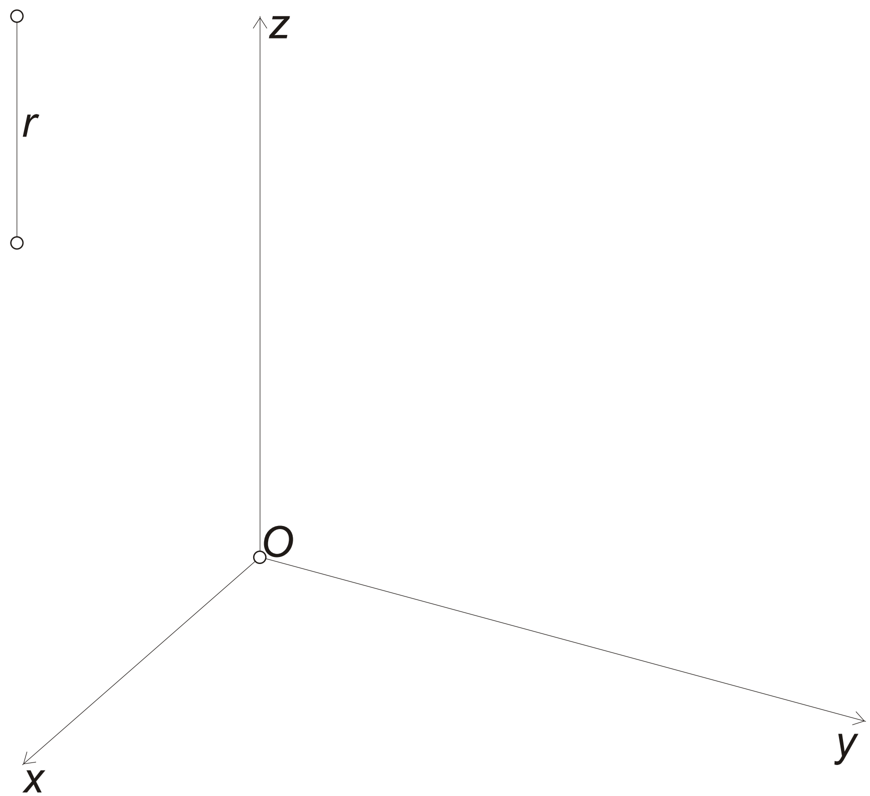
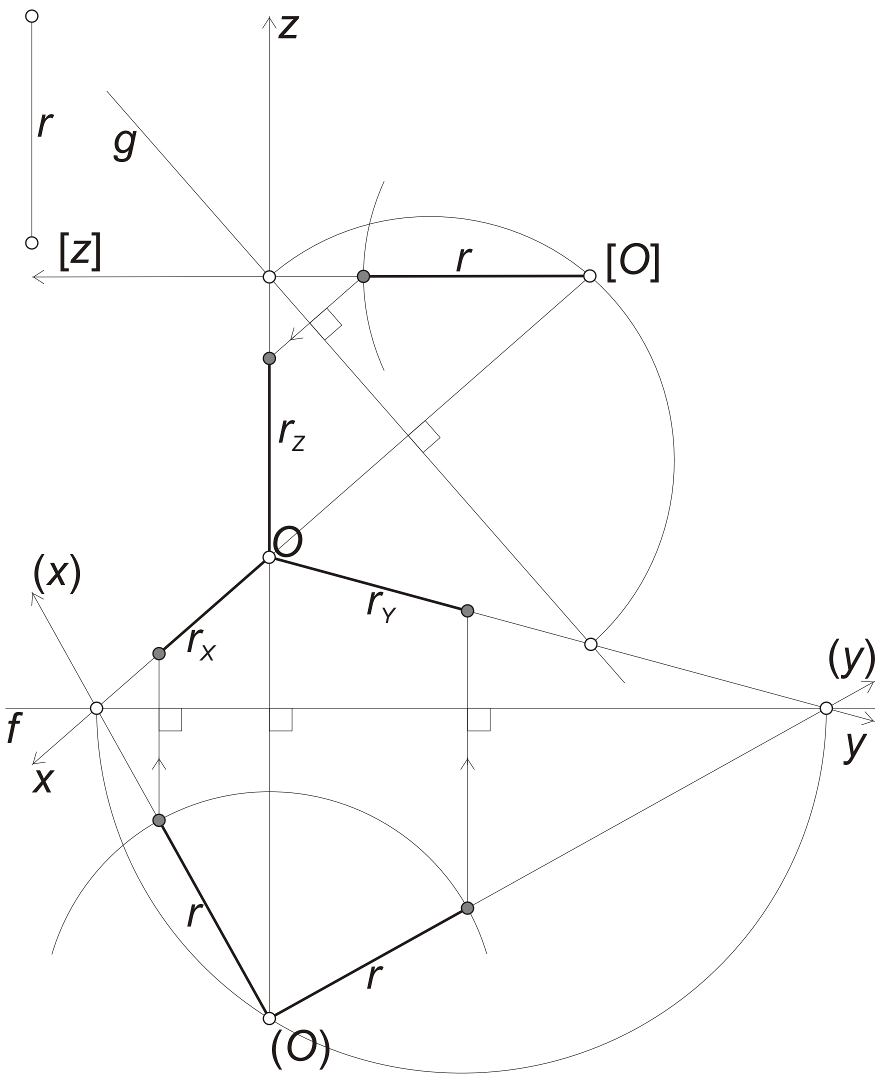
Orthogonal axonometry

