6. MATEMATIKA A2 FELADATSOR

1. Legyen

$$\underline{\underline{A}} = \begin{pmatrix} 2 & 3 \\ 5 & 1 \end{pmatrix} \quad \underline{\underline{B}} = \begin{pmatrix} 4 & -1 \\ 0 & 2 \end{pmatrix} \quad \underline{\underline{C}} = \begin{pmatrix} 1 & 4 & 2 \\ 3 & 1 & 5 \end{pmatrix}$$

Határozza meg

(a) $\underline{\underline{AC}}$

(b) $\underline{\underline{BC}}$

(c) $(\underline{\underline{A}} + \underline{\underline{B}})\underline{\underline{C}}$ mátrixokat!

(a): $\underline{\underline{AC}} = \begin{pmatrix} 11 & 14 & 19 \\ 8 & 21 & 15 \end{pmatrix}$

(b): $\underline{\underline{BC}} = \begin{pmatrix} 1 & 15 & 3 \\ 6 & 2 & 10 \end{pmatrix}$

(c): $(\underline{\underline{A}} + \underline{\underline{B}})\underline{\underline{C}} = \begin{pmatrix} 12 & 26 & 332 \\ 14 & 23 & 35 \end{pmatrix}$

2. Legyen

$$\underline{\underline{A}} = \begin{pmatrix} 1 & 2 & 3 \\ -2 & 5 & 1 \\ 0 & 4 & -3 \end{pmatrix} \quad \underline{\underline{B}} = \begin{pmatrix} 6 & -2 \\ 3 & -2 \\ 1 & 2 \end{pmatrix} \quad \underline{\underline{C}} = \begin{pmatrix} 1 & 4 & 2 \\ 3 & 1 & 5 \end{pmatrix}$$

Határozza meg az alábbi mátrixok közül azokat amelyek léteznek: $\underline{\underline{AB}}$, $\underline{\underline{AC}}$, $\underline{\underline{BA}}$, $\underline{\underline{BC}}$, $\underline{\underline{CA}}$, $\underline{\underline{CB}}$.

$$\underline{\underline{AB}} = \begin{pmatrix} 15 & 0 \\ 4 & -4 \\ 9 & -14 \end{pmatrix}$$

$$\underline{\underline{BC}} = \begin{pmatrix} 0 & 22 & 2 \\ -3 & 10 & -4 \\ 7 & 6 & 12 \end{pmatrix}$$

$$\underline{\underline{CA}} = \begin{pmatrix} -7 & 30 & 1 \\ 1 & 31 & -5 \end{pmatrix}$$

$$\underline{\underline{CB}} = \begin{pmatrix} 20 & -6 \\ 26 & 2 \end{pmatrix}$$

Az $\underline{\underline{AC}}$ és $\underline{\underline{BA}}$ mátrixszorzatok nem léteznek.

3. Legyen

$$\underline{\underline{A}} = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$$

Határozza meg az

(a) $\underline{\underline{A}}^2$;

(b) $\underline{\underline{A}}^3$;

(c) $\underline{\underline{A}}^{2025}$;

(a): $\underline{\underline{A}}^2 = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}$

(b): $\underline{\underline{A}}^3 = \begin{pmatrix} 1 & 0 \\ 3 & 1 \end{pmatrix}$

(c): $\underline{\underline{A}}^{2025} = \begin{pmatrix} 1 & 0 \\ 2025 & 1 \end{pmatrix}$

4. Elemi sormûveletek segítségével határozzuk meg az alábbi mátrixok inverzét.

   (a) $\underline{\underline{A}} = \begin{pmatrix} 2 & 4 \\ 3 & 5 \end{pmatrix}$

   (b) $\underline{\underline{B}} = \begin{pmatrix} 1 & 1 & 5 \\ 2 & 4 & 8 \\ -4 & 2 & -9 \end{pmatrix}$

