

1. Let $G = \langle a \rangle \cong C_{36}$, $N = \langle a^{24} \rangle$, and $g = a^{16}$.
 - a) What is the order of g in G , and of $\bar{g} = Ng$ in $\bar{G} = G/N$?
 - b) What is the number of elements h outside of N for which $o(h) = o(\bar{h})$?
 2. Let G be a group and $H \leq G$. Are the following statements true or false?
 - a) There always exists a group L and homomorphism $\varphi : G \rightarrow L$ such that $\text{Ker } \varphi = H$.
 - b) If $H \triangleleft G$, then there exists a group L and homomorphism $\varphi : G \rightarrow L$ such that $\text{Ker } \varphi = H$.
 - c) There always exists a group L and homomorphism $\varphi : L \rightarrow G$ such that $\text{Im } \varphi = H$.
 - d) If $\varphi : G \rightarrow L$ is a homomorphism then $\varphi(H) \leq L$.
 - e) If $H \triangleleft G$, and $\varphi : G \rightarrow L$ is a homomorphism then $\varphi(H) \triangleleft L$.
 - f) If $H \triangleleft G$, and $\varphi : G \rightarrow L$ is a homomorphism then $\varphi(H) \triangleleft \varphi(G)$.
 - g) If $\varphi : G \rightarrow L$ is a homomorphism then $|G| \mid |\varphi(G)|$.
 - h) If $\varphi : G \rightarrow L$ is a homomorphism then $|\varphi(G)| \mid |G|$.
 - i) A group L is a homomorphic image of G (that is, there exists a surjective homomorphism $G \rightarrow L$) if and only if L is isomorphic to a factor group of G .
 3. Let $H \leq G$ and $M, N \triangleleft G$. Prove that
 - a) $H \cap N \triangleleft H$; $N \cap M \triangleleft G$;
 - b) $HN \leq G$; $NM \triangleleft G$;
 - c) if $M \leq N$ then G/N is a homomorphic image of G/M .
 4. Let $N \triangleleft G$. Prove that the map $H \mapsto H/N := \{Nh \mid h \in H\}$ is a bijection between the subgroups of G containing N and the subgroups of G/N . Show that this map connects normal subgroups with normal subgroups, and it preserves the inclusion of subgroups, that is, $H_1 \leq H_2$ if and only if the subgroup of G/N corresponding to H_1 is contained in the one corresponding to H_2 .
 5. Prove that if $N \triangleleft G$, and $|G : N|$ is even then there is an H with $N \leq H \leq G$ such that $|H : N| = 2$.
 6. Let $N \triangleleft G$, $H \leq G$, $|G| = 24$, $|N| = 4$, and $|H| = 6$. What can be the cardinality of H at the natural homomorphism $G \rightarrow G/N$? Give an example for each case.
 7. What is the number of elements of order 4 in A_8 ?
 8. Prove that the elements (123) and (234) of A_4 are conjugate in S_4 but they are not conjugate in A_4 .
- HW1.** Let $G = \langle a \rangle \cong C_{24}$ and $N = \langle a^{40} \rangle$. Determine the order of the (normal) subgroup N and of the factor group G/N . Find an element $g \in G$ such that $o(\bar{g}) = 4$ in G/N but $o(g) \neq 4$ in G .
- HW2.** Suppose that $N \triangleleft G$, $H \leq G$, $|G| = 100$, $|N| = 20$ and $|H| > 20$. Prove that there is a subgroup $L \leq H$ with $|H : L| = 5$. (Hint: what can be the order of NH ?)