

# Probability 1 - Practice

Tutorial no. 3

Oct 1, 2026

- 3.1** Eggs are sold in packs of 12. Of the packs available in the store, 80% contain no broken eggs, 15% contain exactly one broken egg, and 5% contain exactly two broken. The positions of the broken eggs within the pack are random. I buy a pack of eggs and take an egg (without looking) and am annoyed to find that the egg I took out is broken. What is the probability that there is another broken egg in the pack?
- 3.2** We deal all cards from a well shuffled French deck (52 cards). What is the probability that the first 4 cards are of different suits? ( $\clubsuit, \spadesuit, \heartsuit, \diamondsuit$ )
- 3.3** We deal all the 52 cards of a French deck one-by-one. Which is the most likely position to see the second Ace?
- 3.4** We put  $N$  balls into  $n$  boxes, with all  $n^N$  possibilities having equal probability. Assuming that a given box is not empty, what is the conditional probability that there are  $K$  balls in that box ( $1 \leq K \leq N$ )?
- 3.5** We throw three dice. Assuming that the number on each die is different, what is the conditional probability that at least one of them is a 6?
- 3.6** A student has to fill out a test with 20 questions that can be answered with either "yes" or "no". Let's assume that the student knows the answer to each question independently with probability  $p$ . With probability  $q$  they think they know the answer, but they don't, and with probability  $r$  they know that they don't know the answer ( $p + q + r = 1$ ). If the student knows that they don't know the answer, they randomly write "yes" or "no" with 50-50% probability. What is the probability that they answer at least 19 questions correctly?
- 3.7** Die  $\alpha$  has 4 red and 2 white sides, while die  $\beta$  has 2 red and 4 white sides. We toss a coin. If the outcome is heads, we use die  $\alpha$ , and if the outcome is tails, we use die  $\beta$ . We throw  $n$  times with the die chosen.
- (a) What is the probability, that the  $k$ -th throw lands on red? ( $k = 1, 2, \dots, n$ )
  - (b) Assuming that the first throw was red, what is the conditional probability that the second throw will also be red?
  - (c) Assuming that the first  $k - 1$  throws were red, what is the conditional probability that the  $k$ -th throw will also be red? ( $k = 1, 2, \dots, n$ )
- 3.8** There are 2 balls in an urn. They were painted to either black or gold with probability  $\frac{1}{2} - \frac{1}{2}$  independently.
- (a) We have been told that at least one of the balls is gold. What is the probability of both balls being gold?
  - (b) We pull out a ball and it is golden. What is the probability that the other ball is also golden?
- HW 3.9** We throw with one blue, one yellow, one green and one red die. Let the three numbers thrown be  $B, Y, G$  and  $R$ .
- (a) What is the probability that all four numbers thrown are different?
  - (b) Assuming that all four numbers are different, what is the conditional probability of  $B < Y < G < R$ ?
  - (c) How much is  $\mathbb{P}(B < Y < G < R)$ ?

- ♣ **3.10** A hunter sees a fox at a distance of 40 meters and shoots at it. If the fox survives, it runs away with a speed of 10 m/s. It takes 5 seconds for the hunter to reload and shoot again, continuously. The fox has a  $\frac{1}{4}$  probability of surviving a shot, no matter how many shots have hit him previously. We know that:

$$\mathbb{P}(\text{The hunter hits the fox from a distance of } x \text{ meters}) = \frac{675}{x^2}$$

Does the fox have a positive probability of surviving?

- 3.11** (Monty Hall problem) Suppose you're on a game show, and you're given the choice of three doors: Behind one door is a car; behind the others, goats. You pick a door, say No. 1, and the host, who knows what's behind the doors, opens another door, say No. 3, which has a goat. He then says to you, "Do you want to pick door No. 2?" Is it to your advantage to switch your choice?

**HW 3.12** There are two urns, with red and blue balls. In the first urn, there are 5 red and 4 blue balls, while in the second urn, there are 7 red and 3 blue balls.

(a) Scenario 1: we take a ball from the first urn and put it into the second urn. We then take a ball from the second urn and put it into the first urn. Then, we take another ball from the first urn. What is the probability that this ball is red?

(b) Scenario 2: we take 1-1 ball from the two urns at the same time, and switch them. Then we take a ball from the first urn. What is the probability that this ball is red?

- 3.13** A private facebook group can only be joined if someone from the group invites you. Originally, the group has two members: Adam and Eve. Occasionally, a random person from the group invites a new member. Someone is part of Adam's "clique", if they are Adam themselves, or if someone from Adam's clique invited them.

(a) What is the probability of Adam's clique having  $k$  members when the facebook group has 4 members? ( $k = 1, 2, 3$ )

(b) Let  $X$  be the number of people in Eve's clique when the facebook group has  $n$  members. What is the probability of the event  $\{X = k\}$  for  $k \in \{1, \dots, n-1\}$ ? *Hint:* Use mathematical induction.

- 3.14** 100 passengers are waiting to board an airplane with 100 seats. The first person to board the airplane lost their ticket, and so they sit down randomly in one of the seats. The following people board the plane individually, and try to sit down in the seat written on their ticket. If they see someone already sitting in their seat, they sit down randomly in one of the empty seats. What's the probability that the last person to board the plane ends up sitting in the correct seat?

- 3.15** Among five players,  $A, B, C, D, E$ , the numbers from 1 to 5 are distributed randomly, without repetition. First,  $A$  and  $B$  compete: whoever has the higher number moves on to play against the next opponent. The one who advances in this way will now face  $C$ , then the one who advances from among them will compete with  $D$ , then the winner battles with  $E$ . Let  $X$  be the number of rounds that  $A$  wins. Determine the distribution and expected value of  $X$ .

**HW 3.16** A class of 7 women and 4 men wrote a test. Let's assume that there are no ties in their test scores, and order them from best to worst, with each of the orderings having equal probability. Let  $X$  denote the placing of the highest ranking woman (eg.  $X = 1$  means that the best test was written by a woman). Determine the distribution and the expected value of  $X$ .