Smart Contracts and the Coase Conjecture

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Model

- discrete time $t = 0, 1, 2...$
- seller of a durable good
- buyer with valuation $v \in \{v_l, v_h\}$
- $\mu v_h > v_l$
- payoffs:

$$\delta^T v - \sum_{t=0}^{\infty} \delta^t p_t \text{ and } \sum_{t=0}^{\infty} \delta^t p_t$$

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- dynamic contracts
- renegotiation in each period

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Laffont and Tirole (1988), Doval and Skreta (2020a, 2020b)

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Dynamic Contract

- buyer reports v
- $t = 0$: trade with v_h with prob α at $p \in (v_l, v_h)$
- $t > 0$: trade with v with prob β at v

such that

- at $t > 0$ the static monopoly price is v_l
- seller's continuation value is $> v_l$

Contracting Game

$t = 0$

- seller chooses contract $c_0 \in \mathcal{C}$
- c_0 determines (x_T, p_T) until it is replaced

$t > 0$

- seller decides whether to proceed with c_{t-1}
- or deploys a new contract
- c_t determines (x_T, p_T) until it is replaced

The Contract Space c

$$c = \left(M_T^b, M_T^s, S_T^b, S_T^s, \mathbf{x}_T, \mathbf{p}_T, \rho_T\right)_{T=0}^{\infty}$$

- T : number of *consecutive* periods c is deployed
- M_T^b, M_T^s : message spaces
- S_T^b, S_T^s : signal spaces
- $\mathbf{x}_T, \mathbf{p}_T : \left(M_\gamma^b, M_\gamma^s\right)_{\gamma=0}^{T} \times \left(S_\gamma^b, S_\gamma^s\right)_{\gamma=0}^{T} \to [0,1] \times \mathbb{R}$
- $\left(\rho_T^b, \rho_T^s\right) : \left(M_\gamma^b, M_\gamma^s\right)_{\gamma=0}^{T} \times \left(S_\gamma^b, S_\gamma^s\right)_{\gamma=0}^{T-1} \to \boldsymbol{\Delta}\left(S_T^b, S_T^s\right)$

Buyer Participation

$r \in M_T^b$ for all T

if $m_T^b = r$ then $x_T = p_T = 0$

Simple and Direct Contract $\mathcal{D}$

- $T = 0$: buyer is asked to report his valuation
- no more communication
- if rejected $\Rightarrow$ delay

Simple and Direct Contract $\mathcal{D}$

$$d = (\mathbf{x}_\tau, \mathbf{p}_\tau)_{\tau=0}^{\infty}$$

- τ : number of period where d was not rejected since another contract
- $M_0^b = \{v_l, v_h, r\}$, $M_\tau^b = \{a, r\}$ $\tau > 0$
- $\mathbf{x}_\tau, \mathbf{p}_\tau : \{v_l, v_h\} \to [0, 1] \times \mathbb{R}$
- $S_\tau^s = \{a, r\}$, $\rho_\tau^s = r \Leftrightarrow m_\tau^s = r$

Assumption

$\mathcal{D} \subset \mathcal{C}$

Equilibrium

Weak Perfect Bayesian Equilibria

$+$ seller does not update if she deviates

$+$ seller's beliefs: limit points of beliefs derived by Bayes' rule along a sequence of totally mixed strategy profiles converging to the equilibrium strategy profile

Revelation Principle

- *direct* contract
- incentive compatible
- always deployed
- never rejected

Not operational

Suppose that equilibrium exists for all δ

(If $\mathcal{C} = \mathcal{D}$, this is true)

$\pi(\mathcal{C}, \delta)$: sup of the seller's payoff across all equilibria

Theorem

$\exists \underline{\pi} > v_l$ such that

$$\pi\left(\mathcal{C}, \delta\right) \geq \underline{\pi}.$$

Incentive Compatibility

notation:

$$X_\tau(v) = \mathbf{x}_\tau(v)\,\Pi_{t=0}^{\tau-1}\left(1-\mathbf{x}(v)_t\right)$$

$$P_\tau(v) = \mathbf{p}_\tau(v)\,\mathbf{x}_\tau(v)\,\Pi_{t=0}^{\tau-1}\left(1-\mathbf{x}(v)_t\right)$$

