

Probability 1 - Practice

Sheet 2

24th Sep 2026

Inclusion-exclusion formula for two events:

$$\mathbb{P}(A \cup B) = \mathbb{P}(A) + \mathbb{P}(B) - \mathbb{P}(A \cap B)$$

Inclusion-exclusion formula for three events:

$$\mathbb{P}(A \cup B \cup C) = \mathbb{P}(A) + \mathbb{P}(B) + \mathbb{P}(C) - \mathbb{P}(A \cap B) - \mathbb{P}(A \cap C) - \mathbb{P}(B \cap C) + \mathbb{P}(A \cap B \cap C)$$

Inclusion-exclusion formula for finitely many events $A_i, i \in I$ (where $|I| < \infty$):

$$\mathbb{P}\left(\bigcup_{i \in I} A_i\right) = \sum_{\emptyset \neq J \subseteq I} (-1)^{|J|+1} \cdot \mathbb{P}\left(\bigcap_{j \in J} A_j\right) = \sum_{k=1}^{|I|} \sum_{J \subseteq I: |J|=k} (-1)^{k+1} \mathbb{P}\left(\bigcap_{j \in J} A_j\right)$$

2.1 What has a higher probability? Throwing a die four times, and rolling 6 at least once, or throwing two dice 24 times, and rolling 6 on both together at least once.

2.2 We are watching a horse race with 7 horses. Let A be the event that $HorseA$ is either first, second or third, while B is the event that $HorseB$ is second. What is the probability of $A \cup B$?

2.3 There are 37 pockets on the roulette wheel: 18 black, 18 red and 1 green. We spin the wheel 13 times. What is the probability of rolling each color at least once?

2.4 Santa Claus has 40 dark chocolate bars and 20 white chocolate bars. He distributes the chocolate bars randomly among ten children in a way that each child gets six chocolate bars. What is the probability that each of the ten children get at least one white chocolate bar?

2.5 We use a 52 card French card deck to play bridge. There are 4 players (N,E,S,W) and each get exactly 13 cards. What is the probability that N has exactly k aces? ($k = 0,1,2,3,4$)

HW 2.6 (a) We throw a die 6 times. What is the probability, that 1, 2, 3, 4, 5 and 6 all get thrown?

(b) We throw a dice 13 times. What is the probability that 1, 2, 3, 4, 5 and 6 all get thrown at least once?

2.7 An urn contains 6 red, 4 blue, and 5 green balls. What is the sample space, if a set of 3 balls is randomly selected without replacement?

(a) What is the probability that each of the balls will be of the same color?

(b) What is the probability that each of the balls will be of different colors?

2.8 There are 5 red, 7 white, and 5 blue balls in an urn. We draw 6 without replacement. What is the probability that we draw at least one of each color?

2.9 We deal all cards from a well shuffled French deck (52 cards). What is the probability that

(a) the ace of hearts is the 11th distributed card?

(b) the first distributed ace is the 11th distributed card?

(c) the first 4 cards are of different suits? ($\clubsuit, \spadesuit, \heartsuit, \diamondsuit$)

(d) an ace never follows the king of the same suit directly? (that is eg. $K\clubsuit A\clubsuit$ do not appear in the sequence)

HW 2.10 (3 points) 11 married couples sit down randomly to a round table with 22 chairs around it.

- (a) What is the probability that Anne, Betty and Caroline all sit next to their husbands?
- (b) What is the probability that no husband sits next to his wife?

2.11 We throw a coin until it lands on the same side twice in a row. Show that the probability of every admissible n long sequence is 2^{-n} . Write down the sample space for the experiment. What is the probability of the following events?

$A := \{\text{The experiment ends after less than 6 throws}\}$

$B := \{\text{The experiment ends after an even amount of throws}\}$

2.12 Arnold, Bruce and Clint are equally strong ping-pong players. They play as follows: A and B play the first round. Then the loser of the round steps aside, and the third player, who was waiting, takes the loser's place to play against the previous winner. They repeat this until one of them wins twice in a row, and he becomes the winner of the round-robin match. Let us describe the sample space of the whole game.

- (a) Explain why the probability of a particular outcome of the game with n rounds is 2^{-n} .
- (b) What are the probabilities that A, B, and C, respectively, win the game?

HW 2.13 Ariana, Britney and Cardi, are throwing coins, one-by-one. Cardi starts, then Britney, then Ariana, then Cardi again and so on. They continue, until someone throws heads. The one who throws heads first wins the game.

- (a) What is the sample space (Ω) ?
- (b) Write down the following events as subsets of the event space, and calculate their probabilities.

$$A = \{\text{Ariana wins}\}, \quad B = \{\text{Britney wins}\}, \quad (A \cup B)^c$$

2.14 Three chefs, A, B and C are trying to bake special cakes. They have 0.02, 0.03 and 0.05 probabilities of overcooking a cake, respectively. A bakes 50% of the cakes, B bakes 30% and C bakes the remaining 20% of cakes. What percent of the ruined cakes did A bake?

2.15 0.1% of human fetuses have the Down syndrome. There is a clinical test that for each fetus with the Down syndrome gives a false negative with probability 1%, while for each fetus without the syndrome gives a false positive with probability 5%. The test tells a couple that their fetus has Down syndrome. What is the probability that this is indeed the case?

HW 2.16 Shane spends $\frac{4}{5}$ of his days at the pub. (So, he decides to go to the pub on a day with probability $\frac{4}{5}$ and stays there for the whole day.) There are five bars in the village, and because he has no preference, he picks each pub with an equal probability and spends the day there. We try to find him and visit four bars, but he wasn't in any of them. What is the conditional probability that he is in the fifth pub?