Probability 1 – Exercises

Tutorial no. 5 — 15th Oct 2026

5.1 What is the expected value and the variance of a die roll?

5.2 We're reading Glamour Magazine, and we notice that there are around 0.2 typos each page, on average. What is the probability that the next page has (a) 0 typos (b) 2 or more typos?

5.3 We are rolling a die until getting the second 5.

(a) What is the expected number of rolls we need?

(b) What is the probability that we need more rolls than this expectation?

5.4 We have 10 red, 20 blue and 40 green balls in an urn. We randomly select 7. What is the distribution of the number of blue balls selected? What is their expected number?

5.5 We throw 10 times with a fair coin. Let X be the number of sequences with the same result (For example if $HHTTTTHTHH$ then $X = 5$). What is the distribution of X?

5.6 Let $\mathbb{E}(X) = 1$ and $\mathbb{D}^2(X) = 5$. Calculate the following:

(a) $\mathbb{E}((2+X)^2)$

(b) $(\mathbb{E}(2+X))^2$

(c) $\mathbb{D}^2(4+3X)$

5.7 We have two dice, one red and one green.

(a) What is the expected value of the minimum of the two numbers, and the maximum of the two numbers?

(b) We threw with both dice 3 times. I only looked at the red one, but I saw that it was 3 each time. What is the probability, that it was bigger than the green one at least once?

HW 5.8 We throw three times with a biased coin which gives heads with probability $3/4$. Let U be the number of times we throw the previous result again (so U can be $0, 1$ or 2. For example, if HHH then $U = 2$, if HTH then $U = 0$). Calculate the expected value and variance of U!

HW 5.9 We throw 10 times with a biased coin that has $\frac{2}{3}$ probability of heads. After each throw that lands on tails, we have to pay $200Ft$, however, we get $300Ft$ after each throw that lands on heads. Calculate the expected value and variance of our winnings at the end of the game.

5.10 Let N be a non-negative, integer valued random variable with $\mathbb{E}(N^3) < \infty$.

Show that:

$$\sum_{i=1}^{\infty} i^2 \mathbb{P}(N \geq i) = \frac{\mathbb{E}(N^3)}{3} + \frac{\mathbb{E}(N^2)}{2} + \frac{\mathbb{E}(N)}{6}$$

♣ **5.11** Two people with the same abilities are running a race, on an infinitely long track. Both of their lanes have the same obstacles (ditches), and they both have 2^{-L} probability to jump over a ditch, where L is the width of the ditch, independently of each other or the previous ditches. If one of them falls in a ditch, then the race is over. If both of them fall in the same ditch, then it is a tie, and we start over.

(a) Show that the probability of a tie is less than ε exactly if $L < \log_2(\frac{1+\varepsilon}{1-\varepsilon})$

(b) How big should L be, if we want to minimize the expected number of jumps until someone wins?

5.12 Show that if $X \sim \mathrm{POI}(\lambda)$ and $Y \sim \mathrm{POI}(\mu)$ and if X and Y are independent and if $Z = X + Y$ then $Z \sim \mathrm{POI}(\lambda + \mu)$.

5.13 The police frequently checks the speed of cars on the highway ("radaring"). Experience shows that if they turn on the radar for five minutes then the probability of detecting at least one speeding car is the same as the probability of not detecting any.

(a) What is the probability that after 20 minutes of radaring, (i) exactly 2, (ii) at least 2 speeding cars will be caught?

(b) How long should the police radar for if they want to catch at least 1 speeding car with a 95% probability?

HW 5.14 Leo, when he goes hiking, has a small, equal and independent probability to fall with each step and hit his knee, or fall and hit his elbow. On a 10 kilometer hike, on average, he hits his knee three times, and his elbow two times. At most, how long of a hike can his mother let him go on if she wants a $\frac{2}{3}$ probability for him to not hit his knee or elbow?

5.15 Let's assume that the number of gold ores in a given area has Poi(10) distribution. Each gold ore, independently of the others, is discovered with probability $\frac{1}{50}$. Calculate the probability that

(a) exactly 1,

(b) at least 1, and

(c) at most 1 ore is discovered this time.

Hint: First show that if a random variable has Poi(λ) distribution, and each event is counted with p probability, independently of each other, then the number of events counted is $Poi(\lambda \cdot p)$.

5.16 Ulysses wants to go home to Ithaca, but he angered Poseidon, who put the ocean full of whirlpools. The density of whirlpools is 2 whirlpools in every 10 square kilometers. If a boat gets closer than $150m$ to a whirlpool, it sinks. Otherwise, the ship remains unharmed. Odysseus doesn't know where the whirlpools are, he just goes straight to Ithaca. He studied Probability 1 at BME, so he knows that he has a 1% probability to make it home safely. How far away is Odysseus from Ithaca?

HW 5.17 The average density of trees in a forest is 18 trees in $100m^2$. The trunk of the trees are perfect cylinders, with a diameter of $20cm$ each. We shoot a gun $120m$ away from the edge of the forest, without aiming in the direction of the edge of the forest. What is the probability that we hit a tree? (Note: Disregard the fact that the centre of two trees can't be closer than 20cm to each other).

5.18 Milka introduced a new type of chocolate bar that contains raisins and peanuts. Each chocolate bar consists of 15 squares. Each chocolate bar has 30 raisins in it on average, and the probability that a square of chocolate has no peanuts is $\frac{1}{2}$.

(a) What is the distribution of the peanuts in the chocolate bar?

(b) We break off a piece that consists of two squares. What is the probability that the combined number of peanuts and raisins in this piece of chocolate is less than 2?