Balancing with reaction delay

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MŰEGYETEM 1782

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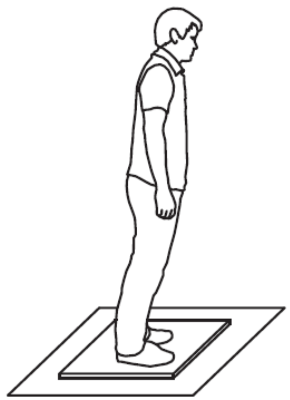
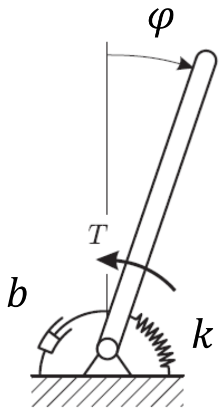
Stick balancing – what is the control law?

Experiments:

- Virtual stick balancing
- Ball and beam
- Pendulum-cart and beam
- Balance board

Postural sway





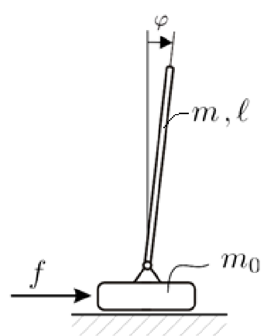
feedback torque

$$\ddot{\varphi}(t) + b\dot{\varphi}(t) + \underbrace{\left(k - \frac{3g}{2l}\right)}_{\approx -0.1\frac{3g}{2l} < 0} \varphi(t) = \frac{12}{ml^2} T(t)$$

(Loram, Lakie, Asai, Nomura)

Upper position: unstable position

Stick balancing



$$\ddot{\varphi}(t) - \frac{6g}{cl}\varphi(t) = -\frac{6}{(m+m_0)lc} f(t)$$

feedback force

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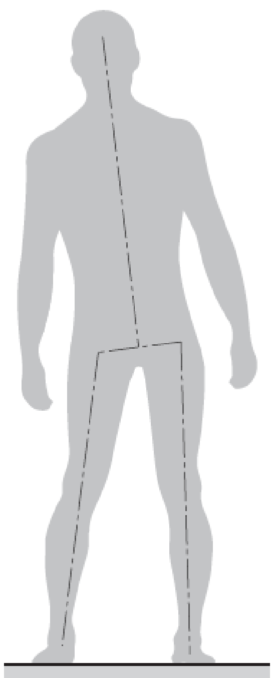
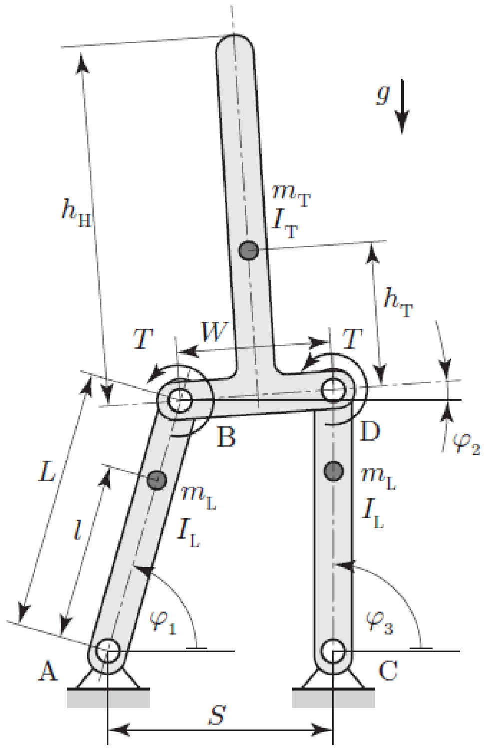
Frontal plane mediolateral balance





$$I\,\ddot{\varphi}_1(t) - G\,\varphi_1(t) = -C\,T(t)$$

(Henry, Fung, Horak, 2001; Bingham, Ting, 2013)

$$I = 2(m_\mathrm{L}L^2 + I_\mathrm{L}) + \frac{m_\mathrm{T}(h_\mathrm{T}\alpha - W\beta)^2 + I_\mathrm{T}\alpha^2}{W^2}$$

$$G = -g\left(\frac{m_\mathrm{T}(h_\mathrm{T}\alpha)^2}{W^2} - \frac{(2lm_\mathrm{L} + Lm_\mathrm{T})(\alpha\beta^2 - L^2S)}{LW\beta}\right)$$

$$C = \frac{S}{W}\left(\frac{\alpha h_\mathrm{H}}{W} - \beta - \frac{\alpha}{W}\right)$$

$$\ddot{\varphi}(t) - a\varphi(t) = -Q(t)$$

Upper position: unstable position

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Motivation

Stick balancing

stick length

reaction time delay

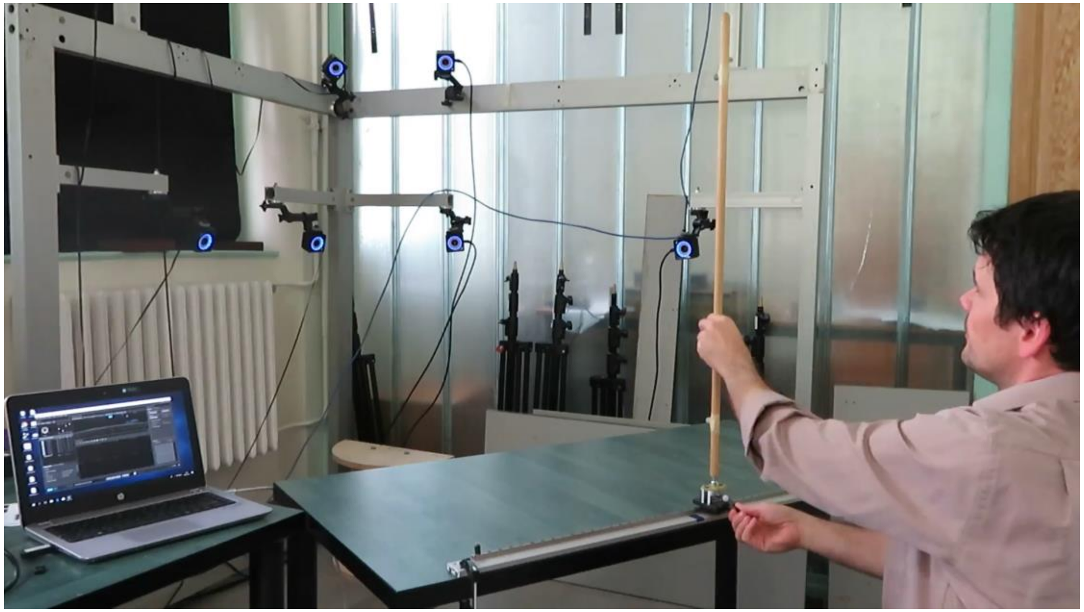
sensory uncertainty

~ critical length?

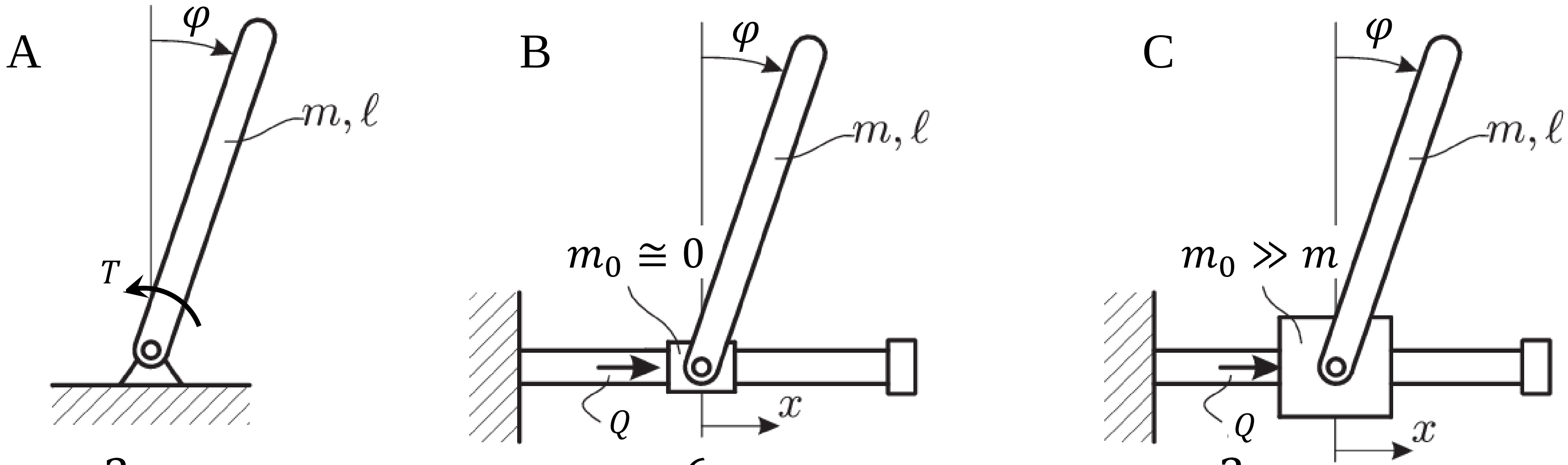
Stick balancing on fingertip

Stick balancing on pingpong racket

Balancing a linearly driven stick



Stick balancing model

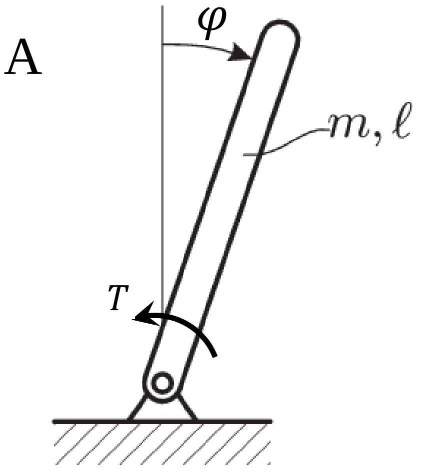


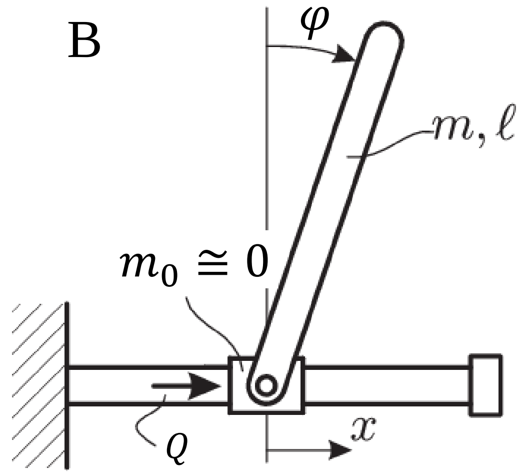
$$\ddot{\varphi} - \frac{3g}{2l}\varphi = c_A T$$

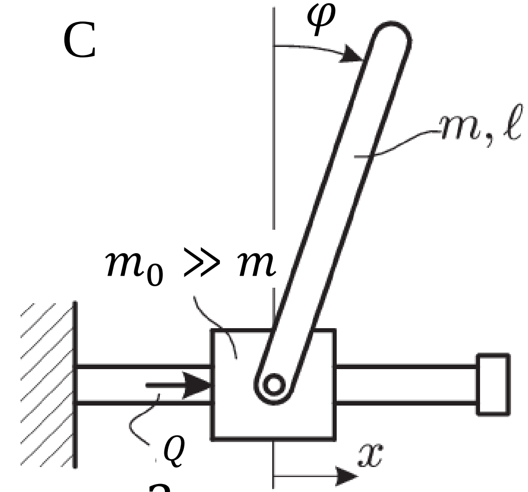
$$\ddot{\varphi} - \frac{6g}{l}\varphi = c_B Q$$

$$\ddot{\varphi} - \frac{3g}{2l}\varphi = c_C Q$$

Stick balancing model







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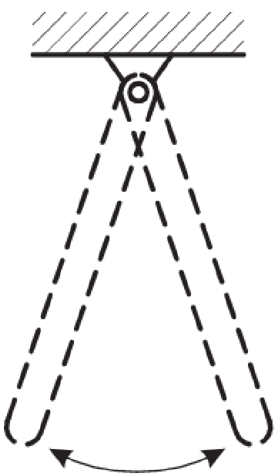
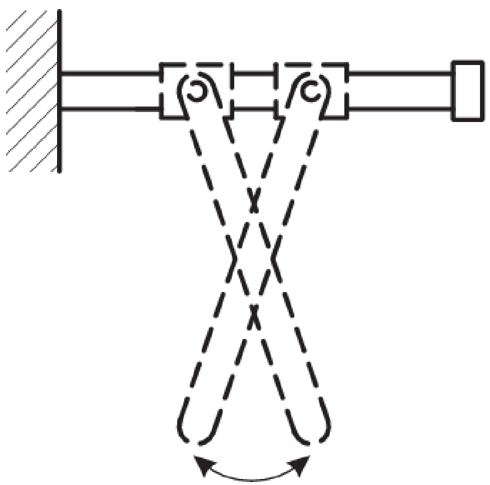
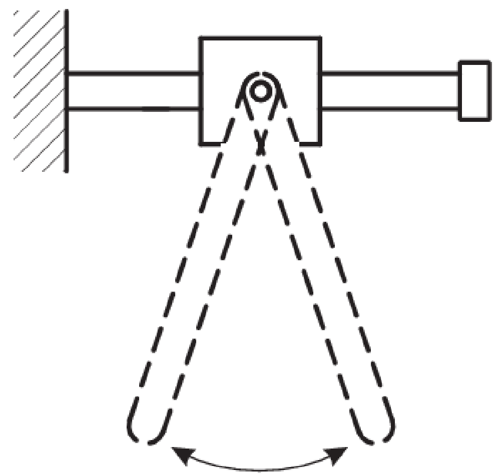






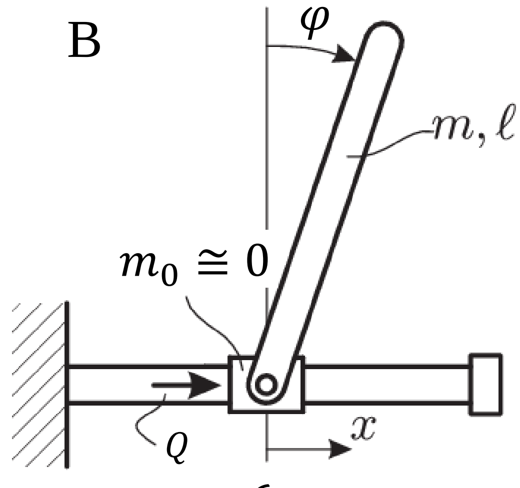
$$T_\mathrm{A} = 2\pi\sqrt{2l/(3g)}$$

$$T_\mathrm{B} = 2\pi\sqrt{l/(6g)} = T_\mathrm{A}/2$$

$$T_\mathrm{C} = 2\pi\sqrt{2l/(3g)} = T_\mathrm{A}$$

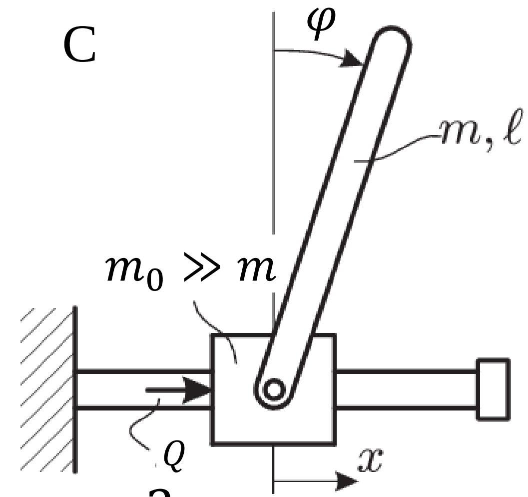
[redacted] model

B

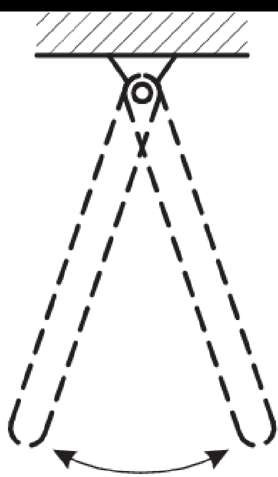


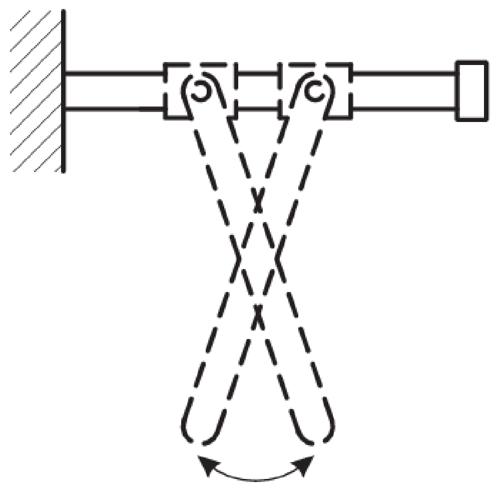
$$\ddot{\varphi} - \frac{6g}{l}\varphi = c_B Q$$

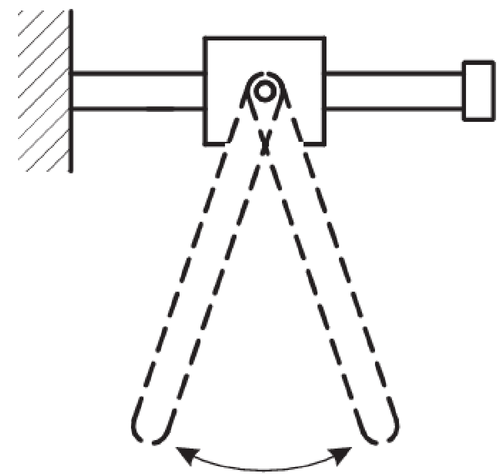
C



$$\ddot{\varphi} - \frac{3g}{2l}\varphi = c_C Q$$







$$T_\mathrm{A} = 2\pi\sqrt{2l/(3g)}$$

$$T_\mathrm{B} = 2\pi\sqrt{l/(6g)} = T_\mathrm{A}/2$$

$$T_\mathrm{C} = 2\pi\sqrt{2l/(3g)} = T_\mathrm{A}$$

Stick balancing on the fingertip

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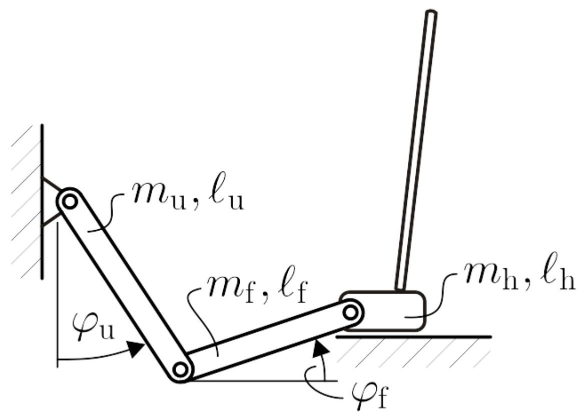

