Probability 1 - Exercises

Tutorial no. 6 — 29th Oct 2026

6.1 We throw with a fair die until each number from 1 to 6 has occurs at least once. What is the expected value of the number of throws that we need for this?

HW **6.2** There are 5 red and 5 blue balls in an urn. We pick 4 balls at random out of the urn. If we picked 2 red and 2 blue, we stop, otherwise we put them back. We continue until we manage to pull out exactly 2 red and 2 blue. What is the expected number of balls that we had to pull from the urn in order to achieve our goal?

6.3 Amy and Bernadette play the following game. Amy throws with a fair die until the second occurrence of the number 6. She does not tell Bernadette which throw she managed to throw 6 for the first time, but she does tells her which throw she managed to land on 6 for the second time. After this, Bernadette needs to guess when the first 6 could have occurred. What should she guess? If she is smart, with what probability that she guesses correctly?

♣ **6.4** Let X be a Poisson distribution with parameter λ. Show that

$$\mathbb{P}(X \text{ is even}) = \frac{1}{2}(1 + \exp(-2\lambda)))$$

6.5 Andrew and Bella are playing the following game. They throw 2 fair dice, and Andrew pays Bella the same amount of forint as the square of the difference of the two numbers, while Bella pays Andrew the sum of the two numbers. Who is favoured in this game?

6.6 Two people are competing in archery. The contestants have a p_1 and p_2 probability to hit the target ($p_1 < p_2$) with each shot. The person who is less skilled starts, and then they alternate. The first person to hit the target wins.

(a) What is the probability that the more skilled person wins?

(b) What is the expected time of the game if there is one shot every minute?

(c) Give a simple expression for the expected time if $p_1 = p_2$. Check that the formula in part (b) gives the same result when $p_1 = p_2$.

6.7 The municipality of Randomburg provides free wifi for the citizens. Each wifi hotspot has $50m$ range, and they placed 3 hotsposts per square kilometer randomly. At the very moment when Bob left his house, the university sent an email to all the students to notify them that all teaching activity is cancelled for today due to a bomb alarm. Bob goes to university on a straight path, and he has 70% chance to receive the notification via the free wifi on the way. How far away is Bob's house from the university?

6.8 How to choose the constants A and B for $F(\cdot)$ to be a cumulative distribution function (CDF)?:

$$F(x) = \exp(-Ae^{-Bx})$$

HW **6.9** Which constants A and B are needed for the following to be a CDF:

(a)

$$F(x) = A \cdot \arctan(x) + B$$

(b)

$$F(x) = \begin{cases} 0, & -\infty < x \leq 0 \\ \frac{A+x}{1+Bx}, & 0 < x < \infty \end{cases}$$

6.10 Which constants C are needed for the following to be a Probability Density Function (PDF)? For these cases, calculate the expected value, and the probability that the corresponding random variable is greater than 2.

$$f(x) = \begin{cases} C \cdot x^{-5}, & 1 \leq x \\ 0, & \text{else} \end{cases}$$

HW 6.11 Which constants C are needed for the following to be a Probability Density Function (PDF)? For these cases, calculate the expected value, and the probability that the corresponding random variable is greater than 2.

(a)

$$f(x) = \begin{cases} C \cdot \cos(\pi x), & \frac{-1}{2} \leq x \leq \frac{1}{2} \\ 0, & \text{else} \end{cases}$$

(b)

$$f(x) = \begin{cases} C \cdot (-\ln(\frac{x}{3})), & 0 < x \leq 3 \\ 0, & \text{else} \end{cases}$$

6.12 A gas station gets their shipment of oil once a week. Their weekly sales (measured in 1000 liters) is a random variable with PDF

$$f(x) = \begin{cases} 5 \cdot (1-x)^4, & 0 < x < 1 \\ 0, & \text{else.} \end{cases}$$

Then how big of a shipment do they need weekly if they want to run out of oil with probability less than 0.01 on a given week?

6.13 The buses arrive every hour at :00, :15, :30 and :45. If we arrive at random between 7:00 and 7:30, then what is the probability that

(a) I have to wait less than 4 minutes?

(b) I have to wait more than 7 minutes?

HW 6.14 Trains go towards city A every 15 minutes, starting from 7:00am, while they go towards city B every 15 minutes, starting from 7:05am.

(a) If a passenger arrives at the station uniformly between 7:00am and 8:00am, and board whichever train leaves earlier, then what is the probability that they end up going to B? (Or A?)

(b) What if the passenger arrives in a uniformly distributed time between 7:10am and 8:10am?

(c) What if the train that goes towards city B leaves every 10 minutes starting from 7:05am?

6.15 A bus goes between city A and city B, which are 100km from each other. The bus breaks down at a uniformly distributed point on the route. If it breaks down, then they call the closest repair center. Currently, there are three repair centers: one in city A, one in city B and one halfway in between them. It has been suggested that it would be better to place the repair centers at distances 25km, 50km and 75km from city A. Do we agree? Why? What would be an even better placement? How do we define "better"?