

1. Let H be a set of at least two elements. Show that the set $P(H)$ of all the subsets of H does not form a group with the operation $\cap$ or $\cup$ but it is a group with the symmetric difference $A \triangle B := (A \cup B) \setminus (A \cap B)$.
 2. Do the following sets form a group with the addition or multiplication of matrices as the operation?
 - a) $n \times n$ real matrices with determinant 1;
 - b) $n \times n$ real matrices with positive determinant;
 - c) $n \times n$ matrices over $\mathbb{Z}$;
 - d) $n \times n$ matrices over $\mathbb{Z}$ with nonzero determinant;
 - e) $n \times n$ matrices over $\mathbb{Z}$ with determinant 1;
 - f) $n \times n$ real upper triangular matrices.
 3. Let G be a group and suppose that $x^2 = 1$ for every $x \in G$. Prove that G is commutative.
 4. What are the possible orders of the elements in the following groups?
 - a) $\mathbb{R}^\times = (\mathbb{R} \setminus \{0\}, \cdot)$;
 - b) $(\mathbb{R}, +)$;
 - c) $\mathbb{C}^\times = (\mathbb{C} \setminus \{0\}, \cdot)$;
 - d) $GL_3(\mathbb{R})$.
 5. Find all the elements of finite order in the group of upper triangular 2×2 invertible matrices over $\mathbb{R}$. Do they form a subgroup?
 6. List all possible cycle structures for the elements of S_4 . What are the orders of these elements, and how many elements are there for each cycle structure (equivalently, for each partition of 4)?
 7. Let $g = (1345)(236)(41)$ and $h = (145)(231)$ be two elements of S_6
 - a) Give the disjoint cycle decomposition of g , h , and gh .
 - b) Calculate g^2 , g^3 and h^{-1} . What is the order of g , h , gh and hg ?
 8.
 - a) What is the number of elements of order 10 and of order 4 in S_{10} ?
 - b) What is the maximum of the orders of the elements in S_8 ?
- HW1.** Let I be the open interval $I = (-1, 1) = \{x \in \mathbb{R} \mid -1 < x < 1\}$. and define $a * b = \frac{a + b}{1 + ab}$. Show that $(I, *)$ is a group. (Do not forget to check that for $a, b \in I$, the number $a * b$ is defined, and it is also in I .)
- HW2.** What is the number of elements of order 6 in S_7 ?