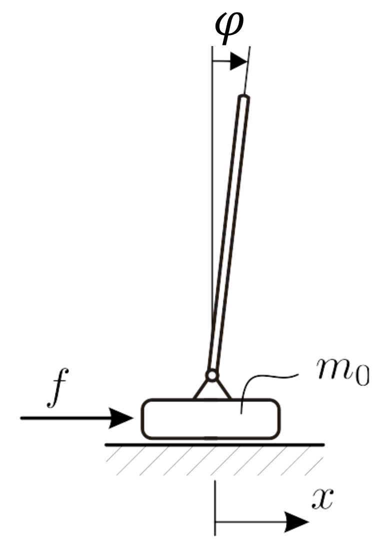

segment	mass	length
upper arm	$m_u = 1.775\text{kg}$	$\ell_u = 0.2874\text{m}$
forearm	$m_f = 1.015\text{kg}$	$\ell_f = 0.2666\text{m}$
hand	$m_h = 1.015\text{kg}$	$\ell_h = 0.0821\text{m}$

(de Leva, 1996)

$\Rightarrow \quad m_0 \cong 2.3\text{kg} \qquad m_0 \gg m$

$\downarrow$

case C

Reaction delay

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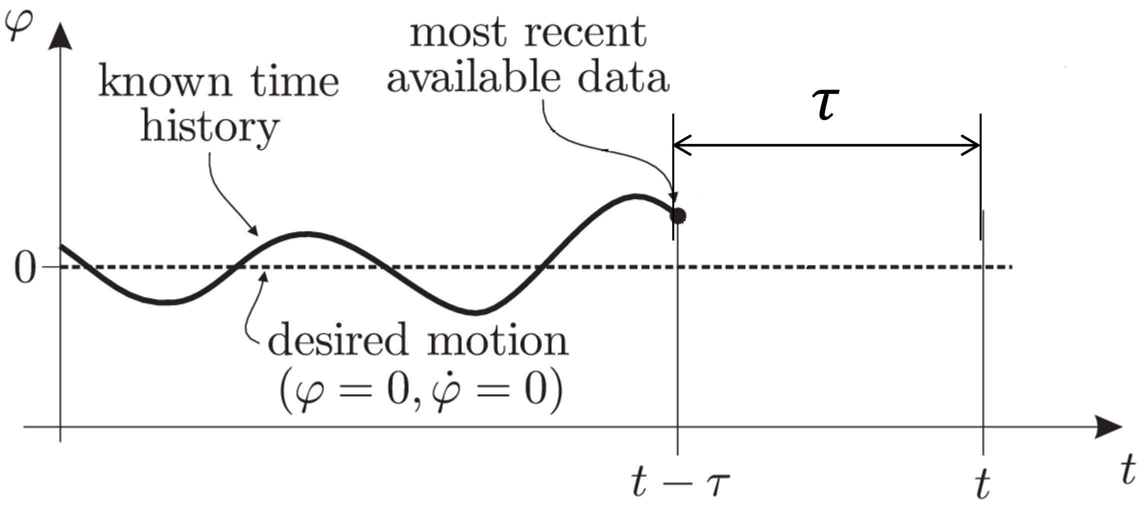
$$\ddot{\varphi}(t) - \frac{3g}{2l}\varphi(t) = -\frac{6}{ml}Q(t)$$

$$Q(t) = f(\varphi(\vartheta), \dot{\varphi}(\vartheta), \ddot{\varphi}(\vartheta)) \qquad \vartheta \in [0, t-\tau]$$

For example $Q(t) = -k_\mathrm{p}\varphi(t-\tau) - k_\mathrm{d}\dot{\varphi}(t-\tau)$



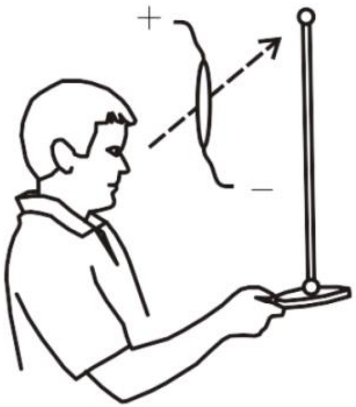
delay for visual tracking
Nasher (1976): 150~250ms
Miall (1993): 200~250ms
Jordan (1996): 100~200ms
Kawato (1999): 150~250ms

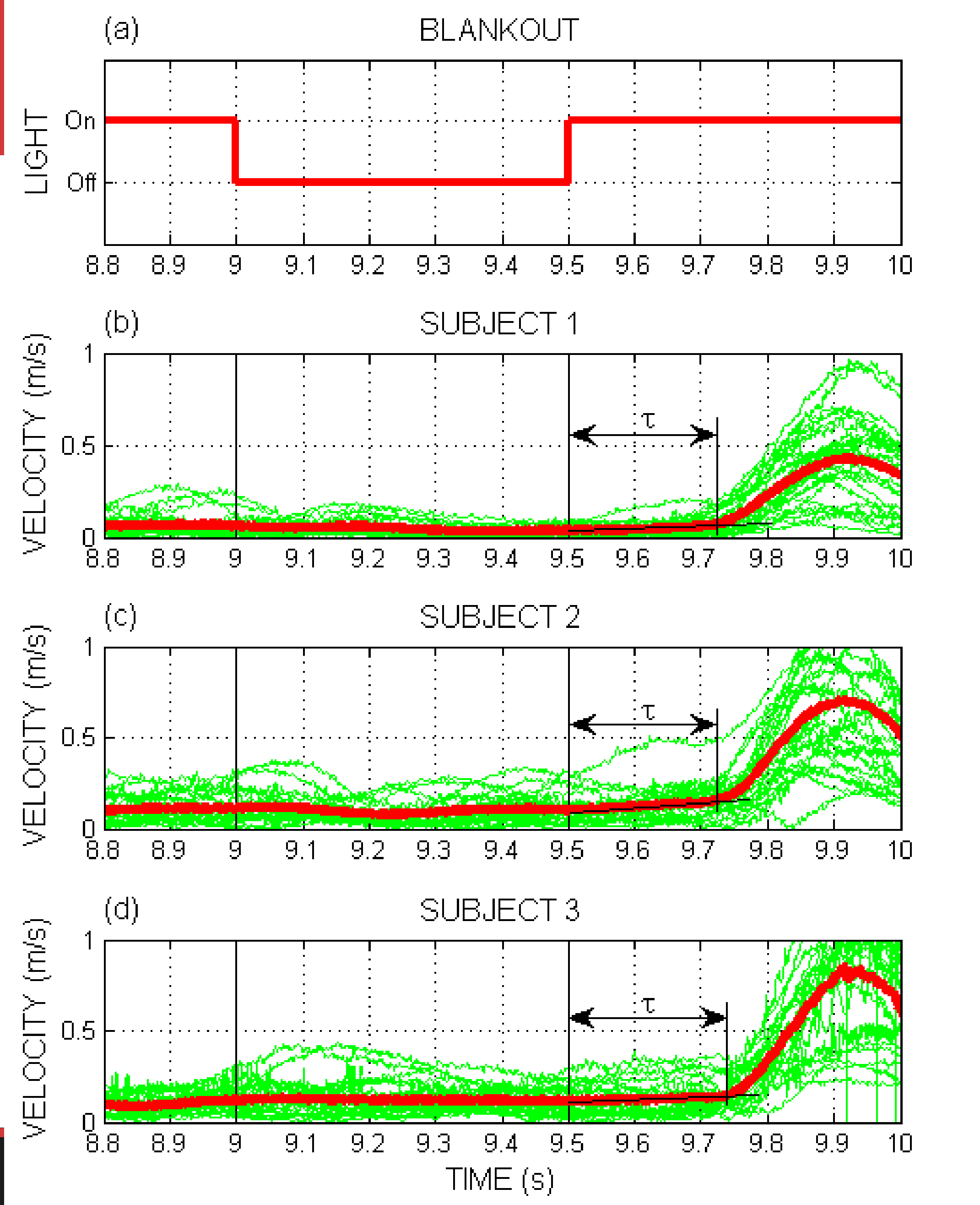
delay for stick balancing using cross-correlation:
Cabrera, Milton (2004): 80~200ms

Reaction delay

blankout tests:
Milton (2011):
$\tau \approx 230\text{ms}$

LCD
shutter





Stick balancing model

$$\ddot{\varphi}(t) - \frac{3g}{2l}\varphi(t) = -\frac{6}{ml}Q(t)$$

$$Q(t) = f(\varphi(\vartheta), \dot{\varphi}(\vartheta), \ddot{\varphi}(\vartheta)) \qquad \vartheta \in [0, t-\tau]$$

What is the control law? $Q(t) = ?$

PD feedback: $Q(t) = -K_\mathrm{p}\varphi(t-\tau) - K_\mathrm{d}\dot{\varphi}(t-\tau)$

PDA feedback: $Q(t) = -K_\mathrm{p}\varphi(t-\tau) - K_\mathrm{d}\dot{\varphi}(t-\tau) - K_\mathrm{a}\ddot{\varphi}(t-\tau)$

Predictor feedback (PF): $Q(t) = -K_\mathrm{p}\varphi_\mathrm{p}(t) - K\dot{\varphi}_\mathrm{p}(t)$

Act-and-wait control:

$$Q(t) = \begin{cases} 0 & \text{if} \quad 0 \le t < t_\mathrm{w} \quad \text{(wait)} \\ -K_\mathrm{p}\varphi(t-\tau) - K\dot{\varphi}(t-\tau) & \text{if} \quad t_\mathrm{w} \le t < t_\mathrm{w} + t_\mathrm{a} = T \ \text{(act)} \end{cases}$$

Delayed PD feedback

$$\ddot{\varphi}(t) - a\varphi(t) = -k_{\mathrm{p}}\varphi(t-\tau) - k_{\mathrm{d}}\dot{\varphi}(t-\tau)$$

$$a = \frac{3g}{2l}$$

(system parameter)

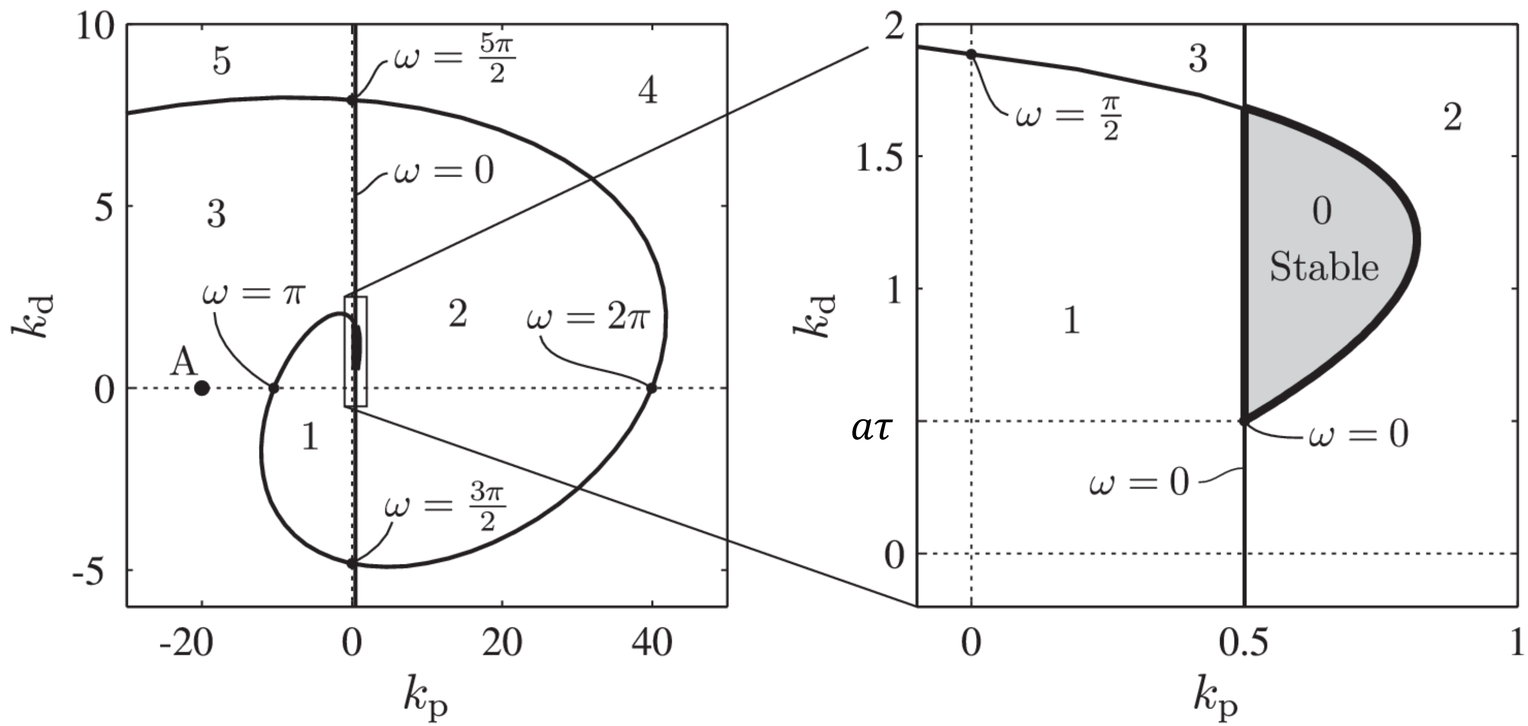
D-subdivision:

$\omega = 0$: $k_{\mathrm{p}} = a$

$\omega \neq 0$: $k_{\mathrm{p}} = (\omega^2 + a)\cos(\omega\tau)$

$$k_{\mathrm{d}} = \frac{\omega^2 + a}{\omega}\sin(\omega\tau)$$

$\tau = 1, a = 0.5$



Delayed PD feedback

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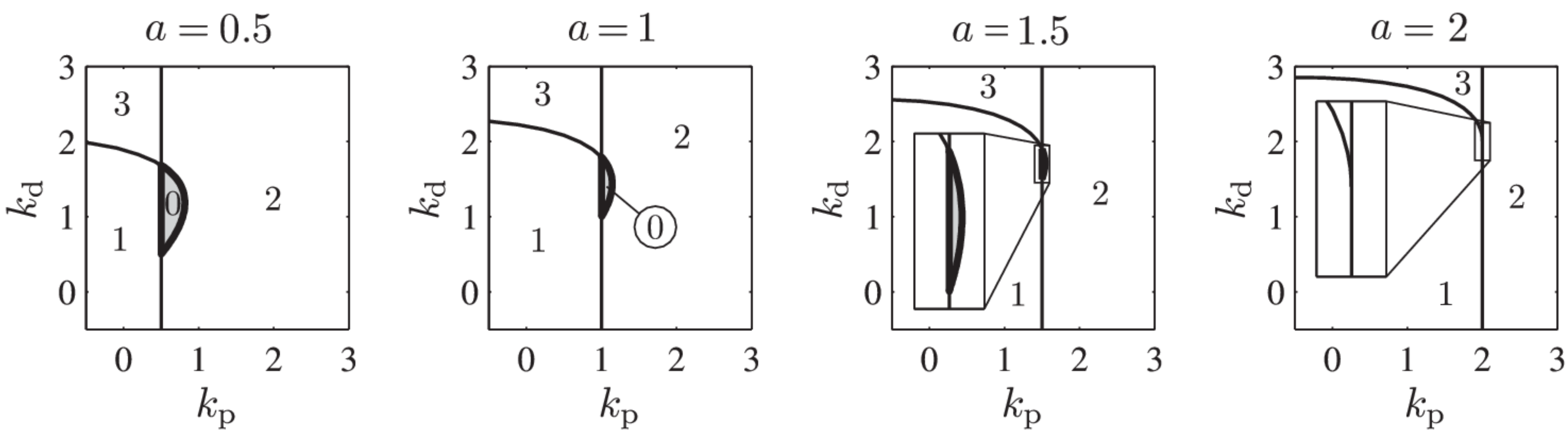
$$\ddot{\varphi}(t) - a\varphi(t) = -k_{\mathrm{p}}\varphi(t-\tau) - k_{\mathrm{d}}\dot{\varphi}(t-\tau)$$

$\tau = 1$



$$a_{\mathrm{crit}} = \frac{2}{\tau^2}$$ (Schürer, 1948)

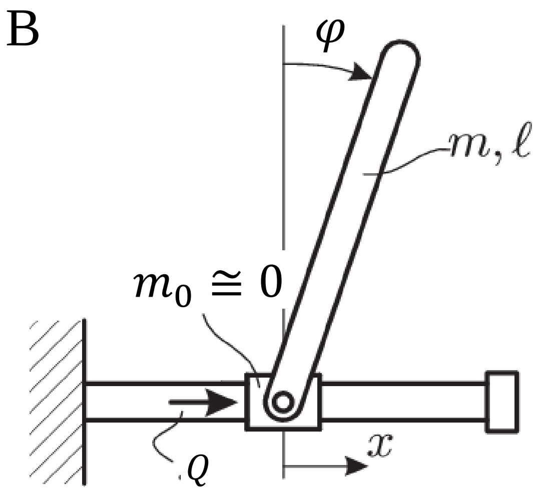
Or, for fixed a, $\tau_{\mathrm{crit}} = \sqrt{\frac{2}{a}} = \frac{T_{\mathrm{p}}}{\pi\sqrt{2}}$, T_{p}: downward oscillation period

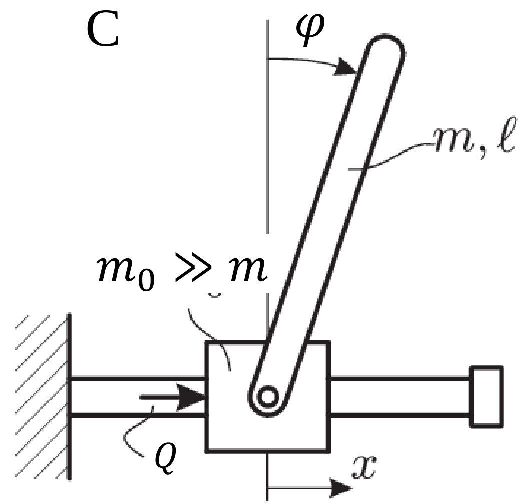
(Stepan, 2009)

