1. a) Recall the definition of orderings and well-ordered sets from the lecture!
   b) Show that an ordered set $(X, <)$ is well ordered if and only if it does not contain an infinite descending chain $(x_1 > x_2 > x_3 > \dots)$.
   c) Prove that if S is well ordered, then all of its subset is well ordered!
   d) Show that if $S_1, S_2, \dots, S_n$ are well ordered subsets of an ordered set $(X, <)$, then so is $\bigcup_{k=1}^{n} S_k$.
2. What is the sum of the first n Fibonacci numbers?
3. Show that $\sum_{k=1}^{n} k \cdot k! = (n+1)! - 1$.
4. What is the number of those sequences of length n of entries 1,2,3 or 4, which starts with 1 and the difference between adjacent numbers in them are 1?
5. Prove that a) $[x] + \left[x + \frac{1}{2}\right] = [2x]$ and b) $\left[\sqrt{[x]}\right] = [\sqrt{x}]$.
6. Write the continued fraction form of $\sqrt{2} \cong 1,414$.

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7. a) Let $S_1, S_2, \dots$ be an infinite chain of well ordered subsets of an ordered set $(X, <)$ such that for all $i < j$, $x \in S_i$ and $y \in S_j$ we have $x < y$. Prove that $\bigcup_{k=1}^{\infty} S_k$ is also well ordered.
   b) Show that $\mathbb{Z}$ (with the standard ordering) is not well ordered and find an ordering $\prec$ such that $(\mathbb{Z}, \prec)$ is well ordered!
8. Let $g_0 = a$, $g_1 = b$ and $g_n = g_{n-1} + g_{n-2}$ for $n \geq 2$ – this is a generalized Fibonacci sequence. Show that $g_n = af_{n-1} + bf_n$ for all n (f_n denotes the n-th Fibonacci number)!
9. What is the number of sequences of length n of entries 0 or 1 in which there are no two adjacent 0-s?
10. Write $\sqrt{3} \approx 1.732$ as a continued fraction! Compute the first 5 approximating rationals we get from it and the bound for the error. Are these the approximations strong (in the sense that whether $|\sqrt{3} - p_k/q_k| < 1/2q_k^2$ hold)?

The problem sheets are available on the homepage of the lecturer: `www.math.bme.hu/~merdelyi/bevalg1/`