Construction of a dice in orthogonal axonometry



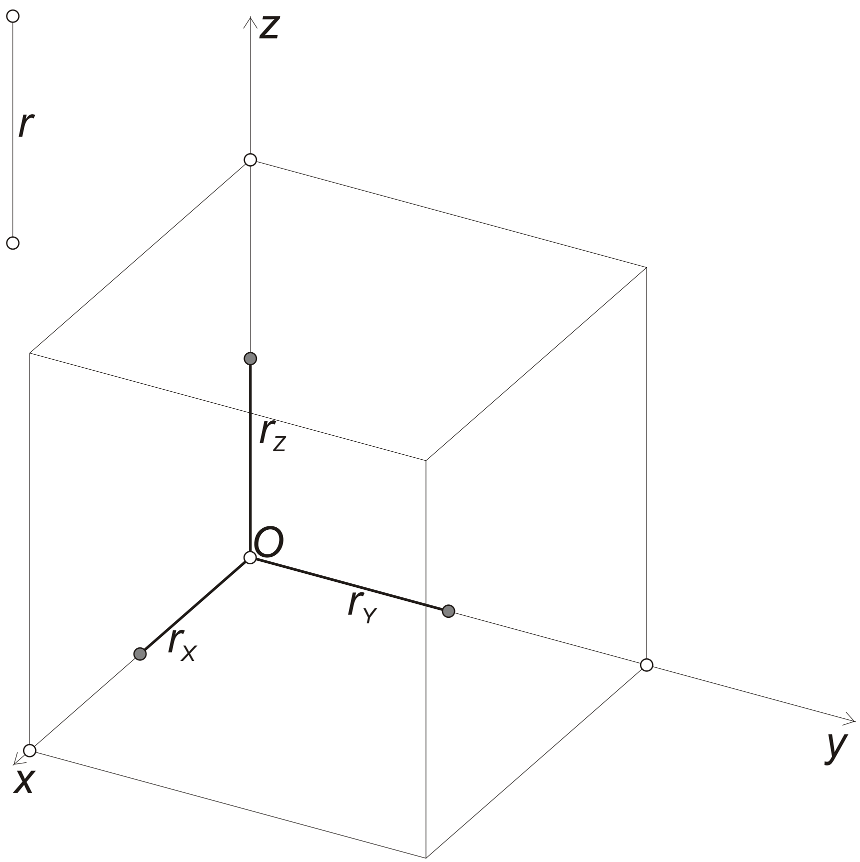
Exercise. *Given the projection of a coordinate system in an orthogonal axonometry, and the distance r. Construct the projection of the cube with edge-length $a = 2r$, with the origin as one of its vertices, and the edges emanating from this lying on the coordinate axes. By constructing the projections of the circles inscribed in the faces of the cube, draw the projection of the intersection of the cube with the ball touching all its edges at their midpoints ("dice"). Show the visibilitz of the solid body.*