Delayed PD feedback

$$\ddot{\varphi}(t) - a\varphi(t) = -k_\mathrm{p}\varphi(t-\tau) - k_\mathrm{d}\dot{\varphi}(t-\tau)$$

$$\tau = 230\mathrm{ms}$$





$$a = \frac{6g}{l}$$

$$a_\mathrm{crit} = \frac{2}{\tau^2}$$

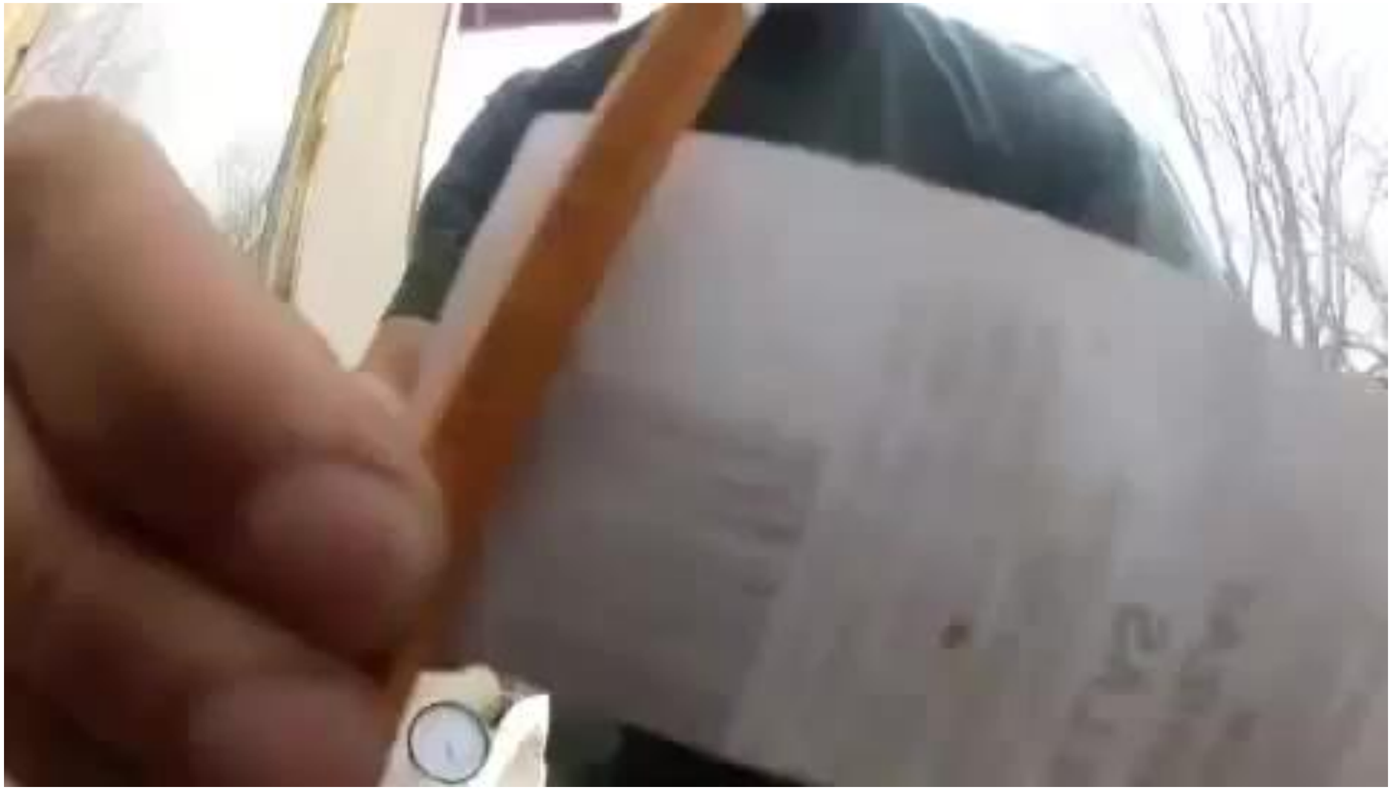
$$a = \frac{3g}{2l}$$

$$l_{\mathrm{crit-B}} = 3g\tau^2 = 156\mathrm{cm}$$

$$l_{\mathrm{crit-C}} = \frac{3}{4}g\tau^2 = 39\mathrm{cm}$$

Experiments: $l_\mathrm{crit} = 25\sim30\mathrm{cm}$ (Milton et al., 1990–)

Delayed PD feedback??



https://www.youtube.com/watch?v=Z6tDflmU0bo&feature=youtu.be

$$l_{\text{crit-C}} = \frac{3}{4} g\tau^2 = \boxed{39\text{cm}}$$

Experiments: $l_{\text{crit}} = 25{\sim}30\text{cm}$ (Milton et al., 1990–)

Stick balancing model

$$\ddot{\varphi}(t) - \frac{3g}{2l}\varphi(t) = -\frac{6}{ml}Q(t)$$

$$Q(t) = f(\varphi(\vartheta), \dot{\varphi}(\vartheta), \ddot{\varphi}(\vartheta)) \qquad \vartheta \in [0, t-\tau]$$

What is the control law? $Q(t) = ?$

PD feedback: $Q(t) = -K_\mathrm{p}\varphi(t-\tau) - k_\mathrm{d}\dot{\varphi}(t-\tau)$

PDA feedback: $Q(t) = -K_\mathrm{p}\varphi(t-\tau) - K_\mathrm{d}\dot{\varphi}(t-\tau) - K_\mathrm{a}\ddot{\varphi}(t-\tau)$

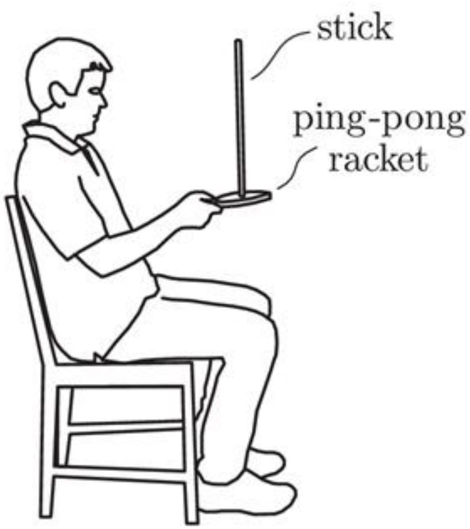
Predictor feedback (PF): $Q(t) = -K_\mathrm{p}\varphi_\mathrm{p}(t) - K\dot{\varphi}_\mathrm{p}(t)$

Act-and-wait control:

$$Q(t) = \begin{cases} 0 & \text{if} \quad 0 \le t < t_\mathrm{w} \quad \text{(wait)} \\ -K_\mathrm{p}\varphi(t-\tau) - K_\mathrm{d}\dot{\varphi}(t-\tau) & \text{if} \quad t_\mathrm{w} \le t < t_\mathrm{w} + t_\mathrm{a} = T \ \text{(act)} \end{cases}$$

Delayed PDA feedback

$$\ddot{\varphi}(t) - a\varphi(t) = -k_{\mathrm{p}}\varphi(t-\tau) - k_{\mathrm{d}}\dot{\varphi}(t-\tau) - k_{\mathrm{a}}\ddot{\varphi}(t-\tau)$$



No sensory feedback from fingertip $\Longrightarrow$ PD

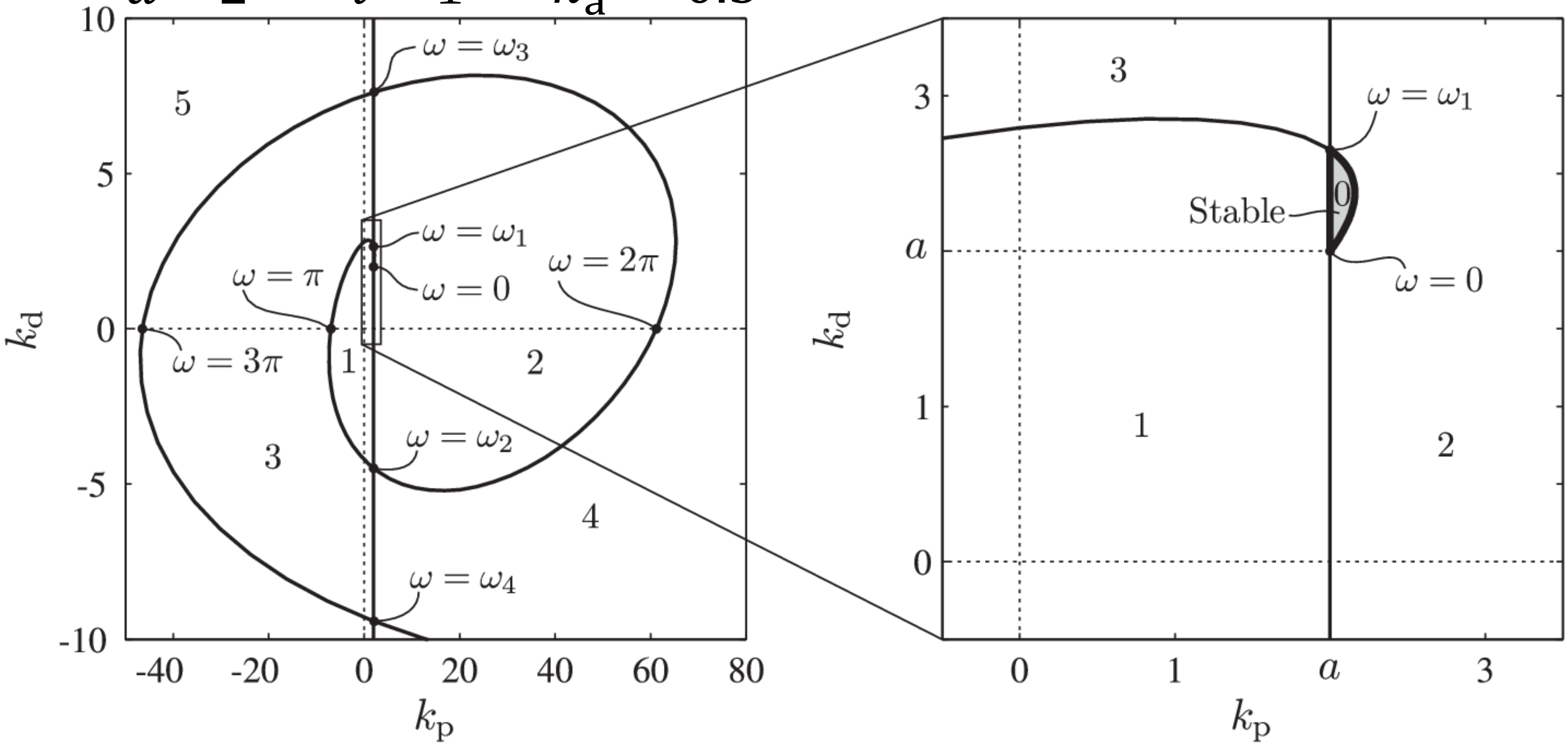
Sensory feedback from fingertip $\Longrightarrow$ PDA (?)

Delayed PDA feedback

$$\ddot{\varphi}(t) - a\varphi(t) = -k_{\mathrm{p}}\varphi(t-\tau) - k_{\mathrm{d}}\dot{\varphi}(t-\tau) - k_{\mathrm{a}}\ddot{\varphi}(t-\tau)$$

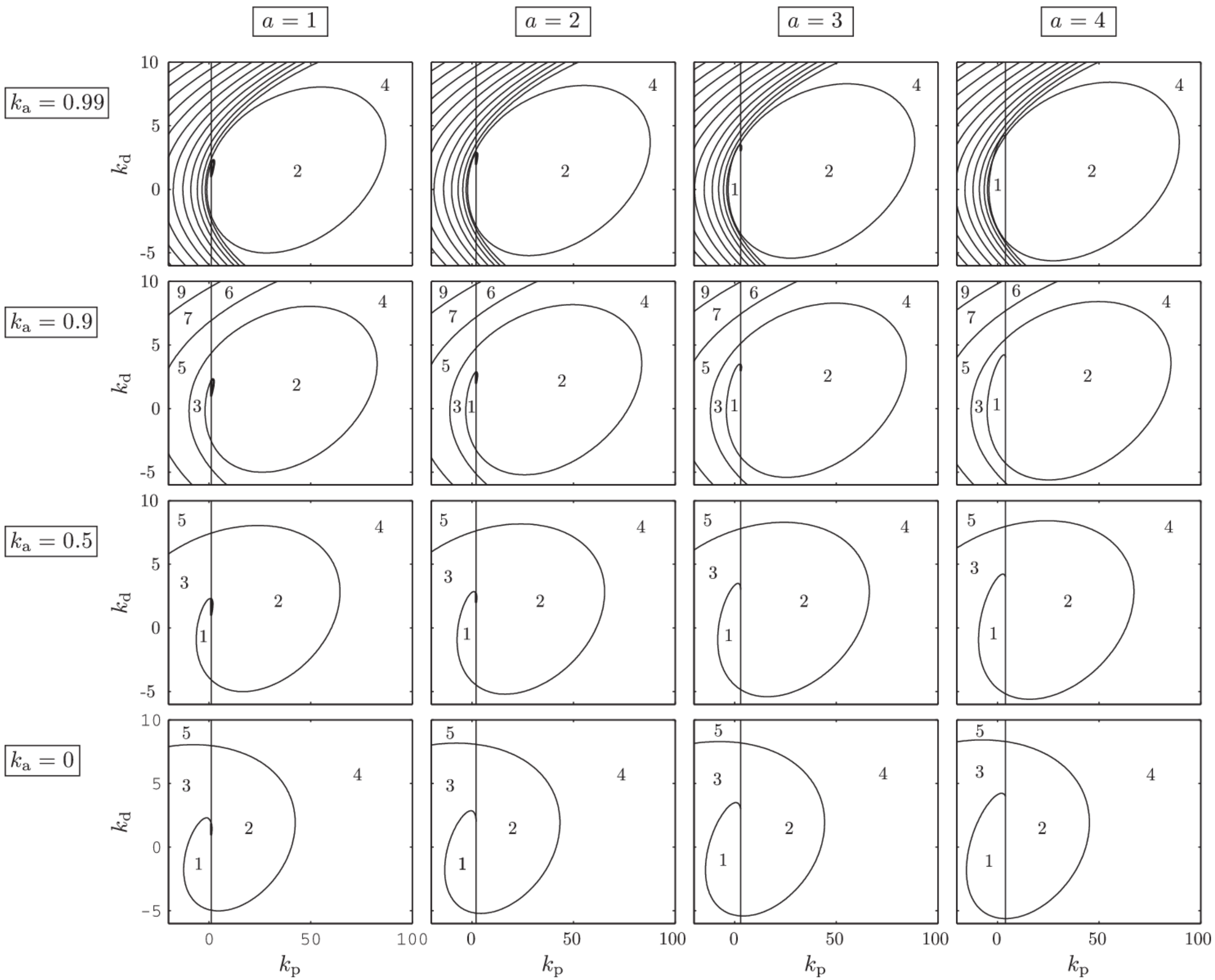
Necessary condition for stability: $|k_{\mathrm{a}}| \leq 1$

$a = 2$ $\tau = 1$ $k_{\mathrm{a}} = 0.5$



Delayed PDA feedback

$$\ddot{\varphi}(t) - a\varphi(t) = -k_\mathrm{p}\varphi(t-\tau) - k_\mathrm{d}\dot{\varphi}(t-\tau) - k_\mathrm{a}\ddot{\varphi}(t-\tau)$$

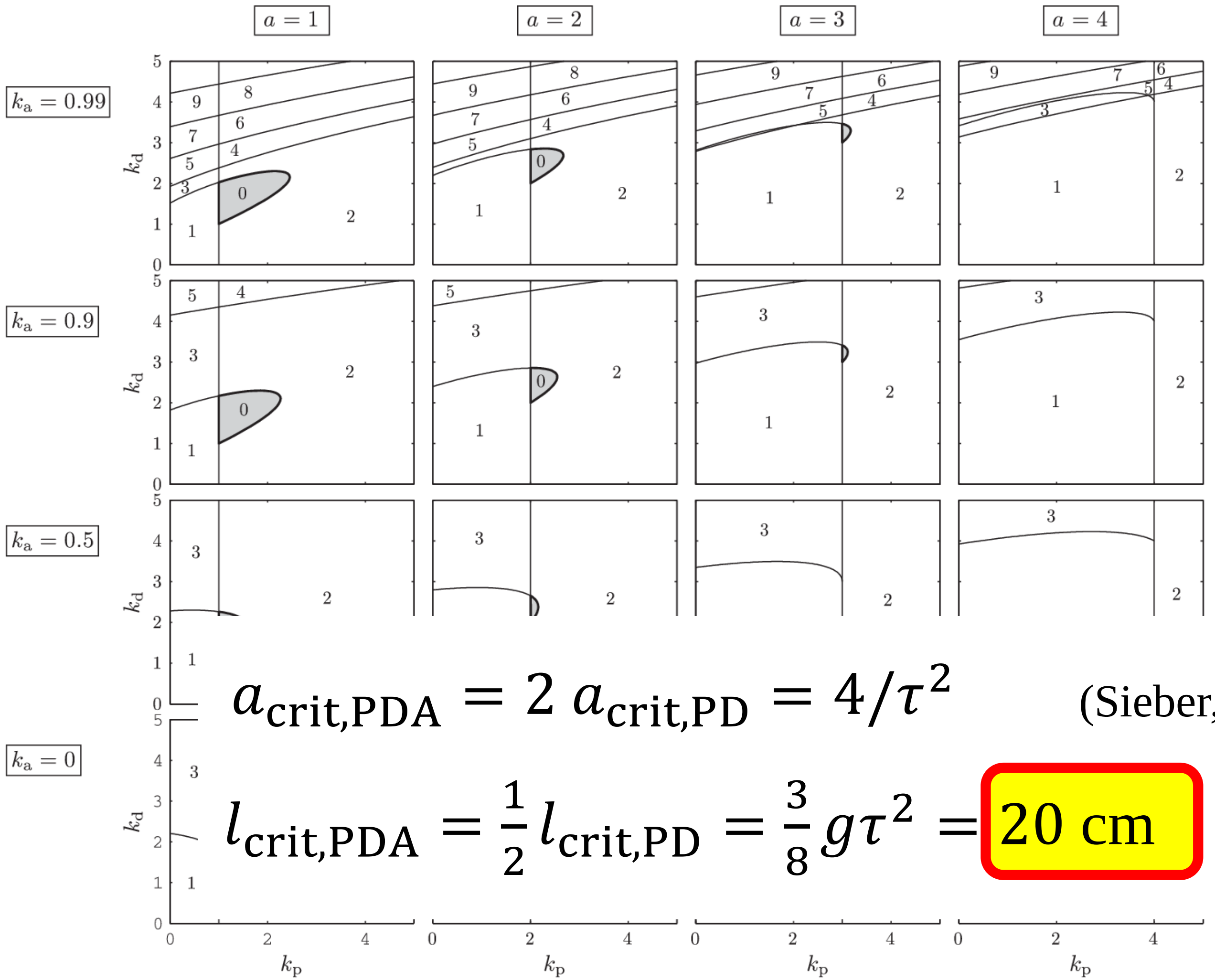


$\tau = 1$

$|k_\mathrm{a}| \leq 1$

Delayed PDA feedback

$$\ddot{\varphi}(t) - a\varphi(t) = -k_\mathrm{p}\varphi(t-\tau) - k_\mathrm{d}\dot{\varphi}(t-\tau) - k_\mathrm{a}\ddot{\varphi}(t-\tau)$$



