VSZ1, PÓTZH1, 2021. NOV. 12.

1 a) $\mathbb{P}(\text{NINCS PIKK}) = \dfrac{\binom{39}{8}}{\binom{52}{8}}$

$\boxed{\text{DE MORGAN}}$

b) $A_i := \{\text{i-EDIK SZÍNBŐL NINCS}\}$, $i = 1,2,3,4$

$$\mathbb{P}(\text{MIND A 4 SZÍNBŐL VAN}) \overset{\text{DE MORGAN}}{=} \mathbb{P}(A_1^c \cap A_2^c \cap A_3^c \cap A_4^c) =$$

$$= 1 - \underbrace{\mathbb{P}\left(\bigcup_{i=1}^{4} A_i\right)}_{= A - B + C - D}$$

$$A = \sum_{i=1}^{4} \mathbb{P}(A_i) = 4 \cdot \frac{\binom{39}{8}}{\binom{52}{8}}$$

$$B = \sum_{1 \le i < j \le 4} \mathbb{P}(A_i \cap A_j) = \underbrace{\binom{4}{2}}_{6} \cdot \overbrace{\mathbb{P}(\text{SE PIKK, SE KŐR})}^{= \frac{\binom{26}{8}}{\binom{52}{8}}}$$

$$C = \sum_{1 \le i < j < k \le 4} \mathbb{P}(A_i \cap A_j \cap A_k) = \underbrace{\binom{4}{3}}_{4} \cdot \frac{\binom{13}{8}}{\binom{52}{8}}$$

$$D = \mathbb{P}(A_1 \cap A_2 \cap A_3 \cap A_4) = 0$$

② a) HA ORSI SÜT $\frac{1}{2}$ KALÁCSOT: $POI(12)$ MAZSOLA

HA PETI — " — " — : $POI(18)$ MAZSOLA

$X :=$ MAZSOLÁK SZÁMA A $\frac{1}{2}$ KALÁCSOMBAN

$$P(X=0) = \underbrace{P(X=0 \mid ORSI)}_{e^{-12}} \cdot \frac{2}{3} + \underbrace{P(X=0 \mid PETI)}_{e^{-18}} \cdot \frac{1}{3}$$

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b) $Y :=$ MAZS SZÁMA EGY SZELETBEN

ORSI SÜTI: $Y \sim POI(2)$ PETI SÜTI: $Y \sim POI(3)$

$$P(ORSI \mid Y=0) = \frac{P(Y=0, ORSI)}{P(Y=0)} = \frac{e^{-2} \cdot \frac{2}{3}}{e^{-2} \cdot \frac{2}{3} + e^{-3} \cdot \frac{1}{3}} = 0.844$$

BÓNUSZ: $Y :=$ AHÁNY FÉLÉV ELTELIK AZ i-EDIK EGYETEM-LEZÁRÁSIG

$Y = X_1 + \dots + X_5$, AHOL $X_j \sim GEO(\frac{2}{3})$, F.A.E.

? $= P(Y$ PÁRATLAN$)$

$$P(X_i \text{ PÁRATLAN}) = \sum_{k=0}^{\infty} P(X_i = 2k+1) = \sum_{k=0}^{\infty} \left(\frac{1}{3}\right)^{2k} \cdot \frac{2}{3} =$$

$$= \frac{2}{3} \cdot \frac{1}{1-\frac{1}{9}} = \frac{2}{3} \cdot \frac{9}{8} = \frac{3}{4}$$

$Z_1 := \sum_{i=1}^{5} \mathbb{1}[X_i \text{ PÁRATLAN}]$

$Z_1 \sim BIN(5, \frac{3}{4})$

$$? = P(Y \text{ PÁRATLAN}) = P(Z_1 \text{ PÁRATLAN}) =$$

$$= E\left(\frac{1}{2} \cdot (1-(-1)^{Z_1})\right) = \frac{1}{2} - \frac{1}{2} E((-1)^{Z_1}) = \frac{1}{2} - \frac{1}{2} \cdot \left(-\frac{1}{2}\right)^5 = \frac{33}{64}$$

$$E((-1)^{Z_1}) = \sum_{k=0}^{5} \binom{5}{k} \cdot \left(\frac{3}{4}\right)^k \cdot \left(\frac{1}{4}\right)^{5-k} \cdot (-1)^k = \sum_{k=0}^{5} \binom{5}{k} \cdot \left(-\frac{3}{4}\right)^k \cdot \left(\frac{1}{4}\right)^{5-k} = \left(\frac{1}{4} - \frac{3}{4}\right)^5$$

2. OLDAL

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