 1 and 3. The point 3 is contained in $BCM$ and $sp$, which also holds for 12, and hence we can connect 3 and 12. By examining the faces containing 12 we can continue the process ...




| | Pyramid | Prism |
|---|---|---|
| 1: | $CDM, DAM;$ | $sp$ |
| 2: | $CDM, DAM;$ | $pq$ |
| 3: | $BCM, CDM;$ | $sp$ |
| 4: | $BCM, CDM;$ | $pq$ |
| 5: | $DAM, ABM;$ | $rs$ |
| 6: | $DAM, ABM;$ | $pq$ |



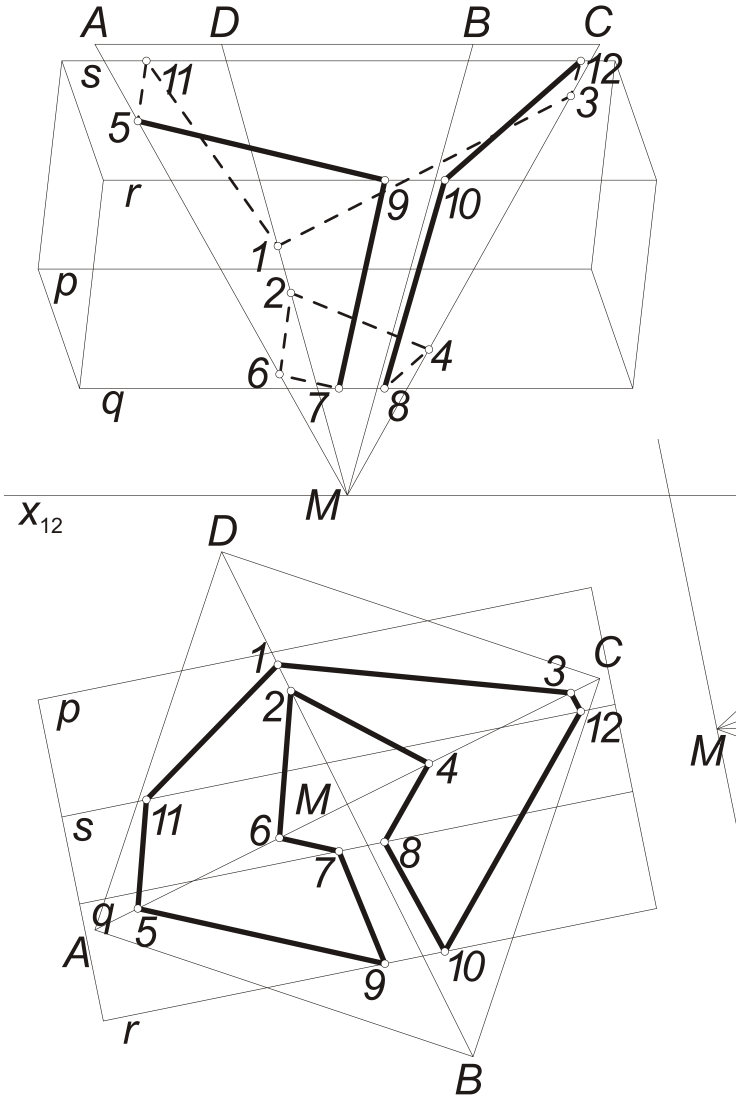

| | Pyramid | Prism |
|---|---|---|
| 7: | $ABM;$ | $pq, qr$ |
| 8: | $BCM;$ | $pq, qr$ |
| 9: | $ABM;$ | $qr, rs$ |
| 10: | $BCM;$ | $qr, rs$ |
| 11: | $DAM;$ | $rs, sp$ |
| 12: | $BCM;$ | $rs, sp$ |

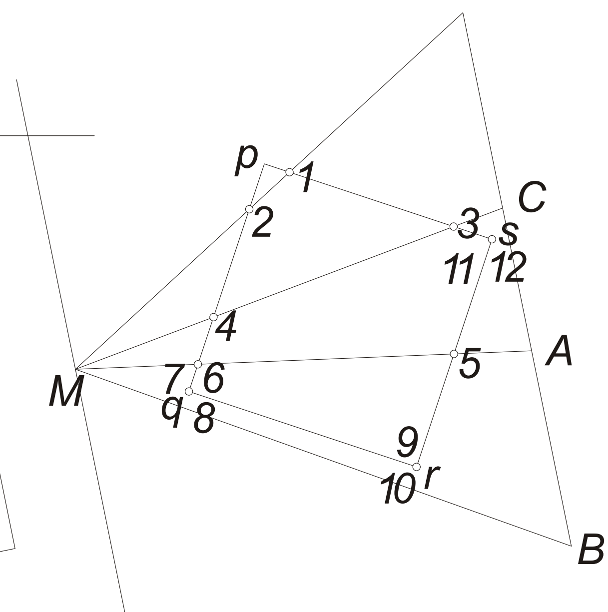
1 – 3 – 12 – 10 – 8 – 4 – 2–
–6 – 7 – 9 – 5 – 11 – 1

We examine visibility corresponding to our conditions. First we determine the visible edges of the intersection polygon. These are the edges that are on visible faces of both polyhedra.