$\tau = 1$

$|k_\mathrm{a}| \leq 1$

$$a_\mathrm{crit,PDA} = 2\, a_\mathrm{crit,PD} = 4/\tau^2$$

(Sieber, Krauskopf, 2005)

$$l_\mathrm{crit,PDA} = \frac{1}{2} l_\mathrm{crit,PD} = \frac{3}{8} g\tau^2 = 20\ \mathrm{cm}$$

Stick balancing model

$$\ddot{\varphi}(t) - \frac{3g}{2l}\varphi(t) = -\frac{6}{ml}Q(t)$$

$$Q(t) = f(\varphi(\vartheta), \dot{\varphi}(\vartheta), \ddot{\varphi}(\vartheta)) \qquad \vartheta \in [0, t-\tau]$$

What is the control law? $Q(t) = ?$

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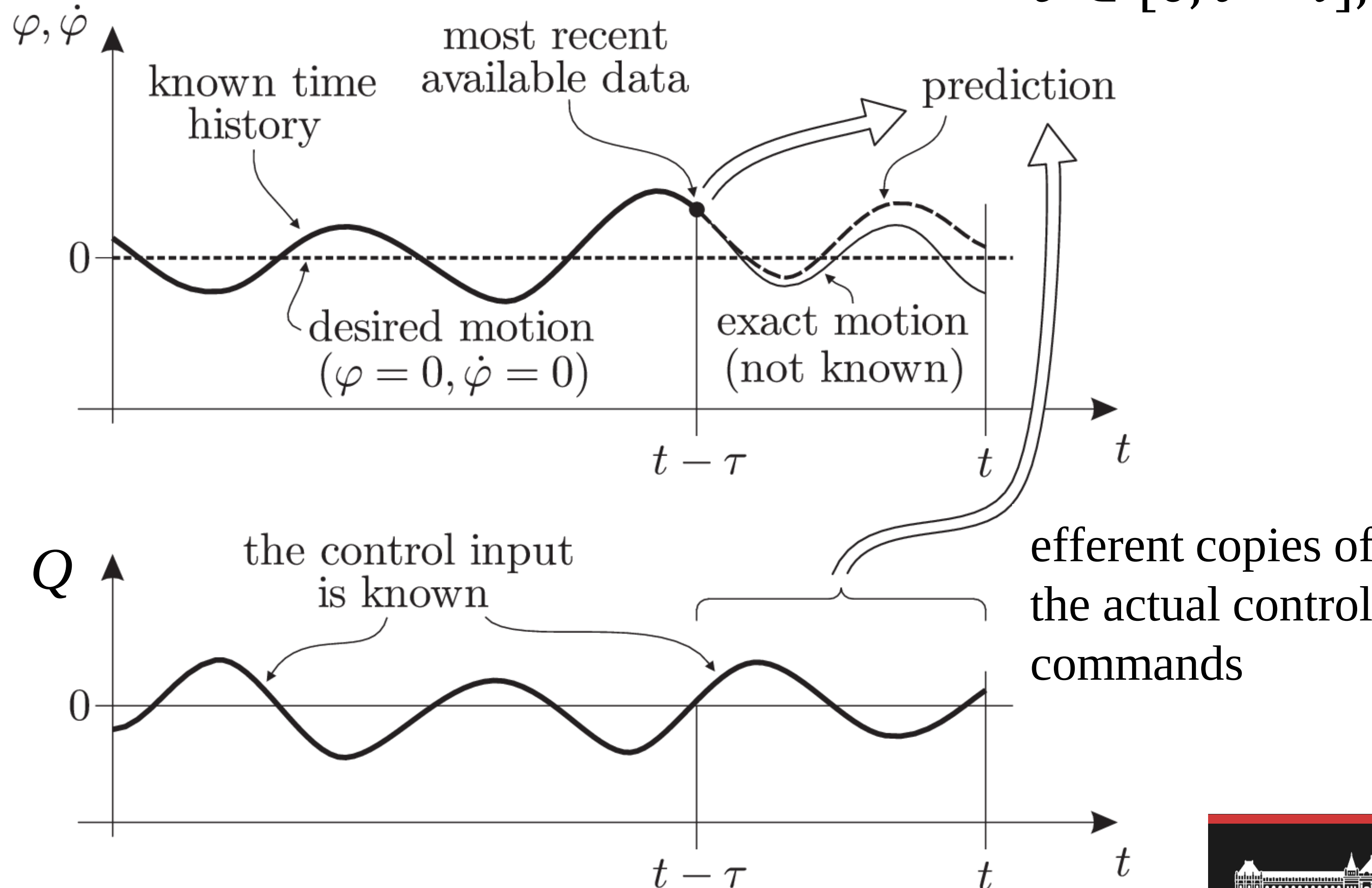
Predictor feedback

$$\ddot{\varphi}(t) - a\varphi(t) = -Q(t) \qquad Q(t) = f\big(\varphi(\vartheta), \dot{\varphi}(\vartheta), \ddot{\varphi}(\vartheta), Q(\xi)\big)$$

$$\vartheta \in [0, t-\tau], \quad \xi \in [0, t]$$



Predictor-based feedback
Finite Spectrum Assignment
Modified Smith predictor

Mayne (1968), Kleinman (1969)
Manitius and Olbrot (1978)
Michiels, Niculescu, Mondie, Krstic, Jankovic, Wang, Karafyllis, Mirkin, Zhong, …

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t-\tau)$$

$$\mathbf{x}(t) = \begin{pmatrix} \varphi(t) \\ \dot{\varphi}(t) \end{pmatrix}, \quad \mathbf{A} = \begin{pmatrix} 0 & 1 \\ a & 0 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} 0 \\ -1 \end{pmatrix}, \quad \mathbf{u}(t-\tau) = Q(t-\tau)$$

Predictor-based feedback
Finite Spectrum Assignment
Modified Smith predictor

Mayne (1968), Kleinman (1969)
Manitius and Olbrot (1978)
Michiels, Niculescu, Mondie, Krstic, Jankovic, Wang, Karafyllis, Mirkin, Zhong, …

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t-\tau)$$

Prediction of $\mathbf{x}(t+\tau)$ from $\mathbf{x}(t)$:

$$\dot{\mathbf{x}}_{\mathrm{p}}(\vartheta) = \widetilde{\mathbf{A}}\mathbf{x}_{\mathrm{p}}(\vartheta) + \widetilde{\mathbf{B}}\mathbf{u}(\vartheta-\tilde{\tau}), \qquad \vartheta \in [t, t+\tilde{\tau}), \qquad \mathbf{x}_{\mathrm{p}}(t) = \mathbf{x}(t)$$

$$\mathbf{x}_{\mathrm{p}}(t+\tilde{\tau}) = \mathrm{e}^{\widetilde{\mathbf{A}}\tilde{\tau}}\mathbf{x}(t) + \int_t^{t+\tilde{\tau}} \mathrm{e}^{\widetilde{\mathbf{A}}(t+\tilde{\tau}-\vartheta)}\widetilde{\mathbf{B}}\mathbf{u}(\vartheta-\tilde{\tau})\mathrm{d}\vartheta$$

Controller:

$$\mathbf{u}(t) = \mathbf{K}\mathbf{x}_{\mathrm{p}}(t+\tilde{\tau}) = \mathbf{K}\mathrm{e}^{\widetilde{\mathbf{A}}\tilde{\tau}}\mathbf{x}(t) + \mathbf{K}\int_t^{t+\tilde{\tau}} \mathrm{e}^{\widetilde{\mathbf{A}}(t+\tilde{\tau}-\vartheta)}\widetilde{\mathbf{B}}\mathbf{u}(\vartheta-\tilde{\tau})\mathrm{d}\vartheta$$

If $\widetilde{\mathbf{A}} = \mathbf{A}, \widetilde{\mathbf{B}} = \mathbf{B}$ and $\tilde{\tau} = \tau$ then $\mathbf{x}_{\mathrm{p}}(t+\tilde{\tau}) = \mathbf{x}(t+\tau)$

$$\Longrightarrow \mathbf{u}(t-\tau) = \mathbf{K}\mathbf{x}(t) \quad \Longrightarrow \quad \dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{K}\mathbf{x}(t) \quad \Longrightarrow \quad l_{\mathrm{crit,PF}} = 0$$

Stick balancing model

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PD feedback: $Q(t) = -K_\mathrm{p}\varphi(t-\tau) - K_\mathrm{d}\dot{\varphi}(t-\tau)$

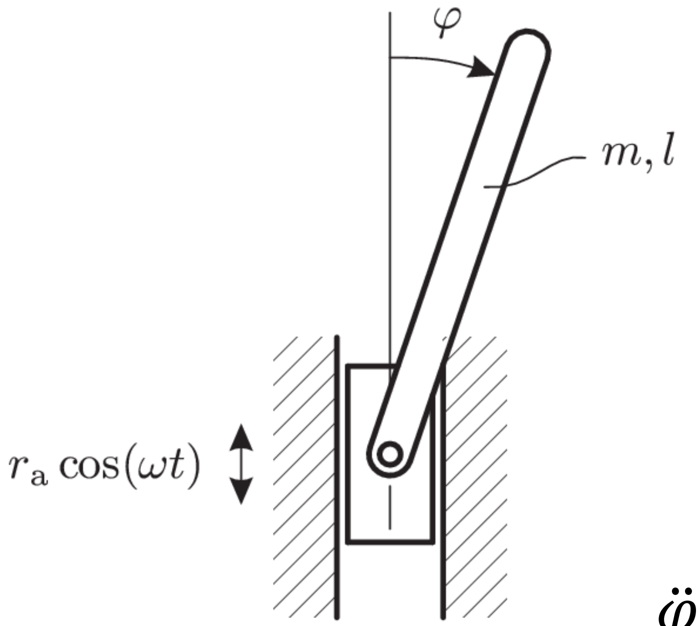
PDA feedback: $Q(t) = -K_\mathrm{p}\varphi(t-\tau) - K_\mathrm{d}\dot{\varphi}(t-\tau) - K_\mathrm{a}\ddot{\varphi}(t-\tau)$

Predictor feedback (PF): $Q(t) = -K_\mathrm{p}\varphi_\mathrm{p}(t) - K_\mathrm{d}\dot{\varphi}_\mathrm{p}(t)$

Act-and-wait control:

$$Q(t) = \begin{cases} 0 & \text{if} \quad 0 \le t < t_\mathrm{w} \ \text{(wait)} \\ -K_\mathrm{p}\varphi(t-\tau) - K_\mathrm{d}\dot{\varphi}(t-\tau) & \text{if} \quad t_\mathrm{w} \le t < t_\mathrm{w} + t_\mathrm{a} = T \ \text{(act)} \end{cases}$$

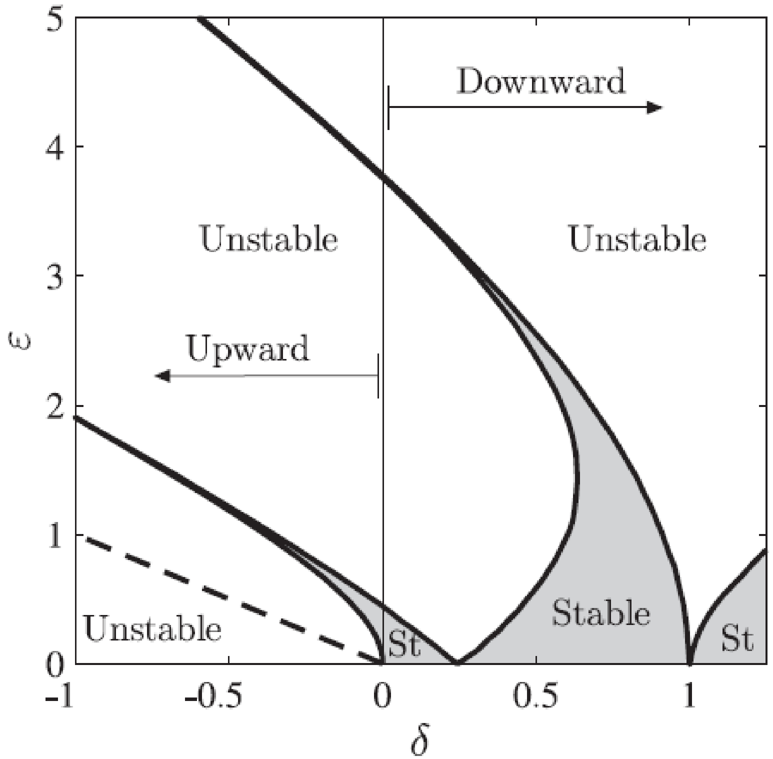
Motivation: parametric forcing of the inverted pendulum

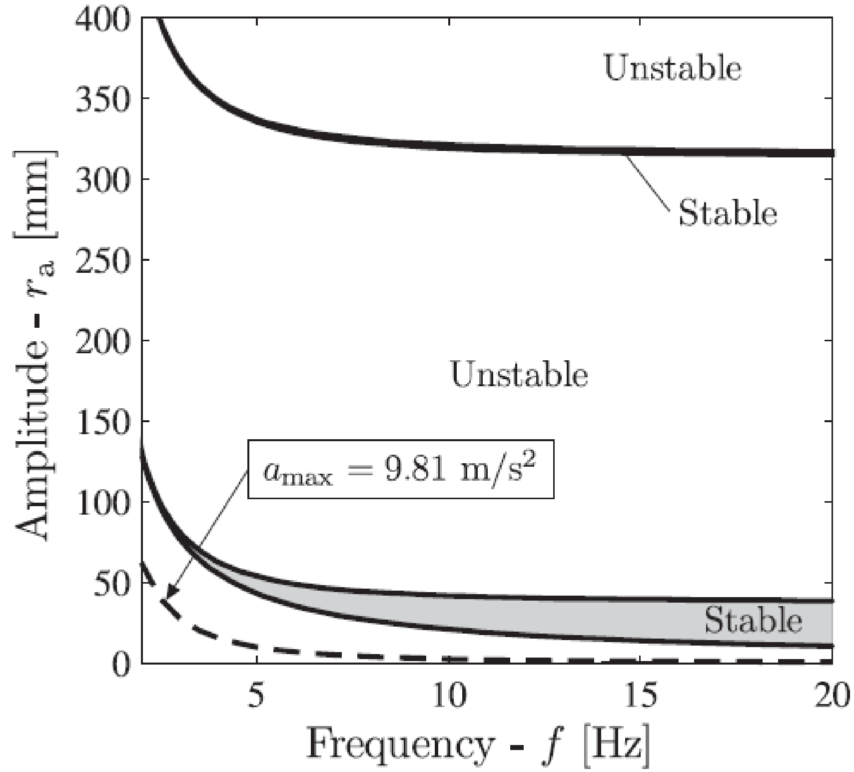


$$\ddot{\varphi}(t) + \left(-\frac{3g}{2l} + \frac{3r_a\omega^2}{2l}\cos(\omega t)\right)\varphi(t) = 0$$

Mathieu equation: $\ddot{\varphi}(t) + (\delta + \varepsilon\cos(\omega t))\varphi(t) = 0$

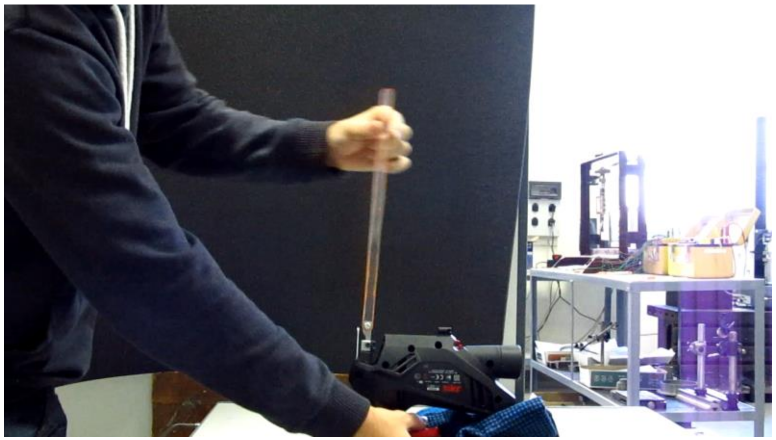
Motivation: parametric forcing of the inverted pendulum

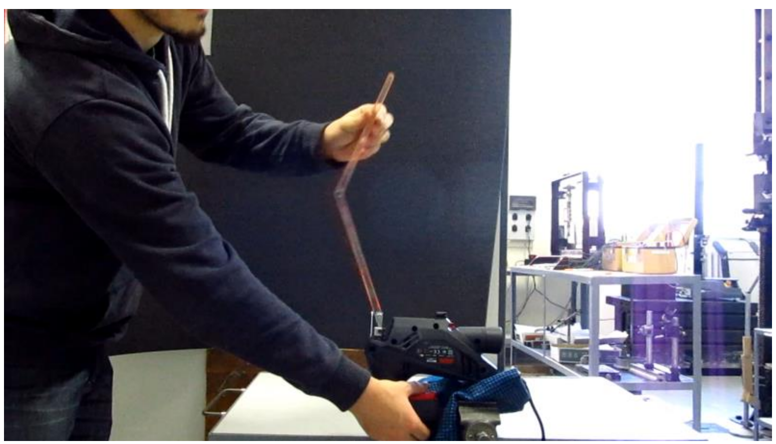




Mathieu equation: $\ddot{\varphi}(t) + (\delta + \varepsilon\cos(\omega t))\varphi(t) = 0$

Motivation: parametric forcing of the inverted pendulum





Act-and-wait control

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t)$$
$$\mathbf{u}(t) = g(t)\mathbf{K}\,\mathbf{x}(t-\tau)$$

Act-and-wait control

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t)$$
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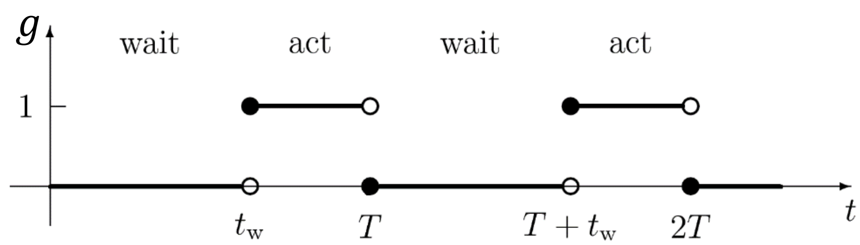

