

1. Prove that $GL_2(\mathbb{R})$ has a subgroup isomorphic to $\mathbb{C}^\times$.
2. Prove that in a group of even order there always exists an element of order 2.
3. Show that 6 is a divisor of $|S_4|$ but S_4 has no element of order 6.
4. Prove that every group of prime order is cyclic.
5. Prove that every cyclic group is commutative. Give an example for a commutative subgroup in S_4 which is not cyclic.
6. Let $A, B \leq G$ and $|G| < \infty$.
Prove that the cardinality of the subset $AB = \{ab \mid a \in A, b \in B\}$ is

$$|AB| = \frac{|A| \cdot |B|}{|A \cap B|}.$$

7. a) Let $A, B \leq G$. Show that AB is also a subgroup if and only if $AB = BA$.
b) Check that for $A = \langle (12) \rangle$, $B = \langle (123) \rangle$ and $C = \langle (13) \rangle$, the subset AB is a subgroup of S_3 but AC is not a subgroup.
c) Show that for $A = \langle (12) \rangle \leq S_4$ and $B = \langle (234) \rangle \leq S_4$ the cardinality $|AB|$ of the subset AB is a divisor of $|S_4|$ but AB not a subgroup of S_4 .
 8. Show that $Hg \leftrightarrow g^{-1}H$ is a bijection between the right and left cosets of $H \leq G$, and for a right transversal R the set R^{-1} is a left transversal.
 9. What are the possible orders of the elements of D_n , and what is the number of elements for each order?
 10. Show that every nontrivial (that is, $\neq 1$) subgroup of C_∞ has a finite index, i.e. it has finitely many cosets.
- HW1.** Let $A, B \leq G$, where G is a finite group. Show that the subgroups A and B are contained in the set AB , and both $|A|$ and $|B|$ are divisors of $|AB|$ (though AB is not necessarily a group).
- HW2.** Prove that the (multiplicative) group of invertible 3×3 upper triangular matrices over $\mathbb{Z}_2$ is not a cyclic group. (For example, you may show that it is not commutative.)