

1. Let $\varphi : G \rightarrow H$ be a group homomorphism, and let $g \in G$ be an element of finite order. Show that $\varphi(g)$ also has a finite order, in fact, $o(\varphi(g)) \mid o(g)$.
 2. a) Suppose $G = \langle a \rangle \cong C_\infty$, and H is an arbitrary group. Prove that for every element h of H there exists exactly one homomorphism $\varphi : G \rightarrow H$ such that $\varphi(a) = h$.
b) Prove that for a **finite** group H , the only homomorphism $H \rightarrow C_\infty$ is the trivial homomorphism, that is, which maps every element to 1.
 3. How many different homomorphisms exist between the two given groups?
a) $C_{10} \rightarrow C_{33}$ b) $C_n \rightarrow C_n$ c) $C_n \rightarrow C_m$
 4. Let G be a group with $|G| = 91$. What is the number of homomorphisms $G \rightarrow G$ such that it maps at least two nonidentity elements of different order to 1?
 5. Let $G = \langle S \rangle$ and $H = \langle T \rangle$ for some $T \subseteq G$. Prove that $H \triangleleft G \Leftrightarrow t^s \in H$ for every $t \in T$ and $s \in S \cup S^{-1}$.
 6. a) Show that in a group G , $ab = ba \Leftrightarrow a^b = a \Leftrightarrow b^a = b$.
b) Show that the center $Z(G) = \{z \in G \mid zg = gz \ \forall g \in G\}$ is a normal subgroup of G , in fact, any subgroup $H \leq Z(G)$ is a normal subgroup in G .
 7. a) Consider S_4 as the group of symmetries of the regular tetrahedron, acting on the four vertices. Show that the orientation preserving isometries of the tetrahedron form a normal subgroup in S_4 . Which conjugacy classes of permutations are in this normal subgroup?
b) Prove that the subset $\{1, (12)(34), (13)(24), (14)(23)\}$ is a normal subgroup in S_4 .
c) There is an embedding φ of D_4 into S_4 mapping each isometry of the square 1234 to the corresponding permutation of the vertices. Prove that $\text{Im } \varphi$ is not a normal subgroup of S_4 .
d) Prove that S_4 has no other normal subgroups than 1, S_4 and the two subgroups defined in part a) and b).
 8. Suppose that $N \triangleleft G$ and $H \leq G$ such that $H \cap N = 1$ and $HN = G$.
a) Prove that H is a (right and left) transversal for N in G .
b) Prove that $G/N \cong H$.
 9. Describe the following factor groups (find a known group or a naturally definable subgroup of such a group isomorphic to the given factor group).
a) $\mathbb{R}^\times / \{\pm 1\}$
b) $\langle a \rangle / \langle a^n \rangle$ if $G = \langle a \rangle \cong C_\infty$
c) $S_4 / \langle (12)(34), (13)(24) \rangle$
 10. a) Determine the conjugacy classes, normal subgroups and factor groups of D_4 .
b) Consider $D_4 \cong D = \langle (1234), (12)(34) \rangle \leq S_4$. Find two elements in D which are conjugate in S_4 (give a conjugating element) and not conjugate in D .
- HW1.** Let $x = (1345)(27)$ and $y = (28)(3156)$ be elements of the symmetric group S_8 . Calculate the expressions xy and $x^y = y^{-1}xy$, and find a permutation $h \in S_8$ such that $x^h = y$.
- HW2.** Suppose that the subgroups $A, B \leq G$ are commutative, and $AB = G$. Prove that $A \cap B \triangleleft G$.