   (a):

$$\left(\begin{array}{cc|cc} 2 & 4 & 1 & 0 \\ 3 & 5 & 0 & 1 \end{array}\right) \sim \left(\begin{array}{cc|cc} 1 & 2 & \frac{1}{2} & 0 \\ 3 & 5 & 0 & 1 \end{array}\right) \sim \left(\begin{array}{cc|cc} 1 & 2 & \frac{1}{2} & 0 \\ 0 & -1 & -\frac{3}{2} & 1 \end{array}\right) \sim \left(\begin{array}{cc|cc} 1 & 2 & \frac{1}{2} & 0 \\ 0 & 1 & \frac{3}{2} & -1 \end{array}\right) \sim \left(\begin{array}{cc|cc} 1 & 0 & -\frac{5}{2} & 2 \\ 0 & 1 & \frac{3}{2} & -1 \end{array}\right)$$

   tehát

$$\underline{\underline{A}}^{-1} = \begin{pmatrix} -\frac{5}{2} & 2 \\ \frac{3}{2} & -1 \end{pmatrix}$$

   (b):

$$\left(\begin{array}{ccc|ccc} 1 & 1 & 5 & 1 & 0 & 0 \\ 2 & 4 & 8 & 0 & 1 & 0 \\ -4 & 2 & -9 & 0 & 0 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 1 & 5 & 1 & 0 & 0 \\ 0 & 2 & -2 & -2 & 1 & 0 \\ 0 & 6 & 11 & 4 & 0 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 1 & 5 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & \frac{1}{2} & 0 \\ 0 & 6 & 11 & 4 & 0 & 1 \end{array}\right) \sim$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 6 & 2 & -\frac{1}{2} & 0 \\ 0 & 1 & -1 & -1 & \frac{1}{2} & 0 \\ 0 & 0 & 17 & 10 & -3 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 0 & 6 & 2 & -\frac{1}{2} & 0 \\ 0 & 1 & -1 & -1 & \frac{1}{2} & 0 \\ 0 & 0 & 1 & \frac{10}{17} & -\frac{3}{17} & \frac{1}{17} \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -\frac{26}{17} & \frac{19}{34} & -\frac{6}{17} \\ 0 & 1 & 0 & -\frac{7}{17} & \frac{11}{34} & \frac{1}{17} \\ 0 & 0 & 1 & \frac{10}{17} & -\frac{3}{17} & \frac{1}{17} \end{array}\right)$$

   tehát

$$\underline{\underline{B}}^{-1} = \begin{pmatrix} -\frac{26}{17} & \frac{19}{34} & -\frac{6}{17} \\ -\frac{7}{17} & \frac{11}{34} & \frac{1}{17} \\ \frac{10}{17} & -\frac{3}{17} & \frac{1}{17} \end{pmatrix}$$

5. Oldjuk meg az alábbi lineáris egyenletrendszereket az $\underline{x} = \underline{\underline{A}}^{-1}\underline{b}$ képletet felhasználval.

   (a) $\begin{array}{rcrcr} x_1 & + & 2x_2 & = & 7 \\ 2x_1 & + & 5x_2 & = & -3 \end{array}$

   (b) $\begin{array}{rcrcrcr} x_1 & + & 2x_2 & + & 3x_3 & = & 5 \\ 2x_1 & + & 5x_2 & + & 5x_3 & = & 7 \\ 3x_1 & + & 5x_2 & + & 8x_3 & = & 14 \end{array}$

   (a): $\underline{\underline{A}} = \begin{pmatrix} 1 & 2 \\ 2 & 5 \end{pmatrix}$, $\underline{x} = \begin{pmatrix} x \\ y \end{pmatrix}$, $\underline{b} = \begin{pmatrix} 7 \\ -3 \end{pmatrix}$. $\underline{\underline{A}}^{-1}$ meghatározása:

$$\left(\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 2 & 5 & 0 & 1 \end{array}\right) \sim \left(\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 0 & 1 & -2 & 1 \end{array}\right) \sim \left(\begin{array}{cc|cc} 1 & 0 & 5 & -2 \\ 0 & 1 & -2 & 1 \end{array}\right)$$

   tehát

$$\underline{\underline{A}}^{-1} = \begin{pmatrix} 5 & -2 \\ -2 & 1 \end{pmatrix}$$

Innen $\underline{\underline{A}}^{-1}\underline{b} = \begin{pmatrix} 41 \\ -17 \end{pmatrix}$, ezért $x = 41$ és $y = -17$.

