Probability 1 – Exercises

Tutorial no. 9 19th Nov 2026

9.1 Let $P = (1, 1)$ and $Q = (U, 0)$, where U is uniformly distributed on an interval of the x-axis. Find the PDF of the distance $\xi = \|P - Q\|$ in each of the following cases: (a) $U \sim \mathrm{UNI}[0, 1]$; (b) $U \sim \mathrm{UNI}[0, 2]$.

9.2 We toss a biased coin ten times, where the probability of heads is p. A run is a maximal sequence of consecutive identical outcomes (heads or tails). Let X denote the number of pure runs. E.g., if the outcome is $HHTHHHHTHT$ then $X = 6$. What is the expected value and the variance of X?

HW 9.3 A group of 20 people plays a game of rock-paper-scissors according to the following rules. They sit in a large circle, and each person simultaneously and independently chooses one of the three options, with a probability of 1/3 for each. Afterward, everyone compares their choice with the neighbor to their left and the neighbor to their right. Rock beats scissors, scissors beat paper, and paper beats rock. In the event of a win, a player receives one peso from the neighbor; in the event of a loss, they pay one peso to the neighbor; in the event of a tie, no money changes hands between them. The same applies to their interaction with the other neighbor. Let ξ denote the number of players who end up 2 pesos richer. What is $\mathbb{E}(\xi)$ and $\mathrm{Var}(\xi)$?

9.4 Let (X, Y) be the two coordinates of a point that is uniformly distributed on the unit disc of the plane. Calculate the conditional probability density function $f_{Y|X}(y \,|\, x)$ of Y given that $X = x$.

9.5 Let the joint PDF of X and Y be

$$f(x, y) = \begin{cases} \frac{1}{y} \exp(-(y + x/y)) & \text{, if } x > 0, y > 0 \\ 0 & \text{, else.} \end{cases}$$

(a) What is the marginal distribution of Y? How much is $\mathbb{E}(Y)$?

(b) What is the conditional density of X, given $Y = y$?

(c) Note that calculating the marginal distribution of X explicitly is problematic. Without doing that, calculate $\mathbb{E}(X)$.

HW 9.6 Let the joint density of (X, Y) be $f(x, y) = \frac{3}{2} xy \mathbf{1}_{\{0<x,\, 0<y,\, x+y<2\}}$.

(a) What is the marginal density of X?

(b) What is the conditional density of Y given $X = x$?

9.7 (a) I go to work each day via train and then bus. First, I get on a train, which arrives at the train station at 7:30am, according to schedule. I have 2 minutes to walk to the bus station, where my bus leaves at 7:37am, according to schedule. However, the actual times when the train arrives and the bus leaves are random. More precisely, the train arrives according to a Normal Distribution with a mean of 7:30am and standard deviation of 3, while the time when the bus leaves is normal with mean 7.37am and standard deviation of 4. What is the probability that I only miss the bus at most once in a given week (which consists of 5 work days)?

(b) In the afternoon, my bus arrives back to the station at 17:40 and my train leaves at 17:49 according to schedule, with the same standard deviations as above. What is the probability that over the 220 workdays of the year, I will miss the afternoon connection more often than my morning connection?

HW 9.8 In an adventure board game, we have three cards: a hobbit, an elf, and a dwarf. The values of these three cards are independent of one another, and each follows normal distribution with an expected value of 30 and a standard deviation of 4. We must battle a dragon, whose energy level (independently of the card values) also follows a normal distribution, with an expected value of 100 and a standard deviation of 4. We defeat the dragon if the sum of the values of the three cards exceeds the dragon's energy level. Calculate the probability of this event.

$\Phi(z)$

z	**0.00**	**0.01**	**0.02**	**0.03**	**0.04**	**0.05**	**0.06**	**0.07**	**0.08**	**0.09**
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.7	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000