$d = (X_\tau, P_\tau)_{\tau=0}^{\infty} \in \mathcal{D}$ is δ-IC if

$$v \in \arg\max_{v' \in \{v_l, v_h\}} \sum_{t=0}^{\infty} \delta^t \left[X_t\left(v'\right) v - P_t\left(v'\right)\right]$$

Abiding Contracts

$d = (X_\tau, P_\tau)_{\tau=0}^{\infty} \in \mathcal{D}$ is δ-*abiding* if it is δ-IC and

(i) $\sum_{t=T}^{\infty} \delta^{t-T} \left[X_t(v)\, v - P_t(v)\right] \geq 0$ for all $v \in \{v_l, v_h\}$ and

(ii) $\max\{v_l, \mu_T(d)\, v_h\} = v_l$

(iii) $\mu_T(d) \sum_{t=T}^{\infty} \delta^{t-T} P_t(v_h) + (1 - \mu_T(d)) \sum_{t=T}^{\infty} \delta^{t-T} P_t(v_l) \geq v_l$

Proof

$v\left(d,\delta\right) = \mu \sum_{t=0}^{\infty} \delta^{t-T} P_t\left(v_h\right) + \left(1-\mu\right) \sum_{t=0}^{\infty} \delta^{t-T} P_t\left(v_l\right)$

Lemma 1

d is δ-abiding $\Rightarrow \pi\left(\mathcal{C},\delta\right) \geq v\left(d,\delta\right)$.

Lemma 2

$\forall\delta\ \exists d_\delta \in \mathcal{D}$ δ-abiding contract such that $v\left(d_\delta,\delta\right) = \underline{\pi} > v_l$.

Lemma 1

d is δ-abiding $\Rightarrow \pi(\mathcal{C}, \delta) \geq v(d, \delta)$

Proof

Take an equilibrium such that seller gets $< v(d, \delta)$

Modify it so that seller gets $v(d, \delta)$

- on path: d is deployed and accepted forever
- if buyer rejects it is deployed again
- off-path: assessment is inherited from the original equilibrium

Lemma 2

$\forall \delta\ \exists d_\delta \in \mathcal{D}$ δ-abiding contract such that $v(d_\delta, \delta) = \underline{\pi} > v_l$

Proof (for large δ) by construction (α, p, β)

- $t = 0$: trade with v_h with prob α at $p \in (v_l, v_h)$
- $t > 0$: trade with v with prob β at v

Key Features

1. seller's posterior

$$\widetilde{\mu}(\alpha) = \frac{(1-\alpha)\mu}{1-\mu+(1-\alpha)\mu} \leq \frac{v_l}{v_h}$$

1. choose $\widetilde{\beta}(\alpha)$ so that seller's continuation payoff

$$\widetilde{\beta}(\alpha)\left[\widetilde{\mu}(\alpha)v_h + (1-\widetilde{\mu}(\alpha))v_l\right] + \left(1-\widetilde{\beta}(\alpha)\right)v_l = \frac{v_l}{\delta}$$

2. choose $\widetilde{p}(\alpha)$ so that v_h-buyer's IC constraint binds:

$$\alpha(v_h - \widetilde{p}(\alpha)) = \frac{\delta}{1-\delta+\widetilde{\beta}(\alpha)\delta}(v_h - v_l)$$

What is the seller's payoff?

$$
\begin{aligned}
& \mu\alpha\widetilde{p}(\alpha) + \delta(1-\mu\alpha)\frac{v_l}{\delta} \\
= \; & \mu\alpha\widetilde{p}(\alpha) + (1-\mu\alpha)v_l \\
= \; & v_l + (v_h - v_l)\left(1 - \frac{1-\mu}{1-\widetilde{\mu}(\alpha)} - \frac{\mu v_l}{\widetilde{\mu}(\alpha)v_h + (1-\widetilde{\mu}(\alpha))v_l}\right)
\end{aligned}
$$

does not depend on δ