In the horizontal projection, because of removing the face $ABCD$, all the remaining faces of the pyramid, and thus all the edges of the intersection polygon are visible.

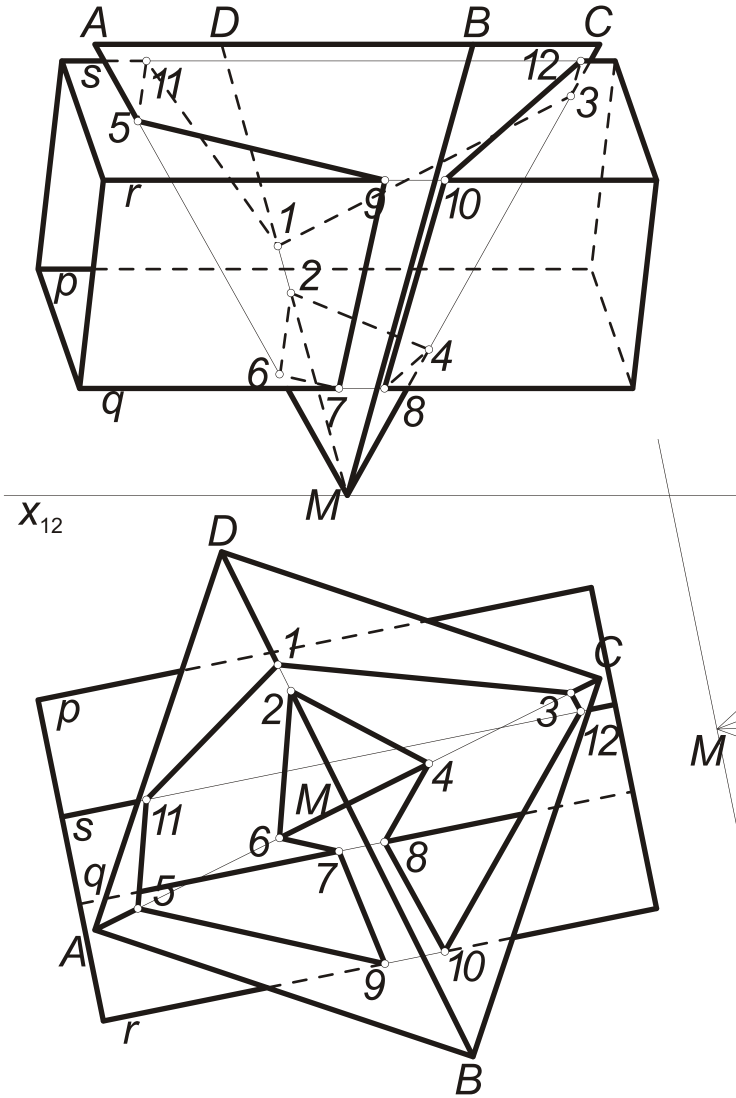
In the vertical projection we consider that the visible faces of the pyramid and the prism are $ABM$ and $BCM$, and $qr$ and $rs$, respectively. All the other edges of the intersection polygon are drawn as existing but invisible lines.

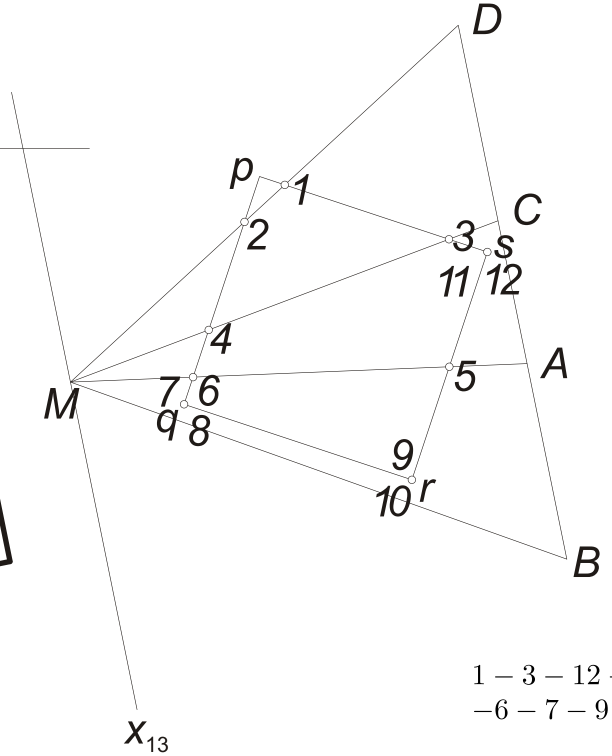



$1 - 3 - 12 - 10 - 8 - 4 - 2-$
$-6 - 7 - 9 - 5 - 11 - 1$

We ignore the parts of the edges that have been removed with some pieces of the surfaces, according to our conditions. These parts do not exist anymore, and we do not need to show them in the construction. We show visibility on the remaining parts using continuous lines for visible, and dashed lines for invisible edges.




$1 - 3 - 12 - 10 - 8 - 4 - 2-$
$-6 - 7 - 9 - 5 - 11 - 1$