(Insperger, Stépán 2006)

$$g(t) = \begin{cases} 0 & \text{if} \quad 0 \le (t \bmod T) < t_w \quad \text{(wait)} \\ 1 & \text{if} \quad t_w \le (t \bmod T) < t_w + t_a = T \ \text{(act)} \end{cases}$$

Act-and-wait control

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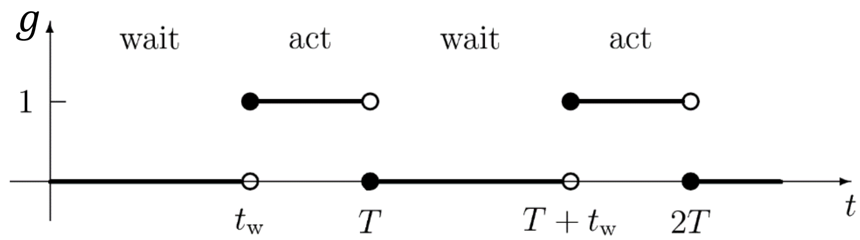
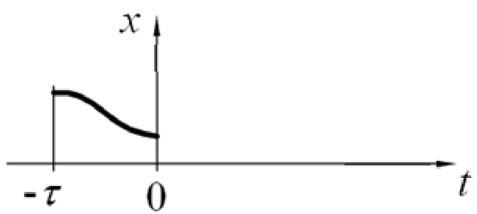


(Insperger, Stépán 2006)

$$\mathrm{g}(t) = \begin{cases} 0 & \text{if} \quad 0 \le (t \bmod T) < t_\mathrm{w} \quad \text{(wait)} \\ 1 & \text{if} \quad t_\mathrm{w} \le (t \bmod T) < t_\mathrm{w} + t_\mathrm{a} = T \text{ (act)} \end{cases}$$

Step-by-step solution ($t_\mathrm{w} \ge \tau$ and $t_\mathrm{a} \le \tau$):



Act-and-wait control

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$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t)$$
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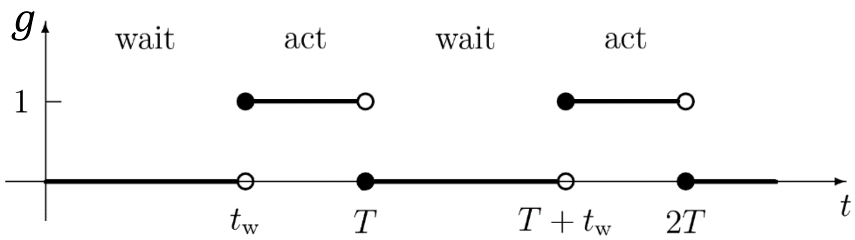
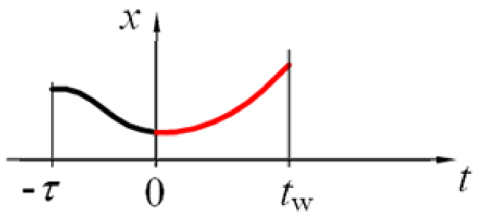


(Insperger, Stépán 2006)

$$\mathrm{g}(t) = \begin{cases} 0 & \text{if} \quad 0 \le (t \bmod T) < t_\mathrm{w} \quad \text{(wait)} \\ 1 & \text{if} \quad t_\mathrm{w} \le (t \bmod T) < t_\mathrm{w} + t_\mathrm{a} = T \ \text{(act)} \end{cases}$$

Step-by-step solution ($t_\mathrm{w} \ge \tau$ and $t_\mathrm{a} \le \tau$):

$$t \in [0, t_\mathrm{w}):\ \ \dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) \ \rightarrow\ \mathbf{x}(t) = \mathrm{e}^{\mathbf{A}t}\mathbf{x}(0)$$



Act-and-wait control

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t)$$
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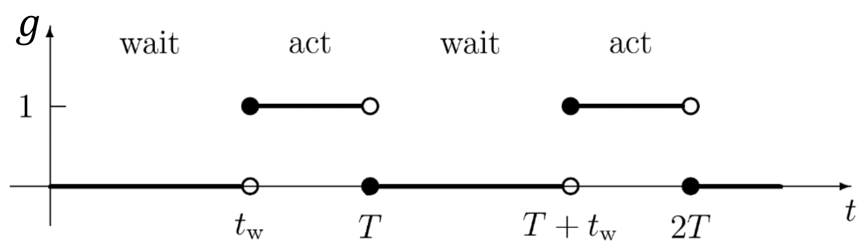
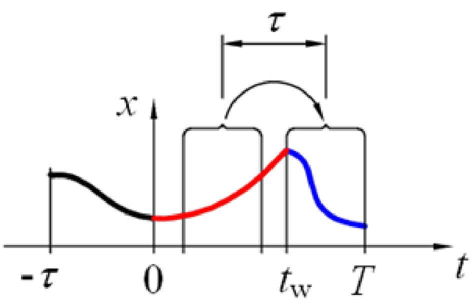


(Insperger, Stépán 2006)

$$\mathrm{g}(t) = \begin{cases} 0 & \text{if} \quad 0 \le (t \bmod T) < t_\mathrm{w} \quad \text{(wait)} \\ 1 & \text{if} \quad t_\mathrm{w} \le (t \bmod T) < t_\mathrm{w} + t_\mathrm{a} = T \ \text{(act)} \end{cases}$$

Step-by-step solution ($t_\mathrm{w} \ge \tau$ and $t_\mathrm{a} \le \tau$):



$$t \in [0, t_\mathrm{w}): \ \dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) \ \rightarrow \ \mathbf{x}(t) = \mathrm{e}^{\mathbf{A}t}\mathbf{x}(0)$$

$$t \in [t_\mathrm{w}, T): \ \dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{K}\,\mathbf{x}(t-\tau) = \mathbf{A}\mathbf{x}(t) + \mathbf{K}\mathrm{e}^{\mathbf{A}(t-\tau)}\mathbf{x}(0)$$

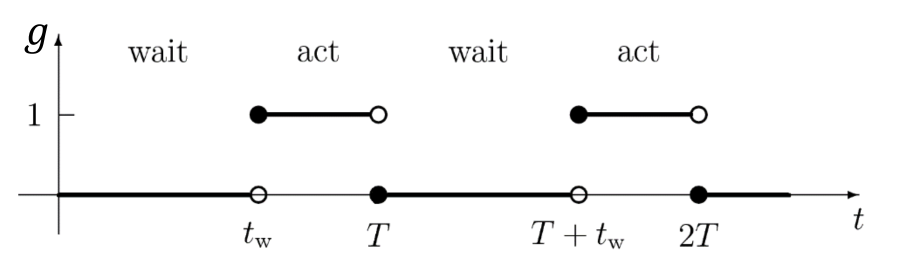
$$\rightarrow \quad \mathbf{x}(T) = \underbrace{\left(\mathrm{e}^{\mathbf{A}T} + \int_{t_\mathrm{w}}^{T} \mathrm{e}^{\mathbf{A}(T-s)}\mathbf{B}\mathbf{K}\mathrm{e}^{\mathbf{A}(s-\tau)}\right)}_{\boldsymbol{\Phi} \in \mathbb{R}^{n\times n}} \mathbf{x}(0)$$

$(\mathbf{x} \in \mathbb{R}^n)$

Finite dimensional map

Act-and-wait control

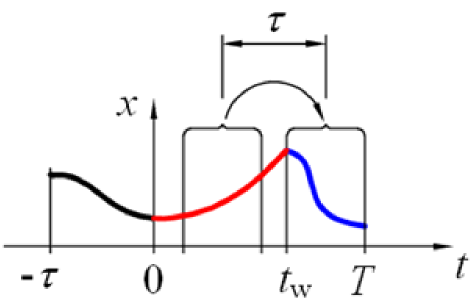
$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t)$$
$$\mathbf{u}(t) = g(t)\mathbf{K}\,\mathbf{x}(t-\tau)$$



(Insperger, Stépán 2006)

$$g(t) = \begin{cases} 0 & \text{if} \quad 0 \le (t \bmod T) < t_w \quad \text{(wait)} \\ 1 & \text{if} \quad t_w \le (t \bmod T) < t_w + t_a = T \text{ (act)} \end{cases}$$

Step-by-step solution ($t_w \ge \tau$ and $t_a \le \tau$):



$$t \in [0, t_w): \;\; \dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) \;\rightarrow\; \mathbf{x}(t) = \mathrm{e}^{\mathbf{A}t}\mathbf{x}(0)$$

$$t \in [t_w, T): \;\; \dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{K}\,\mathbf{x}(t-\tau) = \mathbf{A}\mathbf{x}(t) + \mathbf{K}\mathrm{e}^{\mathbf{A}(t-\tau)}\mathbf{x}(0)$$

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$(\mathbf{x} \in \mathbb{R}^n)$

$l_{\mathrm{crit,AAW}} = 0$

Finite dimensional map

Act-and-wait control

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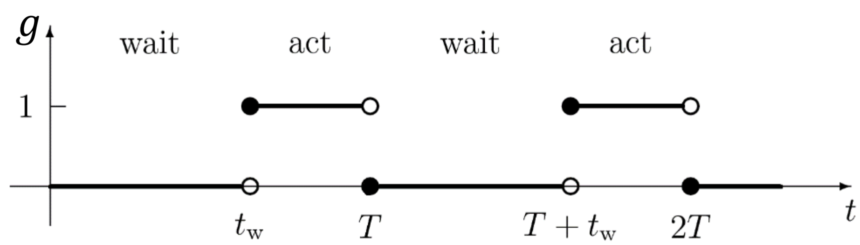

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$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t)$$
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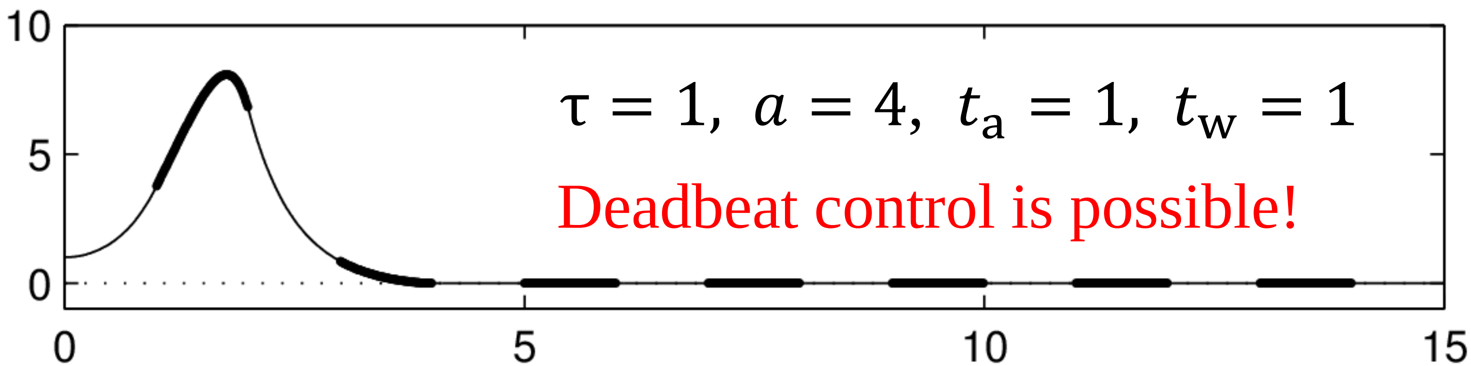
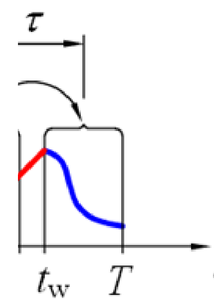




$\mathbf{x}(0)$

$$\rightarrow \quad \mathbf{x}(T) = \underbrace{\left(\mathrm{e}^{\mathbf{A}T} + \int_{t_\mathrm{w}}^{T} \mathrm{e}^{\mathbf{A}(T-s)}\mathbf{B}\mathbf{K}\mathrm{e}^{\mathbf{A}(s-\tau)}\right)}_{\boldsymbol{\Phi} \in \mathbb{R}^{n\times n}} \mathbf{x}(0)$$

$(\mathbf{x} \in \mathbb{R}^n)$

$$l_{\text{crit,AAW}} = 0$$

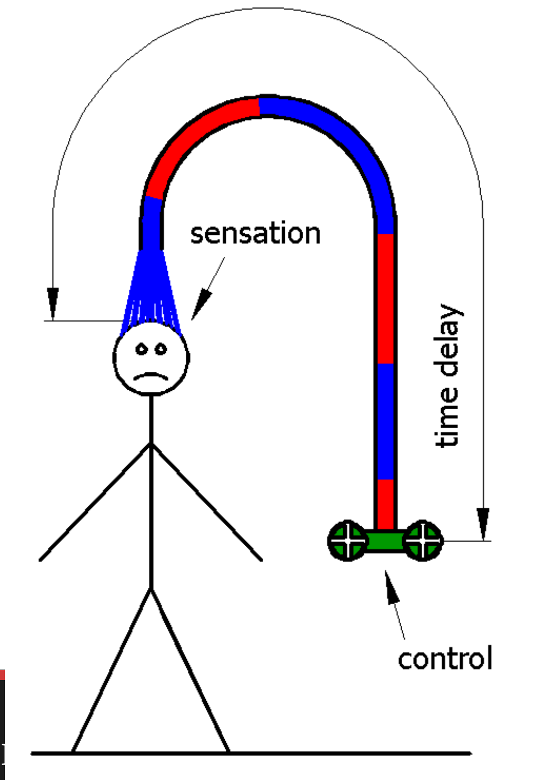
Finite dimensional map

Why wait?

It might seem unnatural not to actuate at all during the wait period in a control process, still...

Why wait?

It might seem unnatural not to actuate at all during the wait period in a control process, still... consider the way you take a shower...



Constant gain control: slow, continuous turning

Act-and-wait: turn and stop, turn and stop

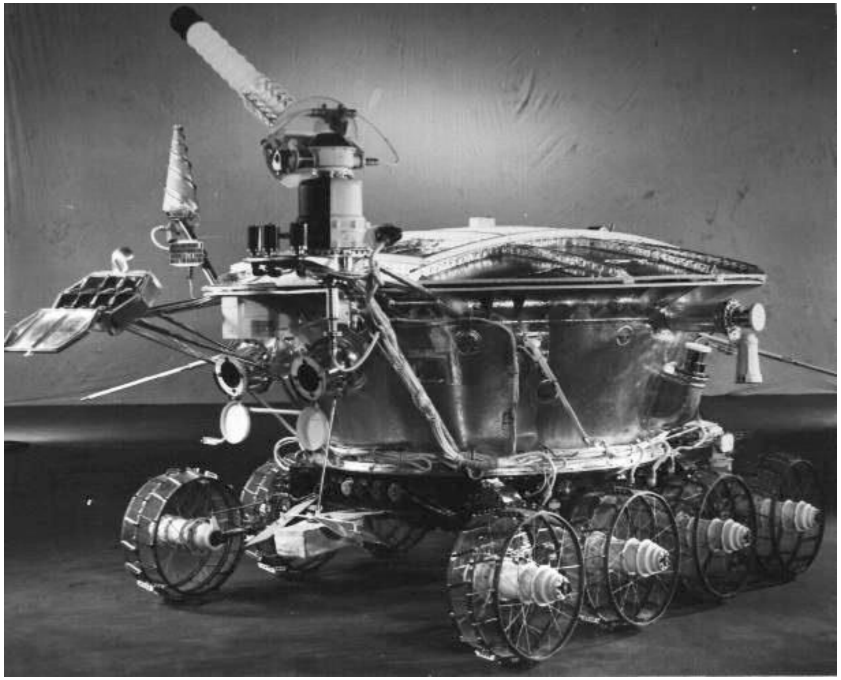
Why wait?

It might seem unnatural not to actuate at all during the wait period in a control process, still... or remember the Lunokhod 2…

January-June, 1973
36 km in 137 days

Earth-Moon-Earth:
2×1.3s = 2.6s

Earth-Mars-Earth:
32min



Act-and-wait control

Why w

It might seem unnatural not to actuate
in a control process, still... or rememb

January-June, 1973
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2×1.3s = 2.6s

Earth-Mars-Earth:
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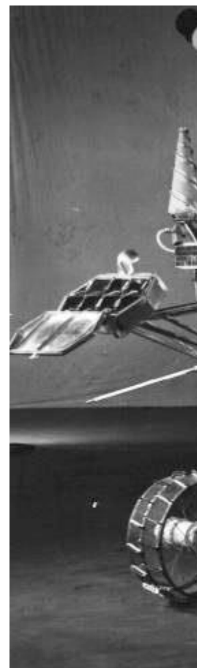
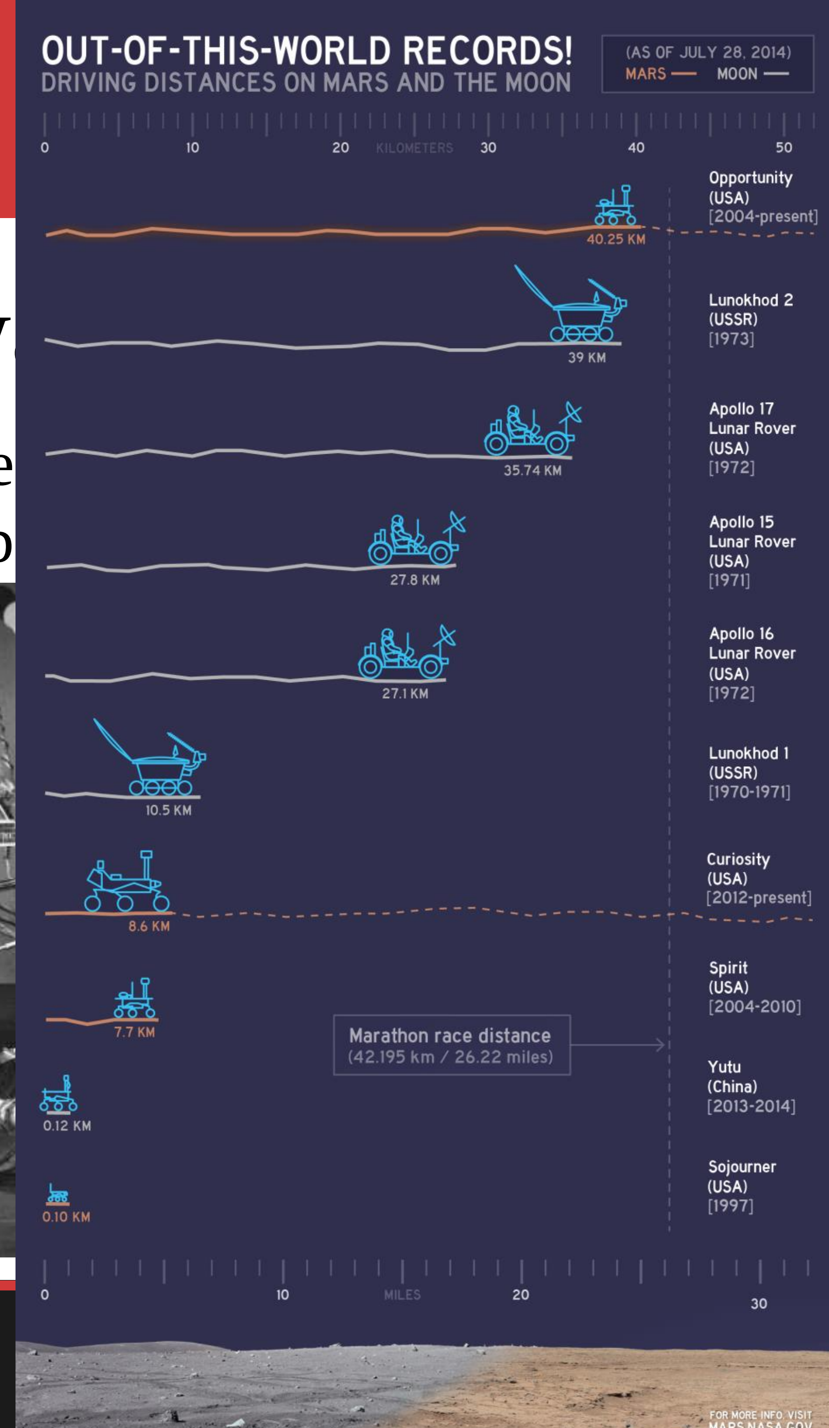




