BSCAI. 2. vizsga 2012.01.11.

1. $\frac{1}{(1-\frac{2}{n})^n}$ (1) $= \left(\frac{n}{n-2}\right)^n = \left(\frac{n-2+2}{n-2}\right)^n = \left(1+\frac{2}{n-2}\right)^{n-2+2} = \Big(\underbrace{\left(\left(1+\frac{1}{\frac{n-2}{2}}\right)^{\frac{n-2}{2}}\right)^2}_{\to e} \underbrace{\left(1+\frac{2}{n-2}\right)^2}_{\to 0}$ (1) $\to e^2$ (1)

$|1+\sin^2 n| \le 2$ (1)

$\frac{1+n^4}{2n^5+n^6} = \frac{\frac{1}{n^6}+\frac{1}{n^2}}{\frac{2}{n}+1}$ (1) $\to 0$ (1) $0 \cdot$ omej $\to 0$ (1)

$a_n \to \frac{1}{e^2}$ (1)

2. $3e^{x+y^2} = x+4y^3$ $/(\,)'$

$3e^{x+y^2}(1+2yy') = 1+12yy'$ (4)

$y' = \frac{3e^{x+y^2}-1}{12y-6ye^{x+y^2}}$ (1), $y'_{(x=-1,y=1)} = \frac{3-1}{12-6} = \frac{1}{3}$ (1)

$y-y_0 = f'(x_0)(x-x_0)$

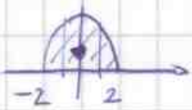
$y-1 = \frac{1}{3}(x+1)$ (2)

3. $f(x) = 3\ln(x^2+1)$; $f'(x) = 3\frac{1}{x^2+1}\cdot 2x$ (1)

$f''(x) = 6\cdot\frac{x^2+1-x\cdot 2x}{x^2+1} = 6\,\frac{1-x^2}{1+x^2}$ (2) $= 0 \iff x_1 = 1,\ x_2 = -1$ (1)

	$(-\infty,-1)$	-1	$(-1,1)$	1	$1,\infty$
f''	−	0	+	0	−
f	$\frown$	prevoj	$\smile$	prevoj	$\frown$

(5)   Odlopodbuit 1-1

4. 

$S(x_s, y_s)$, $x_s = 0$ simetrija urat (1)

$m y_s = M_x$ $m = \rho V \sim \rho T = 1\int_{-2}^{2} 4-x^2\,dx$ (1) $= \left[4x-\frac{x^3}{3}\right]_{-2}^{2} =$

$= 2\left(8-\frac{8}{3}\right) = 16\left(\cdot\frac{2}{3}\right)$ (1)

$M_x \approx \sum_k \frac{f(\xi_k)}{2}\cdot f(\xi_k)\Delta x_k$ (1) $\approx \int_{-2}^{2}\frac{f^2(x)}{2}\,dx$ (1) $= \frac{1}{2}\int_{-2}^{2} 16-8x^2+x^4\,dx$ (1) $=$

$= \frac{1}{2}\left[16x-\frac{8x^3}{3}+\frac{x^5}{5}\right]_{-2}^{2} = 16\cdot 2-\frac{8\cdot 8}{3}+\frac{8\cdot 4}{5} = 8\left(4-\frac{8}{3}+\frac{4}{5}\right) = 8\,\frac{32}{15}$ (1)

$y_s = \frac{8\cdot \cancel{32}\ 3}{15\ \cancel{16}\cdot\cancel{2}} = \frac{8}{5}$ (1)

5. $1+i = \sqrt{2}\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)$ (1)

$(1+i)^5 = (\sqrt{2})^5\left(\cos\frac{5\pi}{4}+i\sin\frac{5\pi}{4}\right)$ (1)

$\sqrt{2}(i-1) = \sqrt{2}\cdot\sqrt{2}\left(\cos\frac{3\pi}{4}+i\sin\frac{3\pi}{4}\right)$ (1)

$\frac{(1+i)^5}{\sqrt{2}(i-1)} = (\sqrt{2})^3\left(\cos\frac{2\pi}{4}+i\sin\frac{2\pi}{4}\right) =$

$= \sqrt{2}^3\left(\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}\right)$ (1)

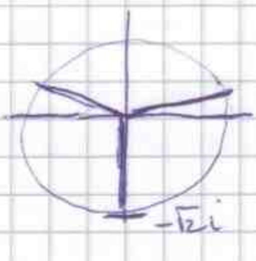
$\sqrt[3]{\frac{(1+i)^5}{i\sqrt{2}-\sqrt{2}}} = \sqrt{2}\left(\cos\frac{\frac{\pi}{2}+2k\pi}{3}+i\sin\frac{\frac{\pi}{2}+2k\pi}{3}\right) = \sqrt{2}\left(\cos\left(\frac{\pi}{6}+\frac{2k\pi}{3}\right)+i\sin\left(\frac{\pi}{6}+\frac{2k\pi}{3}\right)\right)$

$k = 0,1,2$

$k=0$: $\frac{\sqrt{6}}{2}+i\frac{\sqrt{2}}{2}$ (1)

$k=1$: $-\frac{\sqrt{6}}{2}+i\frac{\sqrt{2}}{2}$ (1)

$k=2$: $-\sqrt{2}i$ (1)



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