Introduction to Algebra 1 **Problem sheet 2.** **2023.09.11-14.**

1. Show that any 6-digit number we obtain by repeating the digits of a 3-digit number is divisible by 91. For example by repeating 123 we get $123123 = 1353 \cdot 91$.
2. Show that for all $n \in \mathbb{N}$ we have $133 | 11^{n+1} + 12^{2n-1}$!
3. Compute the value of $p(x) = x^5 - 3x^2 + x + 3$ at $x = 5$ using Horner's method!
4. Convert 26 to base 16, 8, 4, 2, 5, 26!
5. Prove that for any $a, m, n \in \mathbb{N}$, $a > 1$ we have $(a^m - 1, a^n - 1) = a^{(m,n)} - 1$.
6. Using Euclidean algorithm, compute $(288, 204)$ and present it in the form $288m + 204n$ for some $m, n \in \mathbb{Z}$!

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7. Prove that if 23 divides $5a + 9b$ for some integers a, b, then 23 also divides $3a + 10b$!
8. What is the decimal (base 10) representation of the number 120201_3?
9. Convert 2023 to base 2, 8 and 16!
10. Show that $\left\lfloor \frac{n}{ab} \right\rfloor = \left\lfloor \frac{\left\lfloor \frac{n}{a} \right\rfloor}{b} \right\rfloor$ for any $n, a, b \in \mathbb{N}$!
11. Let $x \in \mathbb{R}^+$ and $d \in \mathbb{N}$. Show that the number of integers n which are divisible by d and not greater than x is $\left\lfloor \frac{x}{d} \right\rfloor$.
12. Give all integral and all positive solutions of the following linear equations!
    a) $288x + 204y = 1$,
    b) $288x + 204y = 30$ and
    c) $288x + 204y = 300$.
13. * a) Show that $\mathbb{Z}[\sqrt{2}] = \{a + b\sqrt{2} | a, b \in \mathbb{Z}\} \subset \mathbb{R}$ is a ring.
    b) Which of the following are units in $\mathbb{Z}[\sqrt{2}]$: $\sqrt{2}, 3 - 2\sqrt{2}, 2 + 3\sqrt{2}$?

The problem sheets are available on the homepage of the lecturer: `www.math.bme.hu/~merdelyi/bevalg1/`