TOPOLOGY (TOP) / ALEX KÜRONYA / SPRING 2010

HOMEWORK 11

The problems with an asterisk are the ones that you are supposed to submit. The ones with two asterisks are meant as more challenging, and by solving them you can earn extra credit.

1. * Consider a covering map $p : E \to B$ with B connected. Prove that if for some $x \in B$ the fibre $p^{-1}(x) \subseteq E$ has m elements, then $p^{-1}(b)$ has m elements for every $b \in B$. (In this case the covering map p is called an *m-fold covering* of B.)

2. If $p : X \to Y$ and $q : Y \to Z$ are covering maps, and $(q \circ p)^{-1}(z)$ is finite for every $z \in Z$, then $q \circ p$ is also a covering map.

3. Let $p : E \to B$ be a covering map with E path-connected. Show that if B is simply connected, then p is a homeomorphism.

4. Let $p : E \to B$ be a covering map. Show that if B is Hausdorff/regular, then so is E.

5. If X,Y are topological spaces, $x_0 \in X$, $y_0 \in Y$, then prove that

$$\pi_1(X \times Y, (x_0, y_0)) \simeq \pi_1(X, x_0) \times \pi_0(Y, y_0) \ .$$

6. For a retract $A \subset \mathbb{D}^2$, prove that every continuous map $f : A \to A$ has a fixed point.

7. Let $f : \mathbb{S}^2 \to \mathbb{S}^2$ be a continuous map such that $f(x) \neq f(-x)$ for every $x \in \mathbb{S}^2$. Show that g is surjective.

8. Determine the fundamental group of $\mathbb{S}^1 \times \mathbb{D}^2$ and of $\mathbb{S}^1 \times \mathbb{S}^2$.

9. Accepting the fact that for an arbitrary positive integer n, no antipode preseving map $f : \mathbb{S}^n \to \mathbb{S}^n$ is nullhomotopic, prove the following statements:

(1) There exists no retraction $r : \mathbb{D}^{n+1} \to \mathbb{S}^n$.

(2) There exists no antipode-preserving map $g : \mathbb{S}^{n+1} \to \mathbb{S}^n$.

10. Using the method for determining the fundamental group of the circle, prove that

$$\pi_1(T^2) \simeq \mathbb{Z} \times \mathbb{Z} \ .$$

11. Prove that a nonsingular 3×3 matrix with nonnegative real entries has a positive real eigenvalue.