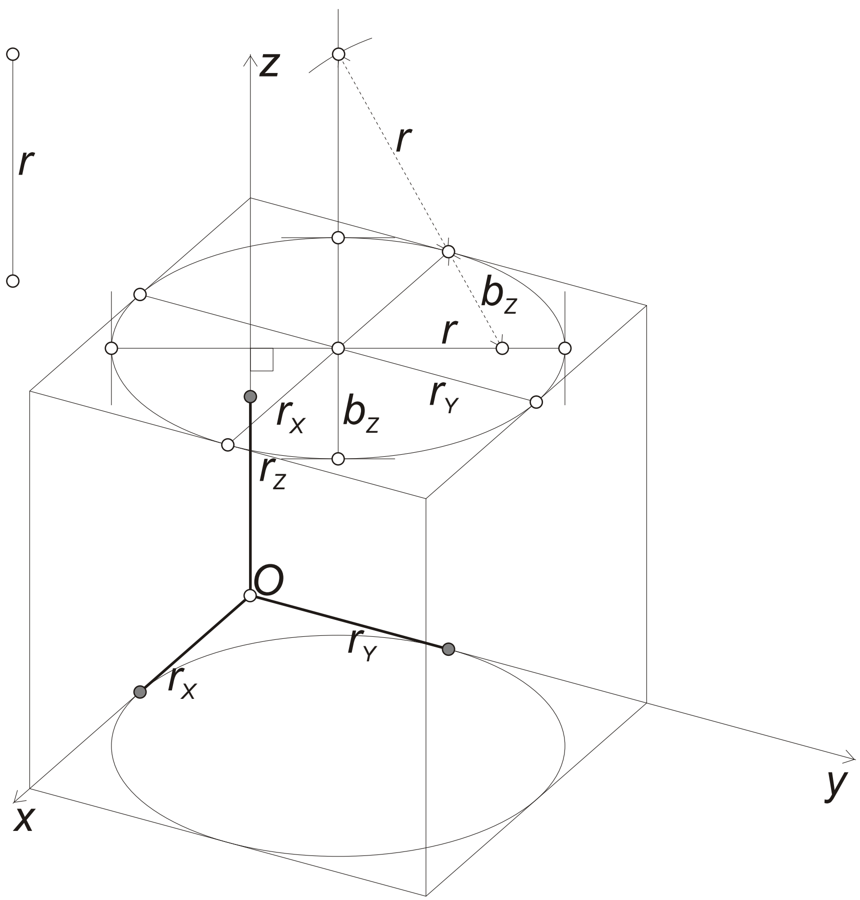
First we construct the foreshortenings of the distance r on the coordinate axes. To do this, first we rotate the $[x, y]$ plane into parallel position. As the axis of the rotation we need to choose a parallel line f. In our case, these line are perpendicular to the z axis. During the rotation, the intersection points of f with the x and y coordinate axes do not move. By drawing the Thales circle of the segment with these two points as its endpoints, we obtain the rotated copy (O) of O as the intersection of the circle with the line of the z-axis. We connect this point with the two fixed points to draw the rotated copies (x) and (y) of the corresponding coordinate axes. We measure from (O) the real distance r on these two half lines, and rotate the points back obtained in this way. Then we have the foreshortenings r_x and r_y of r, on the x and y axes.

To construct the foreshortening r_z on z, we need to rotate also another coordinate plane, say $[y, z]$, into parallel position. As the axis, again we choose a parallel line g, which is now perpendicular to x. The intersections of g with y and z are fixed points, and using their Thales circle, we obtain the rotated copy $[O]$ of O as its intersection with the line of x. Connecting $[O]$ and the intersection point of g and z yields the rotated copy $[z]$ of z. We need to measure the real distance r on this, and rotate it back to obtain r_z.

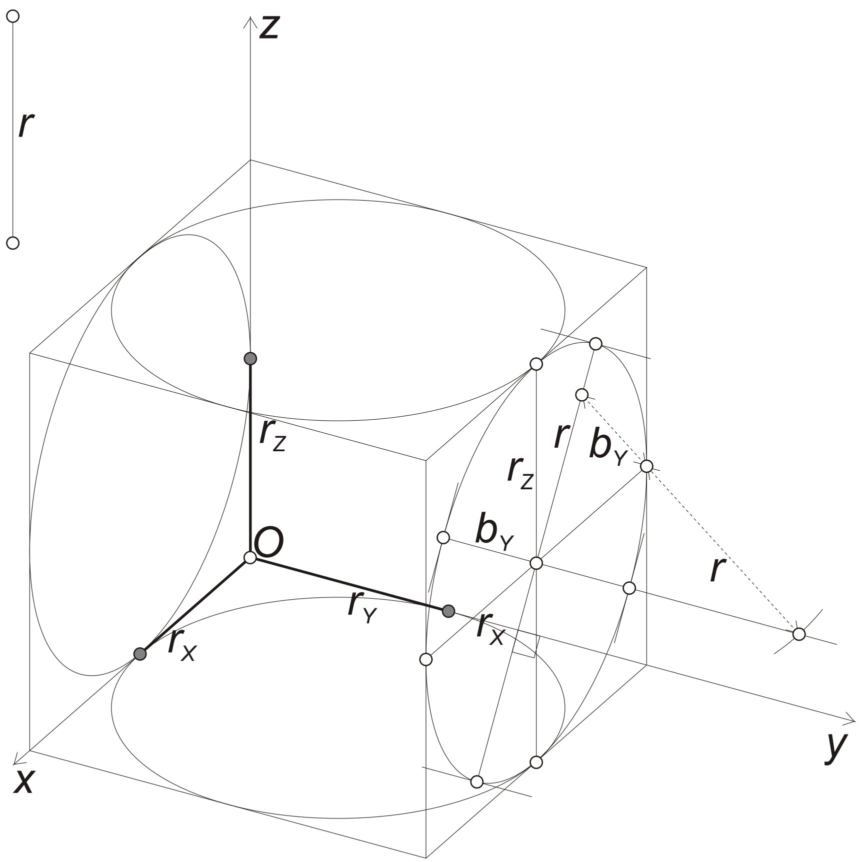
In the remaining part of the construction, we do not need the rotated planes, only the axonometric coordinate system.