(b): (a): $\underline{\underline{A}} = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 5 & 5 \\ 3 & 5 & 8 \end{pmatrix}$, $\underline{x} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$, $\underline{b} = \begin{pmatrix} 5 \\ 7 \\ 14 \end{pmatrix}$. $\underline{\underline{A}}^{-1}$ meghatározása:

$$\left(\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 2 & 5 & 5 & 0 & 1 & 0 \\ 3 & 5 & 8 & 0 & 0 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & -1 & -2 & 1 & 0 \\ 0 & -1 & -1 & -3 & 0 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 0 & 5 & 5 & -2 & 0 \\ 0 & 1 & -1 & -2 & 1 & 0 \\ 0 & 0 & -2 & -5 & 1 & 1 \end{array}\right) \sim$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 5 & 5 & -2 & 0 \\ 0 & 1 & -1 & -2 & 1 & 0 \\ 0 & 0 & 1 & \frac{5}{2} & -\frac{1}{2} & -\frac{1}{2} \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -\frac{15}{2} & \frac{1}{2} & \frac{5}{2} \\ 0 & 1 & 0 & \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ 0 & 0 & 1 & \frac{5}{2} & -\frac{1}{2} & -\frac{1}{2} \end{array}\right)$$

tehát

$$\underline{\underline{A}}^{-1} = \begin{pmatrix} -\frac{15}{2} & \frac{1}{2} & \frac{5}{2} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ \frac{5}{2} & -\frac{1}{2} & -\frac{1}{2} \end{pmatrix}$$

Innen $\underline{\underline{A}}^{-1}\underline{b} = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$, ezért $x = 1$, $y = -1$ és $z = 2$.

6. Határozza meg az alábbi determinánsokat

   (a) $\begin{vmatrix} 5 & 7 \\ 3 & -1 \end{vmatrix}$

   (b) $\begin{vmatrix} 7 & -4 \\ 5 & 2 \end{vmatrix}$

   (c) $\begin{vmatrix} 2 & 3 & 7 \\ 0 & 0 & -3 \\ 1 & -2 & 7 \end{vmatrix}$

   (d) $\begin{vmatrix} 2 & 1 & 1 \\ 4 & 2 & 3 \\ 1 & 3 & 0 \end{vmatrix}$

(a): $\begin{vmatrix} 5 & 7 \\ 3 & -1 \end{vmatrix} = 5 \cdot (-1) - 7 \cdot 3 = -26$

(b): $\begin{vmatrix} 7 & -4 \\ 5 & 2 \end{vmatrix} = 7 \cdot 2 - (-4) \cdot 5 = 34$

(c): Sarrus-szabállyal számolunk:

$$\left|\begin{array}{ccc|cc} 2 & 3 & 7 & 2 & 3 \\ 0 & 0 & -3 & 0 & 0 \\ 1 & -2 & 7 & 1 & -2 \end{array}\right| = \begin{array}{ccccc} 2 & 3 & 7 & 2 & 3 \\ 0 & 0 & -3 & 0 & 0 \\ 1 & -2 & 7 & 1 & -2 \end{array} = 0 + (-9) + 0 - 0 - 12 - 0 = -21$$

(d): Sarrus-szabállyal számolunk:

$$\left|\begin{array}{ccc|cc} 2 & 1 & 1 & 2 & 1 \\ 4 & 2 & 3 & 4 & 2 \\ 1 & 3 & 0 & 1 & 3 \end{array}\right| = \begin{array}{ccccc} 2 & 1 & 1 & 2 & 1 \\ 4 & 2 & 3 & 4 & 2 \\ 1 & 3 & 0 & 1 & 3 \end{array} = 0 + 3 + 12 - 2 - 18 - 0 = -5$$