

Informatics 1

Homework 5

Name

Date

1. Sequences

$$\lim_{n \rightarrow \infty} \frac{2n^2 + 3n + 6}{3n^2 - 1}$$

$$\lim_{n \rightarrow \infty} \frac{n^2 - 10n - 2}{5n^3 + 2n^2 + n + 1}$$

$$\lim_{n \rightarrow \infty} (\sqrt{2n^2 + 5n} - \sqrt{2n^2 - n})$$

$$\lim_{n \rightarrow \infty} (\sqrt{n^2 + 3n + 3} - \sqrt{n^2 + an + 1}) \quad (a \in \mathbb{R})$$

2. Series

$$\sum_{n=1}^{\infty} \frac{1 + \cos n}{n^2}$$

$$\sum_{n=1}^{\infty} \sin\left(\frac{2\pi}{2}\right) \cdot \left(\frac{1}{2}\right)^n$$

$$\sum_{n=1}^{\infty} \frac{2^{2n}}{(-5)^{n+2}}$$

$$\sum_{n=1}^{\infty} \frac{(1+i)^n}{2^{2ni}}$$

$$\sum_{n=1}^{\infty} \frac{3^n - 4^n + 5(-1)^n}{3^{2n}}$$

$$\sum_{n=1}^{\infty} \frac{(3+4i)^n}{4^n(1-i)^{2n}}$$

3. Functions

$$f : A \rightarrow \mathbb{R} \quad x \mapsto \begin{cases} 0, & \text{ha } x < 2 \text{ és } x \notin \mathbb{Q}, \\ x, & \text{ha } x < 2 \text{ és } x \in \mathbb{Q}, \\ 5, & \text{ha } x = 3, \\ 8, & \text{ha } x = 4 \text{ vagy } x = 5 \end{cases} \quad (1)$$

$$f : \mathbb{R} \rightarrow \mathbb{R} \quad x \mapsto \begin{cases} 0, & \text{ha } x \notin \mathbb{Q} \setminus \{0\}, \\ \frac{1}{q_x}, & \text{ha } x = 4 \text{ vagy } x = 5 \end{cases} \quad (2)$$

4. Integrals

$$\begin{aligned} \int \cos 2x \cos 5x \, dx & \quad \int_0^{\frac{\pi}{2}} \sin x \, dx = 1 & \quad \int_0^1 \frac{1}{1+e^x} \, dx = 1 + \ln \frac{2}{1+e} \\ \int x e^{-x^2} \, dx & \quad \int_0^1 (1-2x)^{19} \, dx = 0 & \quad \int_{-1}^1 \frac{x}{\sqrt{5-4x}} \, dx = \frac{1}{6} \end{aligned}$$

5. Matrices

$$\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = 1 \quad (3)$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 1/2 & 1 & 0 \\ 1/4 & 1/2 & 1 \end{bmatrix} \begin{bmatrix} y_{11} & y_{12} & y_{11} \\ y_{21} & y_{22} & y_{23} \\ y_{31} & y_{32} & y_{33} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1/2 & 1 & 0 \\ 1/4 & 1/2 & 1 \end{bmatrix} \quad (4)$$

$$\begin{aligned} \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} &= \begin{vmatrix} a & 0 & 0 \\ 0 & e & 0 \\ 0 & 0 & i \end{vmatrix} + \begin{vmatrix} a & 0 & 0 \\ 0 & 0 & f \\ 0 & h & 0 \end{vmatrix} + \begin{vmatrix} 0 & d & 0 \\ d & 0 & 0 \\ 0 & 0 & i \end{vmatrix} \\ &+ \begin{vmatrix} 0 & b & 0 \\ 0 & 0 & f \\ g & 0 & 0 \end{vmatrix} + \begin{vmatrix} 0 & 0 & c \\ d & 0 & 0 \\ 0 & h & 0 \end{vmatrix} + \begin{vmatrix} 0 & 0 & c \\ 0 & e & 0 \\ g & 0 & 0 \end{vmatrix} \end{aligned} \quad (5)$$

$$\begin{pmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{pmatrix} \quad (6)$$

6. Other

$$\lim_{n \rightarrow \infty} \sqrt[n]{\prod_{k=1}^n \left(1 + \frac{ak}{n}\right)} = \frac{(1+a)^{1+\frac{1}{a}}}{e} \quad (7)$$

$$\sqrt[A]{\prod_{i=1}^n x_i^{a_i}} + \sqrt[A]{\prod_{i=1}^n y_i^{a_i}} \leq \sqrt[A]{\prod_{i=1}^n (x_i + y_i)^{a_i}} \quad (8)$$

$$\min_{1 \leq k \leq n} a_k \leq n \left(\sum_{k=1}^n a_k^{-1} \right)^{-1} \leq \sqrt[n]{\prod_{k=1}^n a_k} \leq \frac{1}{n} \sum_{k=1}^n a_k \leq \max_{1 \leq k \leq n} a_k \quad (9)$$