Comparison of different control concepts

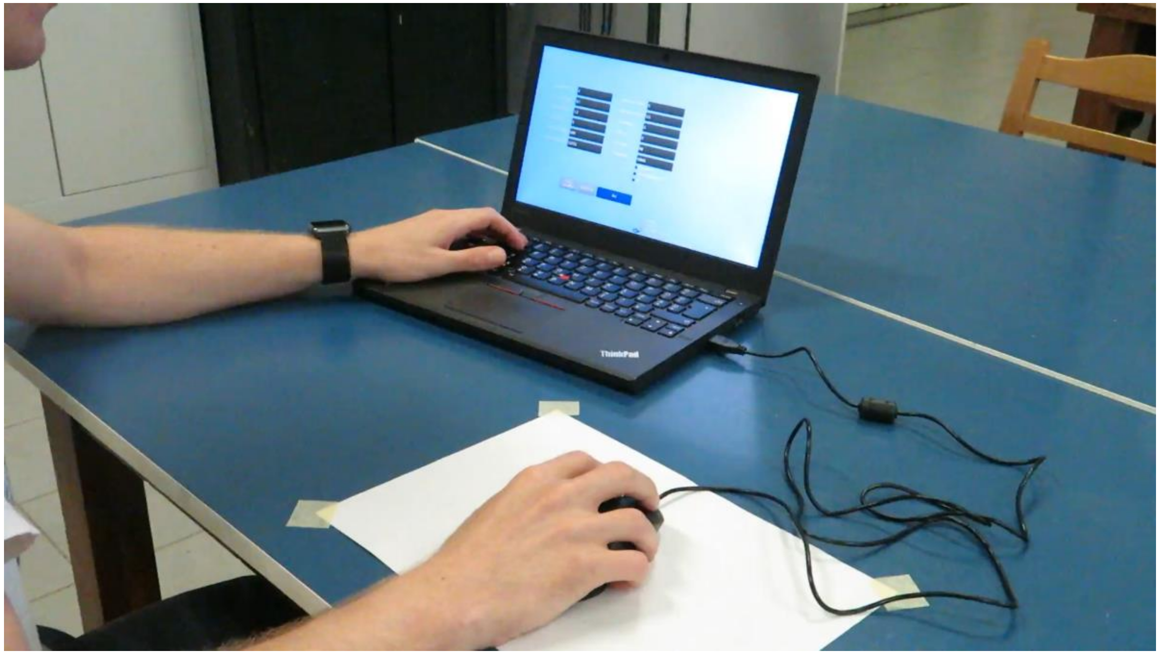
- proportional-derivative (PD) — $l_{\mathrm{crit,PD}} = 39\ \mathrm{cm}$
- proportional-derivative-acceleration (PDA) — $l_{\mathrm{crit,PDA}} = 20\ \mathrm{cm}$
- predictive feedback (PF) — $l_{\mathrm{crit,PF}} = 0\ \mathrm{cm}$
- act-and-wait (AAW) — $l_{\mathrm{crit,AAW}} = 0\ \mathrm{cm}$

Other effects:

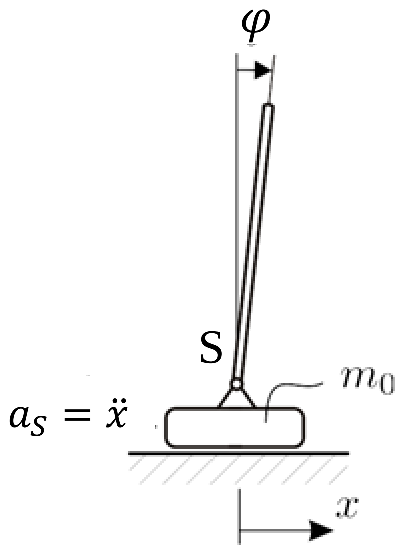
- nonlinearities (sensory threshold, saturation, quantization)
- parameter uncertainties (time-dependent/invariant)
- motor noise
- neural model of reaction delay

Contents

Virtual stick balancing



Virtual stick balancing



$$\ddot{\varphi}(t) - \frac{3g}{2l}\varphi(t) = -\frac{3}{2l}a_S(t)$$

a_S: acceleration of stick's bottom

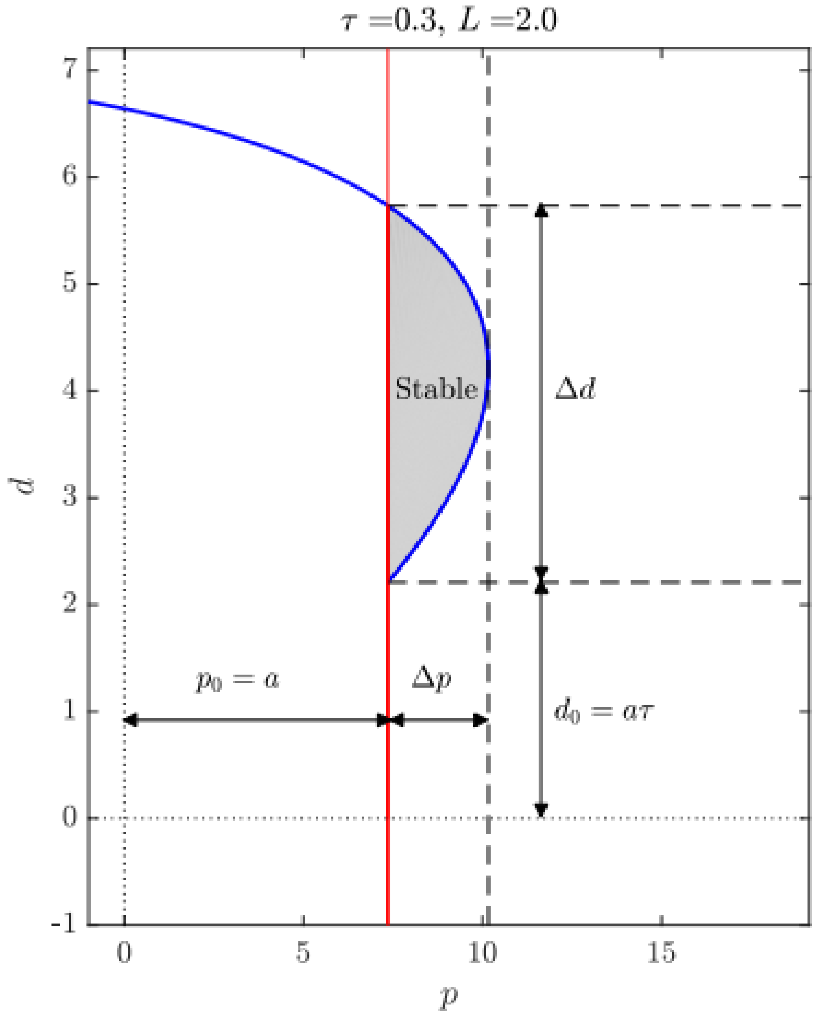
$$a_S(t) = -k_\mathrm{p}\varphi(t-\tau) - k_\mathrm{d}\dot{\varphi}(t-\tau)$$

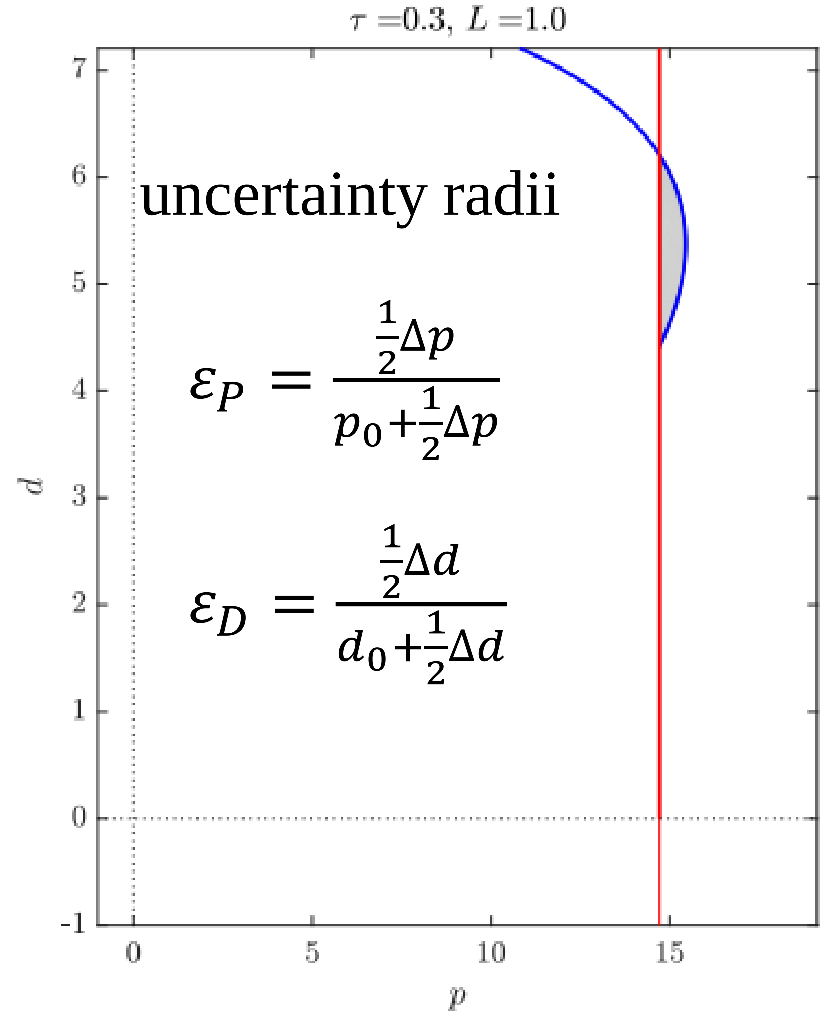
$$\ddot{\varphi}(t) - a\varphi(t) = -p\,\varphi(t-\tau) - d\,\dot{\varphi}(t-\tau)$$

$$a = \frac{3g}{2l}$$

Virtual stick balancing

$$\ddot{\varphi}(t) - a\varphi(t) = -p\,\varphi(t-\tau) - d\,\dot{\varphi}(t-\tau)$$





uncertainty radii

$$\varepsilon_P = \frac{\frac{1}{2}\Delta p}{p_0 + \frac{1}{2}\Delta p}$$

$$\varepsilon_D = \frac{\frac{1}{2}\Delta d}{d_0 + \frac{1}{2}\Delta d}$$

Virtual stick balancing

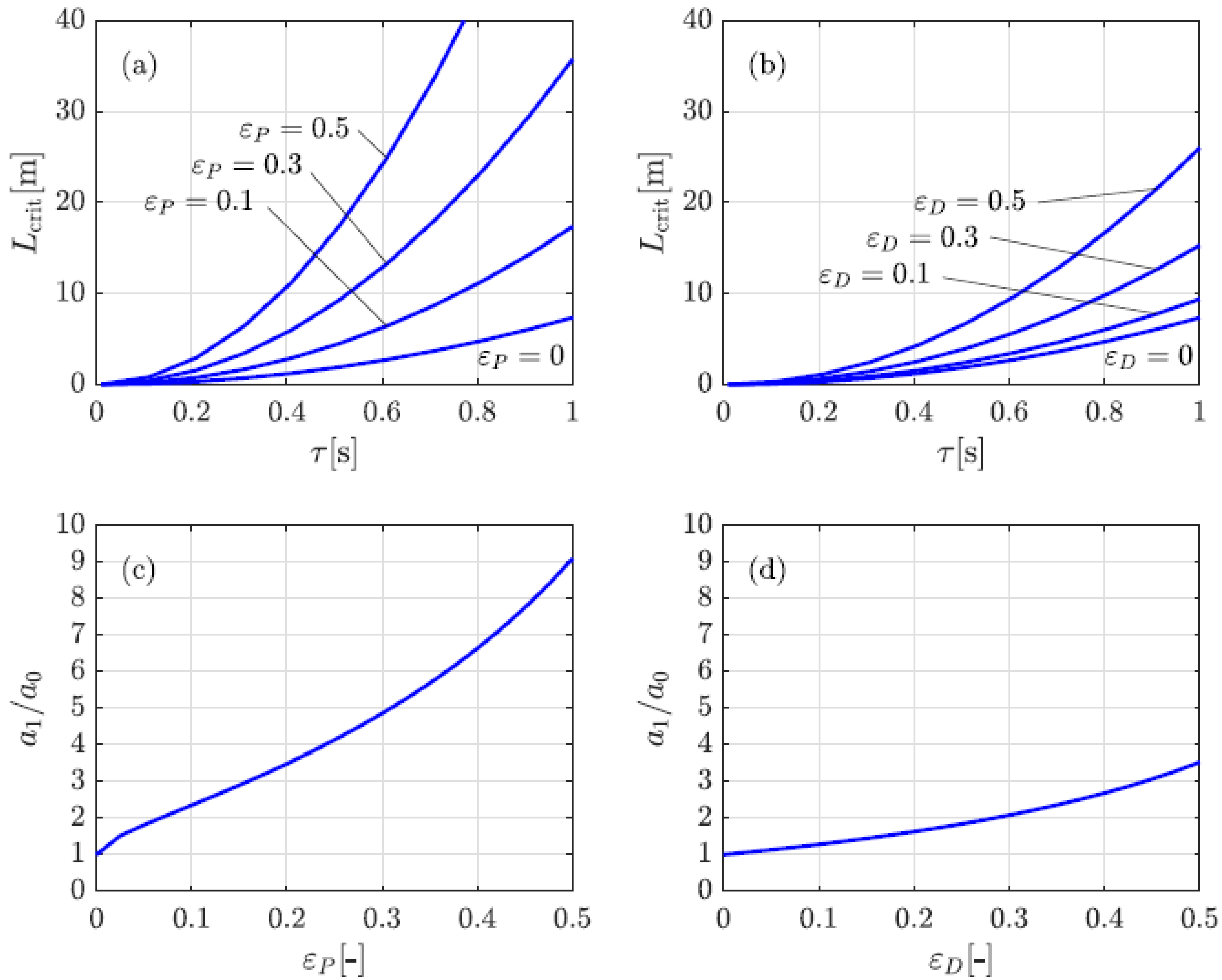
$$\ddot{\varphi}(t) - a\varphi(t) = -p\,\varphi(t-\tau) - d\,\dot{\varphi}(t-\tau)$$

$$l_{\text{crit}}\Big|_{\varepsilon=0} = a_0\tau^2$$

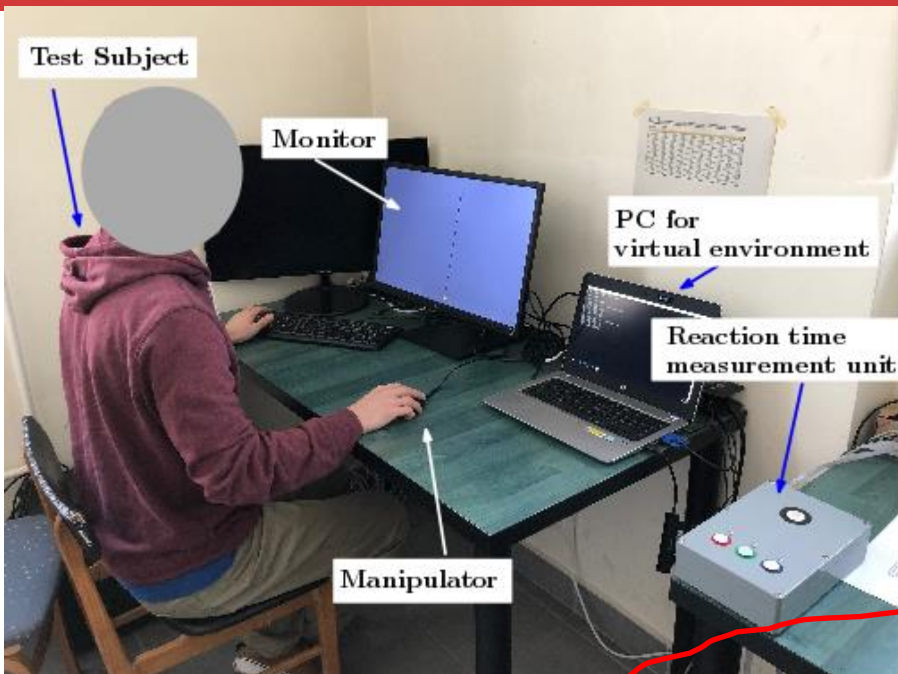
$$a_0 = \frac{3}{4}g$$

$$l_{\text{crit}}\Big|_{\varepsilon>0} = a_1\tau^2$$

$$\frac{a_1}{a_0} = ?$$

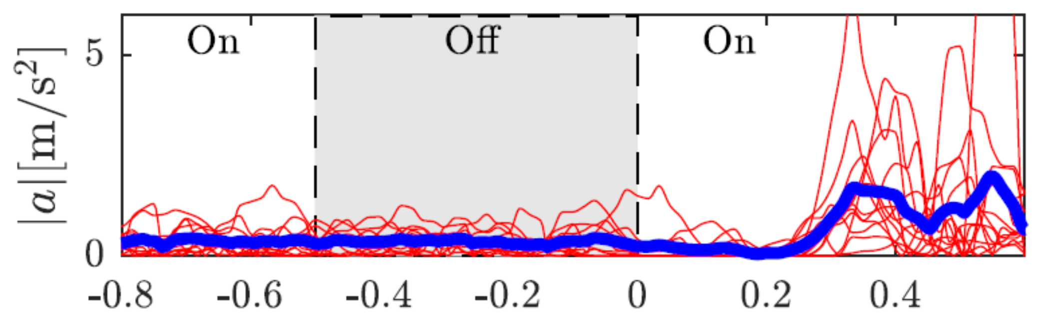


Virtual stick balancing



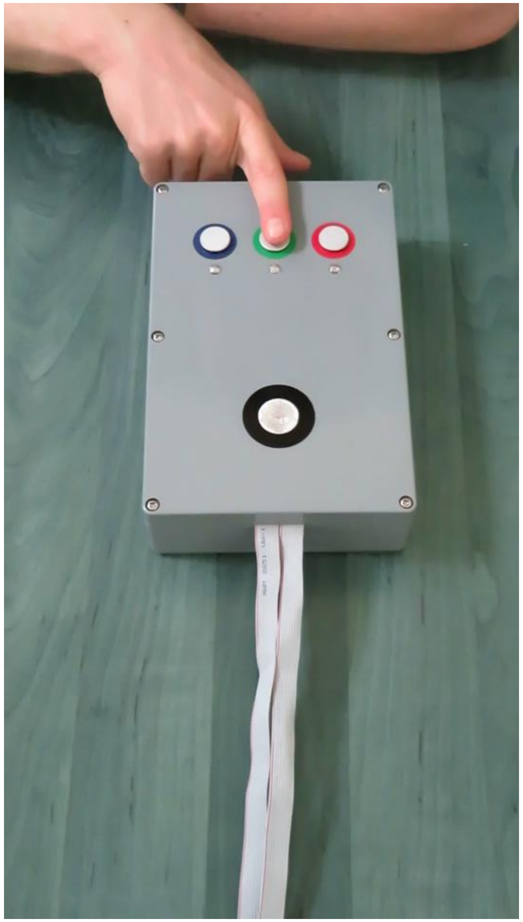
$$\tau = \tau_{\text{Machine}} + \tau_{\text{Neural}} + \tau_{\text{Added}}$$