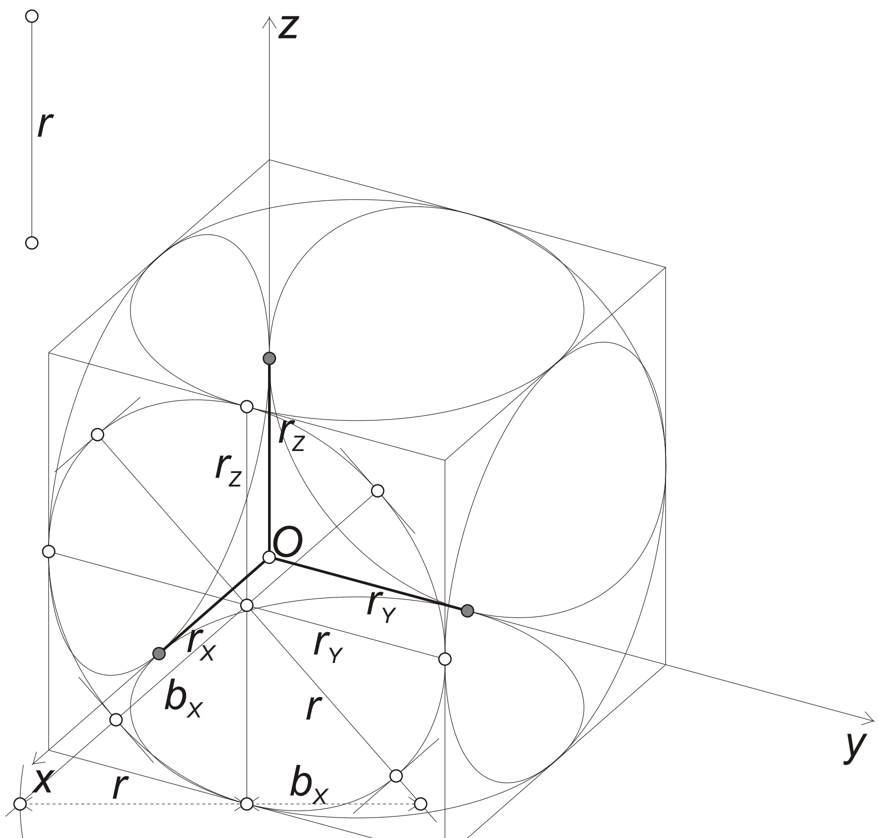
We construct the projections of the faces of the cube. Since the edge-length of the cube is $2r$, we need to measure $2r_x$, $2r_y$ and $2r_z$, respectively, on the coordinate axes from O. We draw the vertices of the cube on the three coordinate axes in this way. In the $[x, y]$ plane, completing the three vertices into a parallelogram, we have the projection of the corresponding face. Translating the edge on the z-axis into the endpoints of this face we draw the lateral edges of the cube. Finally, connecting the upper vertices of these edges we construct the edges of the roof of the solid.



