1. Let $\varphi : G \to H$ be a group homomorphism, and let $g \in G$ be an element of finite order. Show that $\varphi(g)$ also has a finite order, in fact, $o(\varphi(g)) \mid o(g)$.

2. a) Suppose $G = \langle a \rangle \cong C_\infty$, and H is an arbitrary group. Prove that for every element h of H there exists exactly one homomorphism $\varphi : G \to H$ such that $\varphi(a) = h$.
b) Prove that for a **finite** group H, the only homomorphism $H \to C_\infty$ is the trivial homomorphism, that is, which maps every element to 1.

3. How many different homomorphisms exist between the two given groups?
a) $C_{10} \to C_{33}$ b) $C_n \to C_n$ c) $C_n \to C_m$

4. Let G be a group with $|G| = 91$. What is the number of homomorphisms $G \to G$ such that it maps at least two nonidentity elements of different order to 1?

5. Let $G = \langle S \rangle$ and $H = \langle T \rangle$ for some $T \subseteq G$. Prove that $H \triangleleft G \Leftrightarrow t^s \in H$ for every $t \in T$ and $s \in S \cup S^{-1}$.

6. a) Show that in a group G, $ab = ba \Leftrightarrow a^b = a \Leftrightarrow b^a = b$.
b) Show that the center $Z(G) = \{\, z \in G \mid zg = gz \ \forall g \in G \,\}$ is a normal subgroup of G, in fact, any subgroup $H \leq Z(G)$ is a normal subgroup in G.

7. a) Consider S_4 as the group of symmetries of the regular tetrahedron, acting on the four vertices. Show that the motions (orientation preserving isometries) of the tetrahedron form a normal subgroup in S_4. Which conjugacy classes of permutations are in this normal subgroup?
b) Prove that the subset $\{\, 1, (12)(34), (13)(24), (14)(23) \,\}$ is a normal subgroup in S_4.
c) There is an embedding φ of D_4 into S_4 mapping each isometry of the square 1234 to the corresponding permutation of the vertices. Prove that $\operatorname{Im} \varphi$ is not a normal subgroup of S_4.
d) Prove that S_4 has no other normal subgroups than 1, S_4 and the two subgroups defined in part a) and b).

8. a) Determine the conjugacy classes, subgroups and normal subgroups of D_4.
b) Embed D_4 into S_4 by restricting the isometries on the set of the vertices $\{\, 1, 2, 3, 4 \,\}$ of the square, and describe the elements of the image as permutations. Find two elements which are conjugate in S_4 but not conjugate in D_4, and give a conjugating permutation.

HW1. Let $x = (1345)(27)$ and $y = (28)(3156)$ be elements of the symmetric group S_8. Calculate the expressions xy and $x^y = y^{-1}xy$, and find a permutation $h \in S_8$ such that $x^h = y$.

HW2. Suppose that the subgroups $A, B \leq G$ are commutative, and $AB = G$.
Prove that $A \cap B \triangleleft G$.