122 ms ~250 ms $k \times 50$ ms

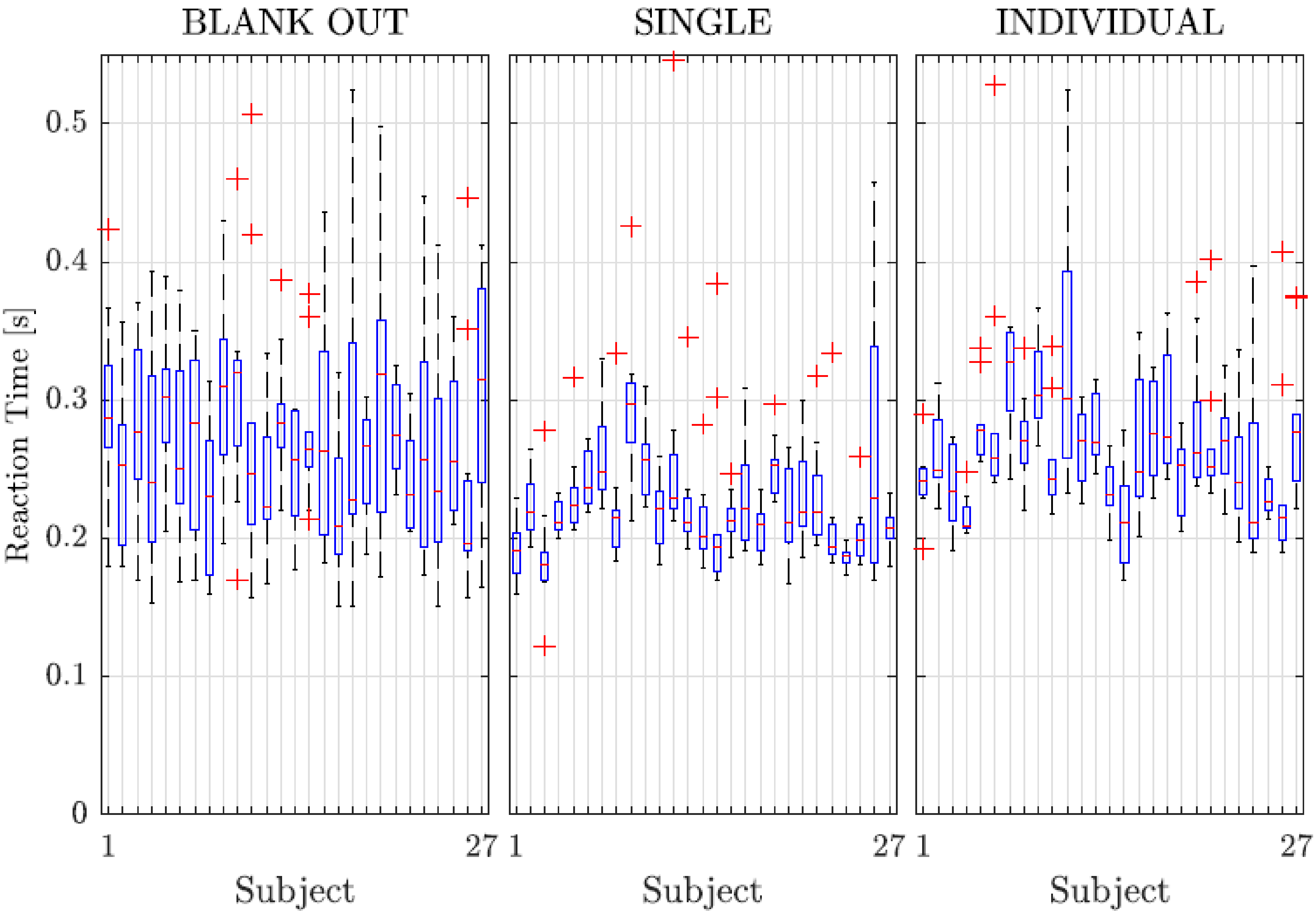


Virtual stick balancing

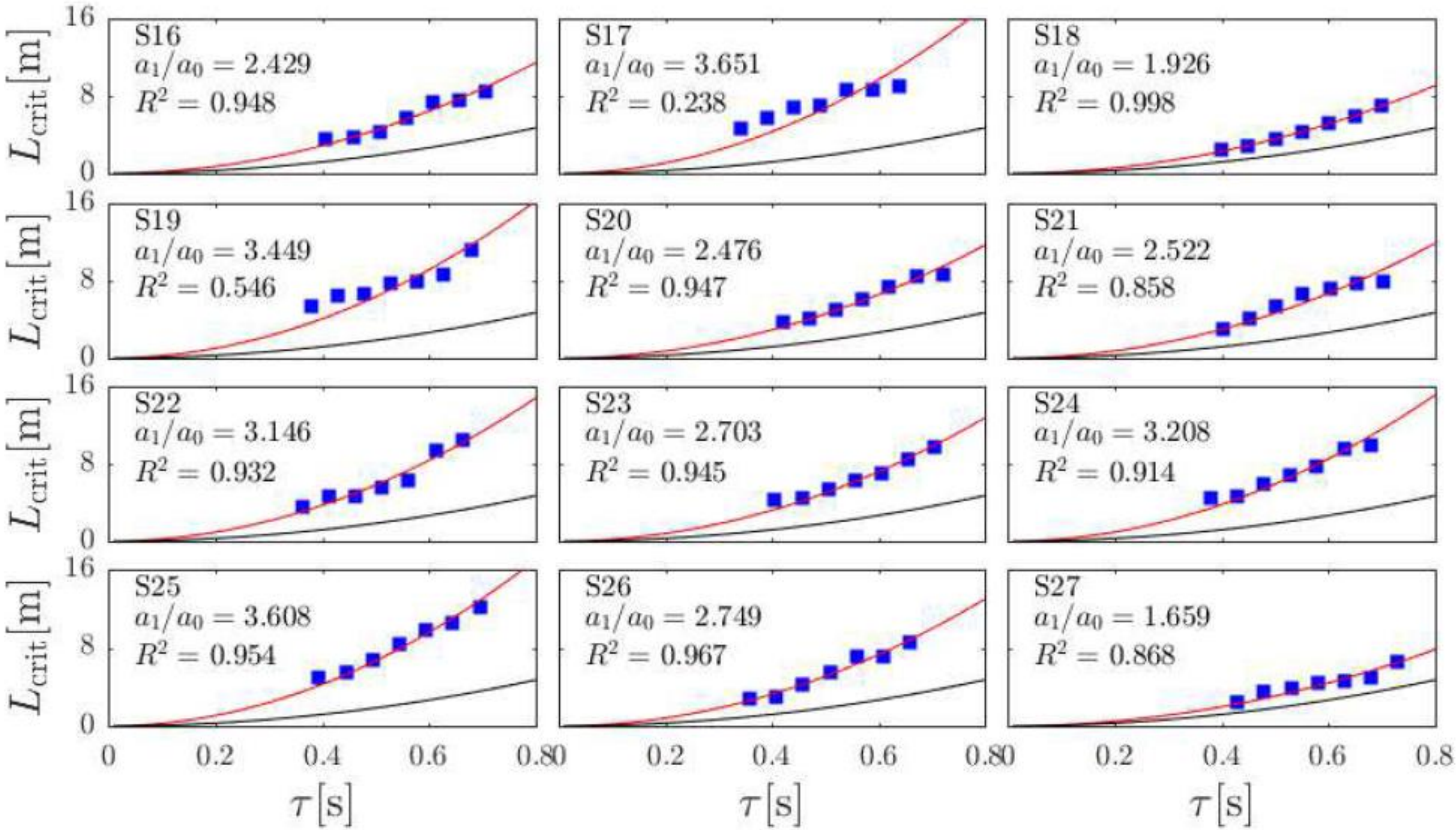
SINGLE



INDIVIDUAL


BLANK OUT
SINGLE
INDIVIDUAL
Reaction Time [s]
0.5
0.4
0.3
0.2
0.1
0
1
27
Subject

Virtual stick balancing

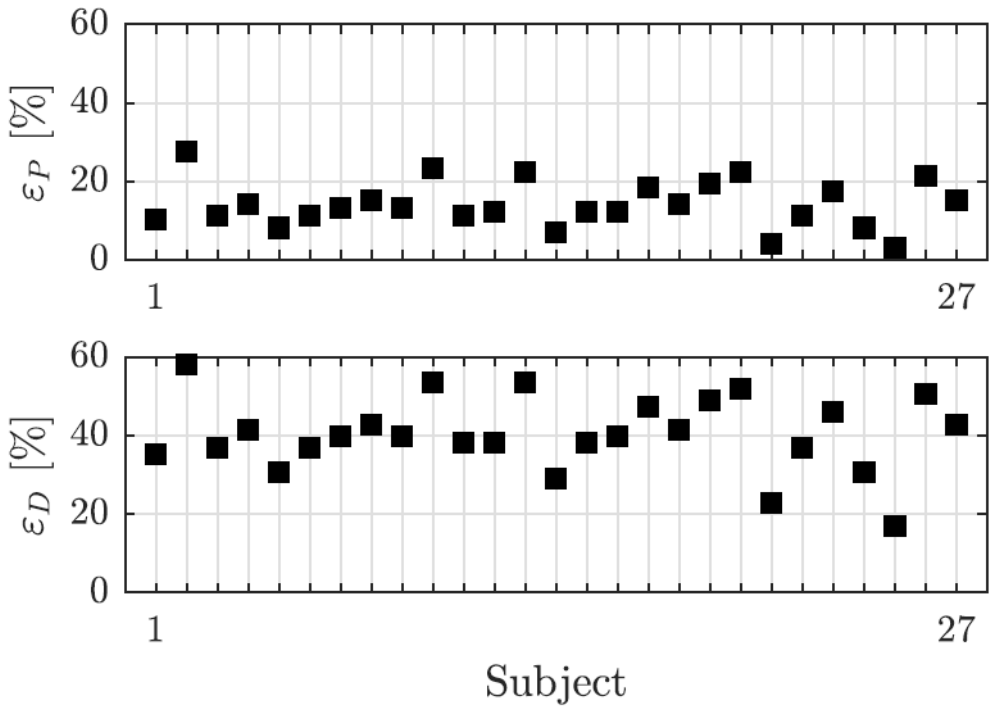


$$l_{crit}\Big|_{\varepsilon=0} = a_0\tau^2$$

$$l_{crit}\Big|_{\varepsilon>0} = a_1\tau^2$$

Virtual stick balancing

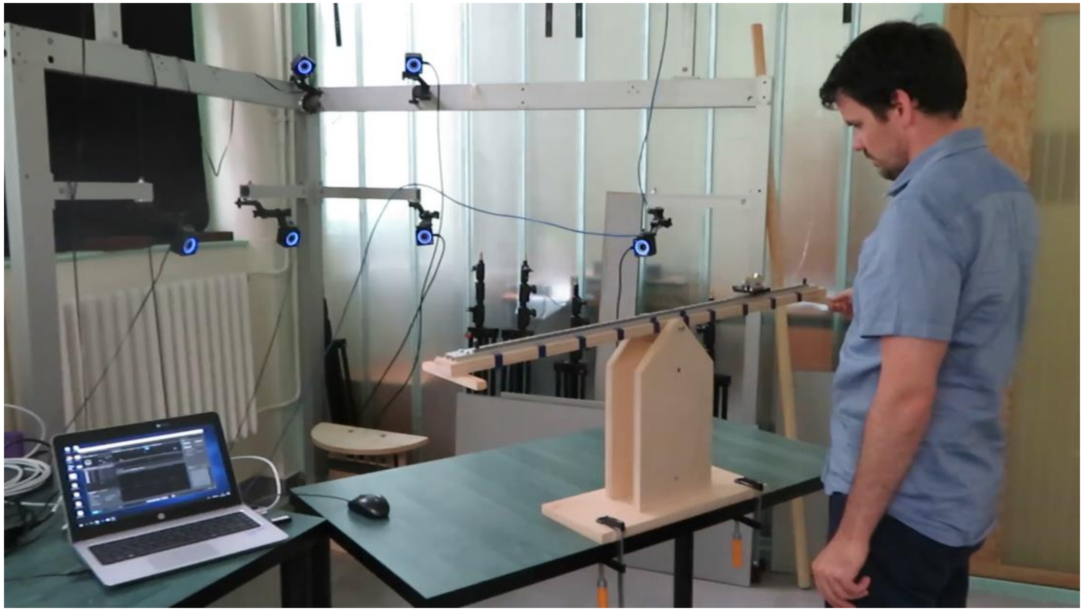
Uncertainty radii



3.1~27.6%
mean: 14.1%

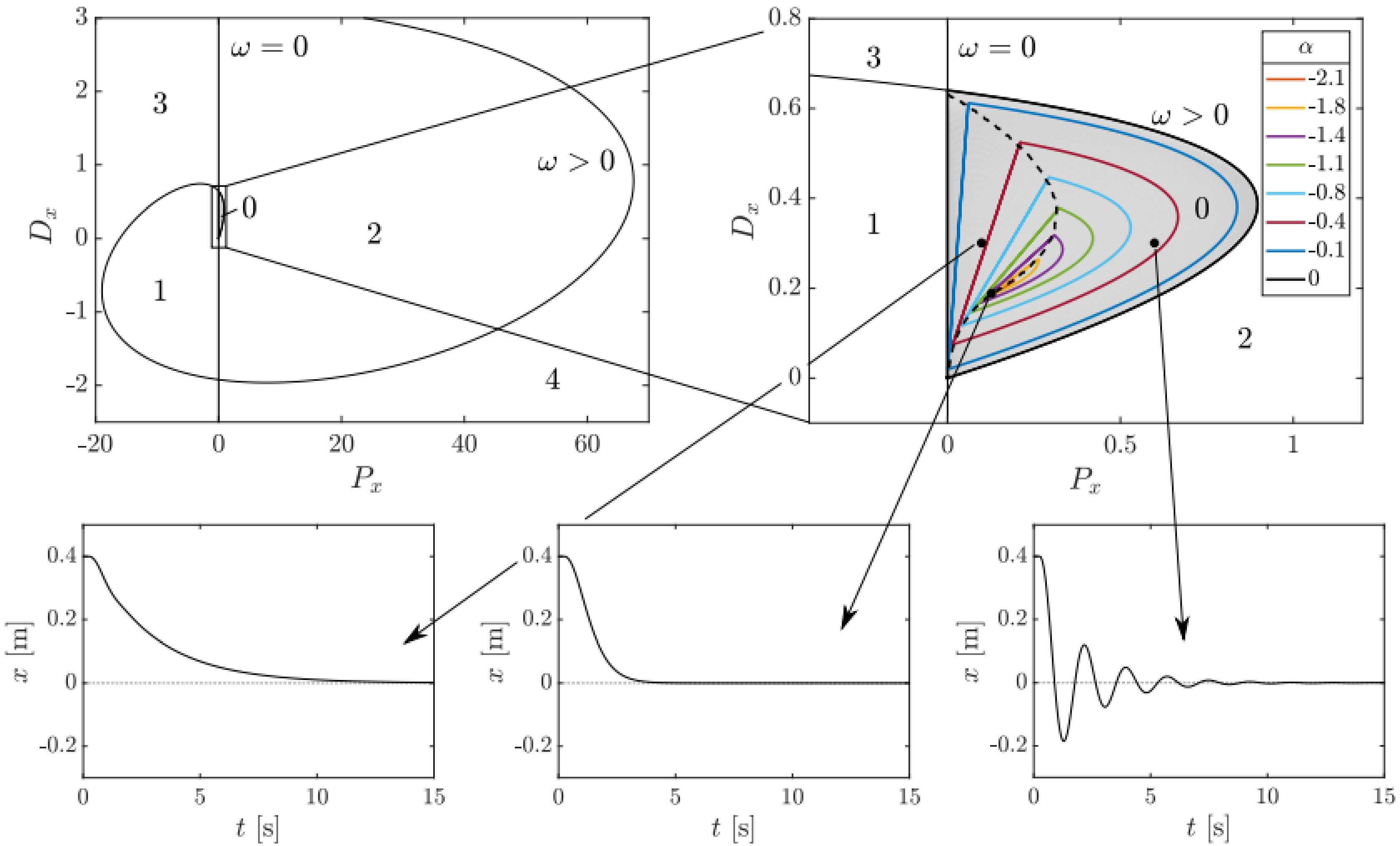
16.8~58.2%
mean: 40.3%

Rolling cart on a see-saw



Ball and beam

$$\ddot{x}(t) + gD_x\dot{x}(t-\tau) + gP_x x(t-\tau) = 0$$



Ball and beam - experiments

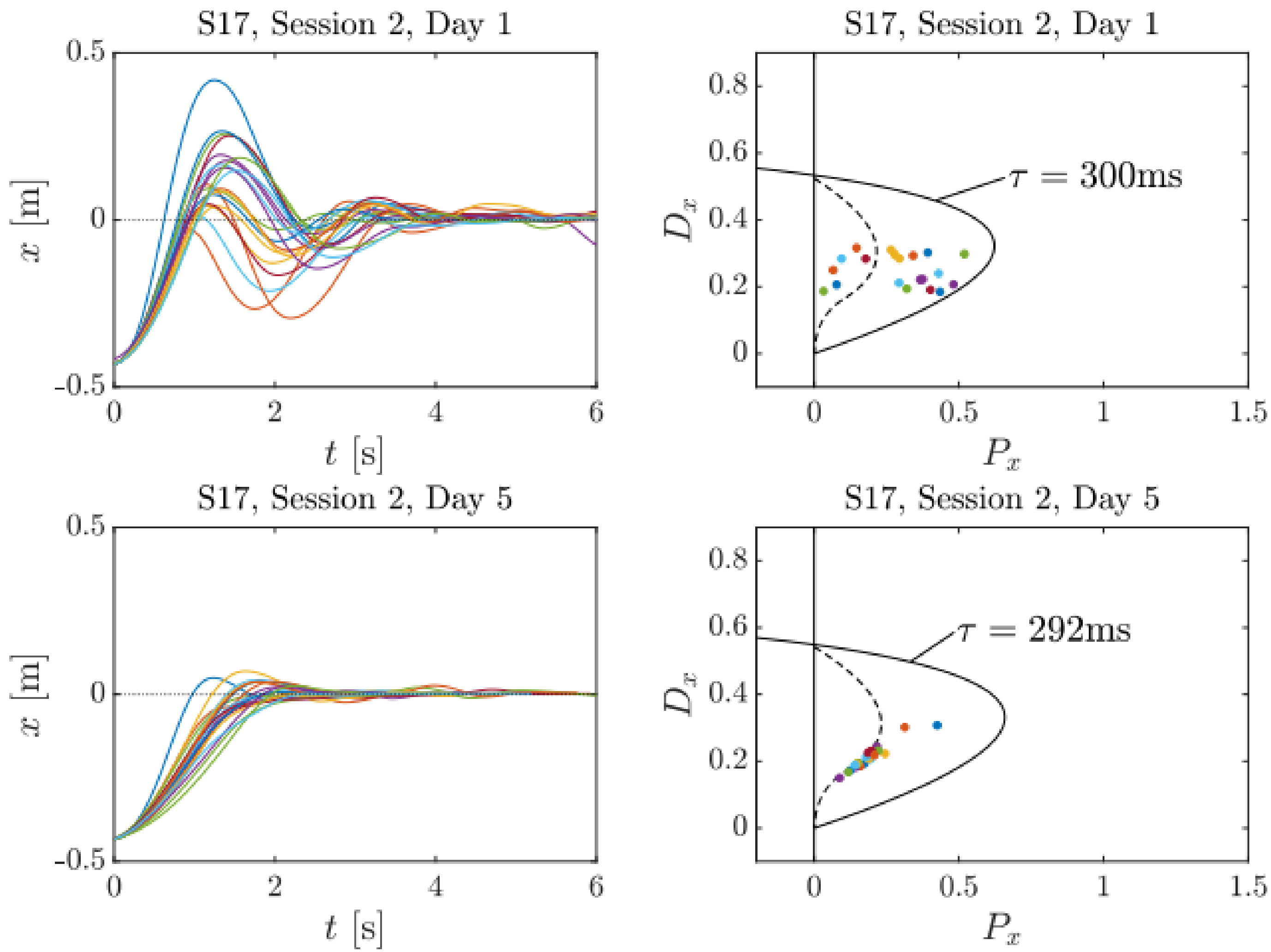
Tests by 22 subjects

5-day test series

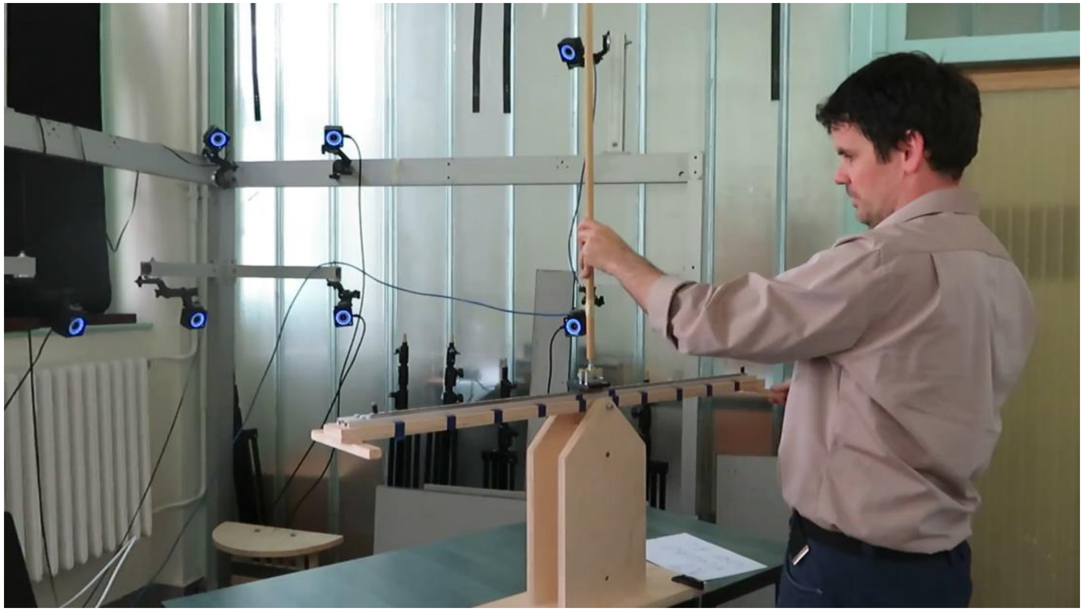
20 trials per day

Settling time decreases

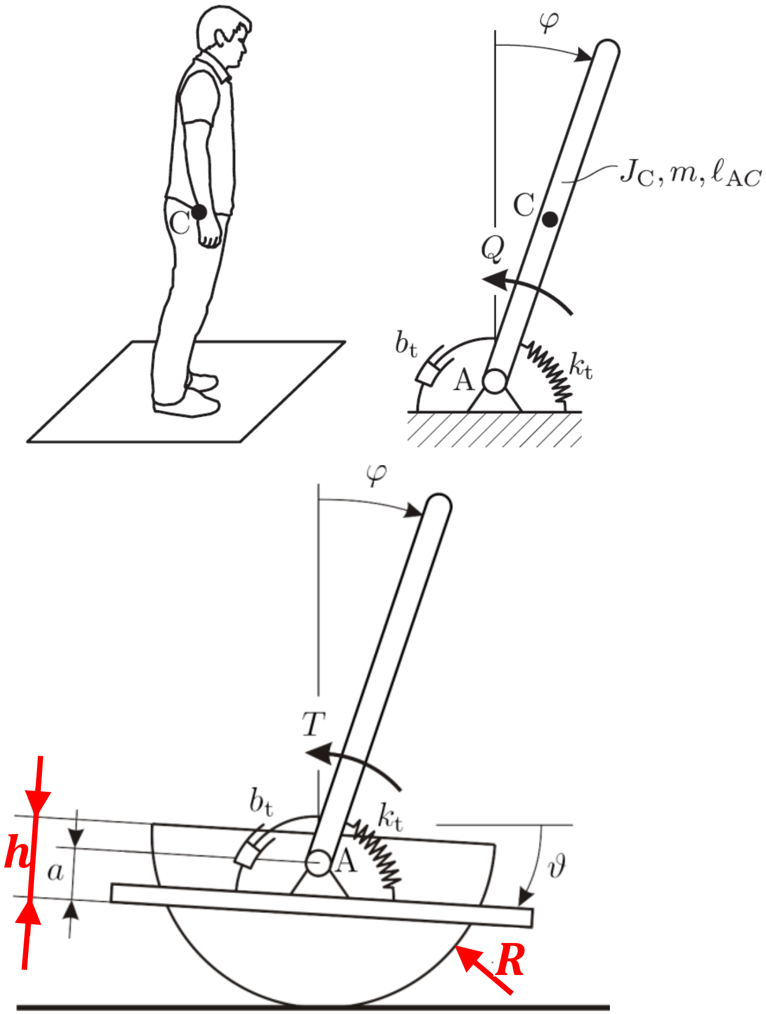
Overshoot decreases



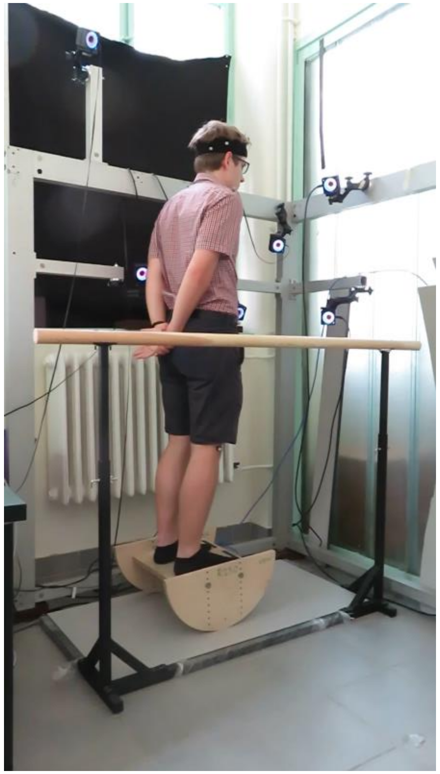
Pendulum-cart rolling on a see-saw



