BSCA VI. 2012. jan. 4.

1. $n^2\left(1-\sqrt{1-\frac{1}{n^2}}\right) = \frac{n^2\left(1-\sqrt{1-\frac{1}{n^2}}\right)\left(1+\sqrt{1-\frac{1}{n^2}}\right)}{\left(1+\sqrt{1-\frac{1}{n^2}}\right)} \; (1) = \frac{n^2\left(1-\left(1-\frac{1}{n^2}\right)\right)}{1+\sqrt{1-\frac{1}{n^2}}} = \frac{n^2\frac{1}{n^2}}{1+\sqrt{1-\frac{1}{n^2}}} \mapsto \frac{1}{2} \quad (1)$

$\left(1+\frac{1}{2n}\right)^{n+1} = \left(1+\frac{1}{2n}\right)^{n}\cdot\left(1+\frac{1}{2n}\right) = \left(\underbrace{\left(1+\frac{1}{2n}\right)^{2n}}_{\to e}\right)^{\frac{1}{2}}\cdot\left(1+\underbrace{\frac{1}{2n}}_{\to 0}\right) \; (1) \mapsto \sqrt{e} \quad (1)$

$\underbrace{3\sqrt[n]{2}\cdot\sqrt[n]{n}}_{\to 3} = 3\sqrt[n]{2n} \le \underbrace{3\cdot\sqrt[n]{2n+n^2}}_{\to 3 \; (1)} \le 3\sqrt[n]{3n^2} = \underbrace{3\cdot\sqrt[n]{3}\cdot\sqrt[n]{n}\cdot\sqrt[n]{n}}_{\to 3}$ rendőrelv + indoklás (2)

$a_n \mapsto \frac{1}{2}+\sqrt{e}+3 \quad (1)$

2. $\lim_{x\to 0}\frac{\sin x - x}{x^3} \overset{\left(\frac{0}{0}\right)}{\underset{B-L}{=}} \lim_{x\to 0}\frac{\cos x - 1}{3x^2} \; (2) \overset{\left(\frac{0}{0}\right)}{\underset{B-L}{=}} \lim_{x\to 0}\frac{-\overbrace{\sin x}^{\to 1 \; (1)}}{6x} \; (2) = -\frac{1}{6} \quad (1)$

$c=-\frac{1}{6}$ (1) választással folytonossá tehető (1)

3. $f(x)=e^x(x^2+1)^2 \qquad f'(x)=e^x(x+1)^2+2e^x(x+1) \overset{(2)}{=} e^x(x+1)(x+1+2)=0$

$x_1=-1$ (1) $x_2=-3$

	$(-\infty,-3)$	-3	$(-3,-1)$	-1	$(-1,\infty)$
f'	+	0	−	0	+
f	↗	lok max	↘	lok min	↗

(5) (oszloponként 1-1 pont)

4. $\frac{-2}{(x^2+1)(x-1)} = \frac{Ax+B}{x^2+1}+\frac{C}{x-1} \; (2) = \frac{Ax^2-Ax+Bx-B+Cx^2+C}{(x^2+1)(x-1)}$

$A+C=0 \Rightarrow C=-A$

$-A+B=0 \Rightarrow B=A$

$C-B=-2 \Rightarrow C=+B-2 \Rightarrow -A=A-2 \Rightarrow 2A=2 \Rightarrow A=1, \; B=1, \; C=-1$ (1) (1) (1)

$\int\frac{-2}{(x^2+1)(x-1)}dx = \int\frac{x+1}{x^2+1}-\frac{1}{x-1}dx = \int\frac{x}{x^2+1}+\frac{1}{x^2+1}-\frac{1}{x-1}dx =$

$= \underbrace{\frac{1}{2}\ln(x^2+1)}_{(1)} + \underbrace{\operatorname{arctg} x}_{(1)} - \underbrace{\ln(x-1)}_{(1)} + C$

5. $z^4 = 8i\frac{(-8+6i)}{3+4i} = 16i\frac{(-4+3i)}{3+4i} = 16\frac{(-4i+3\overset{-1}{i^2})}{3+4i} = -16 \quad (1)$

$z=\sqrt[4]{-16} = \sqrt[4]{2^4(\cos\pi + i\sin\pi)} \; (1) = 2\cdot\left(\cos\frac{\pi+2k\pi}{4}+i\sin\frac{\pi+2k\pi}{4}\right) \; (3)$

$k=0,1,2,3$

$z_1=2\left(\frac{\sqrt{2}}{2}+i\frac{\sqrt{2}}{2}\right)=\sqrt{2}+i\sqrt{2}$

$z_2=2\left(-\frac{\sqrt{2}}{2}+i\frac{\sqrt{2}}{2}\right)=-\sqrt{2}+i\sqrt{2}$

$z_3=2\left(-\frac{\sqrt{2}}{2}-i\frac{\sqrt{2}}{2}\right)=-\sqrt{2}-i\sqrt{2}$

$z_4=2\left(\frac{\sqrt{2}}{2}-i\frac{\sqrt{2}}{2}\right)=\sqrt{2}-i\sqrt{2}$

(2)