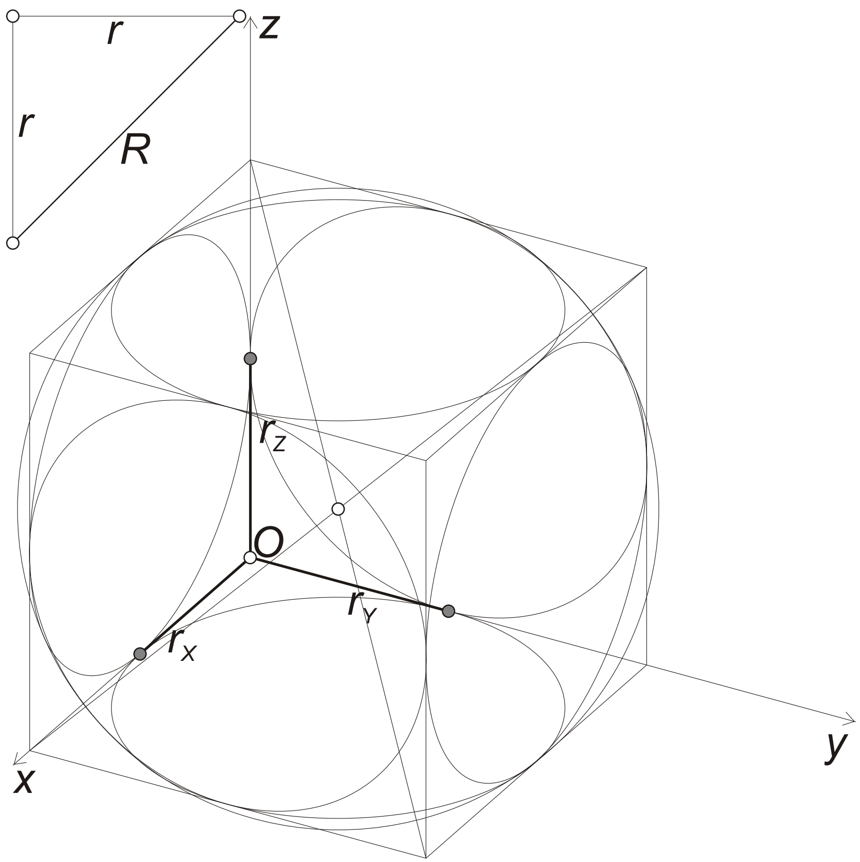
We construct the projections of the incircles of the floor and the roof of the cube. In our case the diameters of the ellipses parallel to the coordinate axes (x and y) are the medians of the faces, and the tangent lines here are the edges of the parallelograms.
The major axes of the ellipses are projections of parallel lines of the planes, and thus, they are perpendicular to the z axis; their length is $2r$, since in this direction the real lengths are not shortened.
The minor axes are the ellipses are perpendicular to the major ones, and hence, they are parallel to the z axis. Their foreshortened length can be constructed, for example, using the *ellipsograph* method. We can choose, e.g. an endpoint of the diameter parallel to the x axis as a point of the ellipse. Drawing a circle around it with radius r, we construct the intersetion point of the circle and the line of the minor axis. Constructing line containing this point and the chosen point of the ellipse, the length of the piece between the ellipse point and the major axis shows half the length of the minor axis, denoted by b_z.
We draw the tangent lines at the endpoints of the two axes, and then, using the constructed points and tangent lines we can draw the ellipse.
Using a similar construction, or translating all the points and their tangent lines with $2r_z$ in the direction of the z axis, we can draw the ellipse contained in the projection of the opposite face of the cube.



To draw the projections of the incircles of the faces perpendicular to the y axis can be obtained similarly. The diameters parallel to the x and the z axes are the medians of the faces, and the tangent lines here are the edges of the cube. The major axes are perpendicular to y, and their length is $2r$. The minor axes lie on lines parallel to the y axis, and their length can be constructed using the ellipsograph method, for example choosing an endpoint of a diameter parallel to the x axis. Using the constructed points and their tangent lines we draw the ellipse. Finally, we translate the points and the tangent lines by $2r_y$, and draw the projection of the incircle in the opposite face of the cube.



Finally, we construct the projections of the incircles of the faces perpendicular to the x axis. The diameters parallel to the y and the z axes are the medians. The major axes are perpendicular to the x axis, and their length is $2r$. The minor axes are parallel to x, and their length b_x can be constructed using the ellipsograph method. Using these points and their tangent lines, we draw the ellipse. We translate the points and the tangent lines by $2r_x$, and draw the projection of the incircle of the opposite face.



Projecting the cube in the direction of one of the edges, we obtain a square of edge-length $2r$. The circumscribed circle of this square is the projection of the sphere touching all edges at their midpoints. Thus, the radius R of this sphere is $\sqrt{2}$ times half the length of the edge. In our case R is the hypotenuse of the isosceles right triangles with legs of length r: $R = \sqrt{2}r$. The contour of the sphere is again a circle of radius R with the center of the cube as its center: the center can be constructed as the intersection of two body diagonals. Hence, the contour circle is the circle of radius R around this center. If we have drawn the ellipses precisely, this circle touches three of them. When showing visibility, we may need some arcs of the contour circle as well.

Finally, we show the visibility.