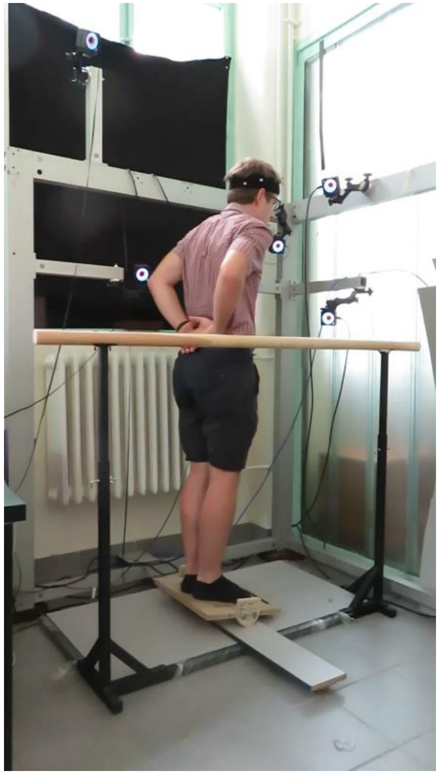
Balance Board



„easy”



„difficult”

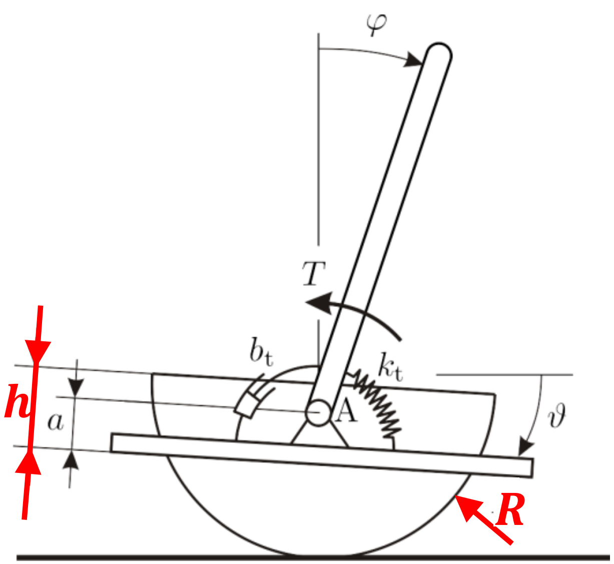


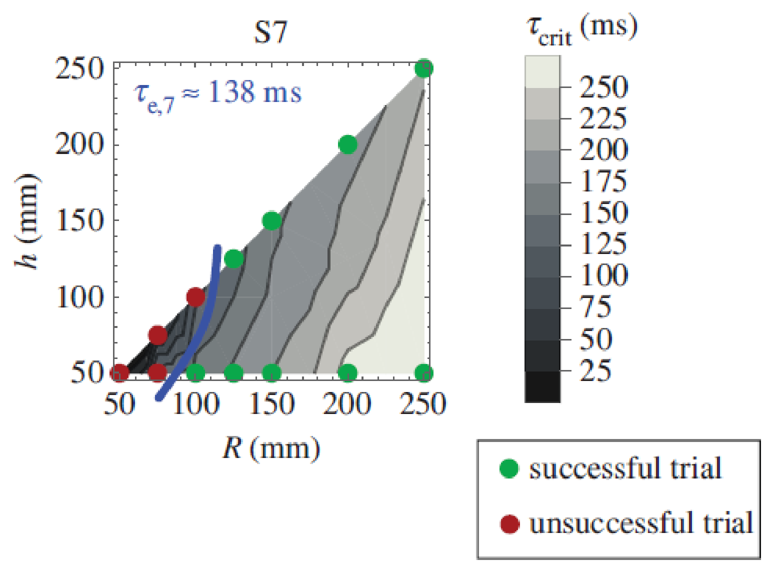
Balance Board

$$\left(\tfrac{1}{4}l^2 m_h + J_h\right)\ddot{\varphi} + b_t\dot{\varphi} + \left(k_t - \tfrac{1}{2}glm_h\right)\varphi + \left(\tfrac{1}{2}lm_hR + \tfrac{1}{2}alm_h - \tfrac{1}{2}hlm_h\right)\ddot{\vartheta} - b_t\dot{\vartheta} - k_t\vartheta = -T$$

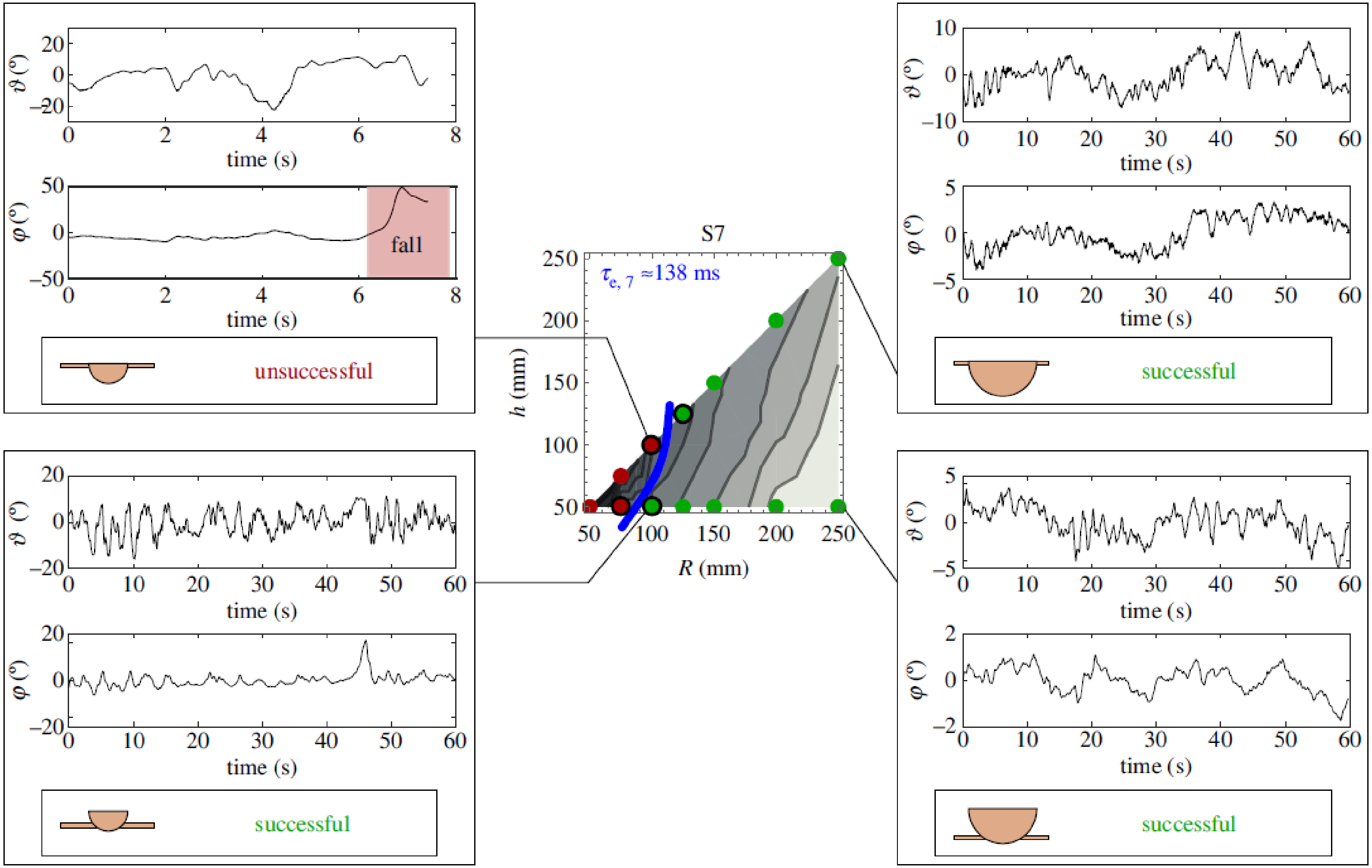
$$\left(\frac{1}{2}lm_hR + \frac{1}{2}alm_h - \frac{1}{2}hlm_h\right)\ddot{\varphi} - b_t\dot{\varphi} - k_t\varphi + \left(m_bR^2 + J_b + m_hR^2 + a^2m_h - 2ahm_h + h^2m_h\right.$$
$$\left. + {l_b}^2m_b + 2am_hR - 2hm_hR\right)\ddot{\vartheta} + b_t\dot{\vartheta} + (-agm_h + ghm_h - gl_bm_b + gm_bR + k_t)\vartheta = T$$

$$T = P_\varphi\varphi(t-\tau) + D_\varphi\dot{\varphi}(t-\tau) + P_\vartheta\vartheta(t-\tau) + D_\vartheta\dot{\vartheta}(t-\tau)$$





Balance Board



Thank you!

https://www.youtube.com/watch?v=lV-iP1jSMlI