1. Describe the following factor groups (find a known group or a naturally definable subgroup of such a group isomorphic to the given factor group).
a) $\mathbb{R}^\times / \langle \pm 1 \rangle$
b) $\langle a \rangle / \langle a^n \rangle$ if $G = \langle a \rangle \cong C_\infty$
c) $S_4 / \langle (12)(34), (13)(24) \rangle$

2. Determine the factor groups of D_4.

3. Let G be a group and $H \leq G$. Are the following statements true or false?
a) There always exists a group L and homomorphism $\varphi : G \to L$ such that $\operatorname{Ker}\varphi = H$.
b) If $H \lhd G$, then there exists a group L and homomorphism $\varphi : G \to L$ such that $\operatorname{Ker}\varphi = H$.
c) There always exists a group L and homomorphism $\varphi : L \to G$ such that $\operatorname{Im}\varphi = H$.
d) If $\varphi : G \to L$ is a homomorphism then $\varphi(H) \leq L$.
e) If $H \lhd G$, and $\varphi : G \to L$ is a homomorphism then $\varphi(H) \lhd L$.
f) If $H \lhd G$, and $\varphi : G \to L$ is a homomorphism then $\varphi(H) \lhd \varphi(G)$.
g) If $\varphi : G \to L$ is a homomorphism then $|G| \mid |\varphi(G)|$.
h) If $\varphi : G \to L$ is a homomorphism then $|\varphi(G)| \mid |G|$.
i) A group L is a homomorphic image of G (that is, there exists a surjective homomorphism $G \to L$) if and only if L is isomorphic to a factor group of G.

4. Let $H \leq G$ and $M, N \lhd G$. Prove that
a) $H \cap N \lhd H$; $N \cap M \lhd G$;
b) $HN \leq G$; $NM \lhd G$;
c) if $M \leq N$ then G/N is a homomorphic image of G/M.

5. Let $N \lhd G$. Prove that the map $H \mapsto H/N := \{ Nh \mid h \in H \}$ is a bijection between the subgroups of G containing N and the subgroups of G/N. Show that this map connects normal subgroups with normal subgroups, and it preserves the inclusion of subgroups, that is, $H_1 \leq H_2$ if and only if the subgroup of G/N corresponding to H_1 is contained in the one corresponding to H_2.

6. Prove that if $N \lhd G$, and $|G : N|$ is even then there is an H with $N \leq H \leq G$ such that $|H : N| = 2$.

7. Let $N \lhd G$, $H \leq G$, $|G| = 24$, $|N| = 4$, and $|H| = 6$. What can be the cardinality of H at the natural homomorphism $G \to G/N$? Give an example for each case.

8. What is the number of elements of order 4 in A_8?

9. Prove that the elements (123) and (234) of A_4 are conjugate in S_4 but they are not conjugate in A_4.

HW1. Let $G = \langle a \rangle \cong C_{24}$ and $N = \langle a^{40} \rangle$. Determine the order of the (normal) subgroup N and of the factor group G/N. Find an element $g \in G$ such that $o(\overline{g}) = 4$ in G/N but $o(g) \neq 4$ in g.

HW2. Suppose that $N \lhd G$, $H \leq G$, $|G| = 100$, $|N| = 20$ and $|H| > 20$. Prove that there is a subgroup $L \leq H$ with $|H : L| = 5$. (Hint: what can be the order of NH?)