Introduction to Algebra 1 **Problem sheet 6.** **2023.10.09-12.**

1. Using Horner's method
   a) write $a(x) = x^3 + 3x^2 + 1$ as $(x-3)q(x) + r$ and
   b) find $b(x)$ such that $a(x) = b(x-3)$!
2. What are the irreducible polynomials
   a) of degree 4 over $\mathbb{Z}_2$ and
   b) of degree 2 and 3 over $\mathbb{Z}_3$?
3. How to choose $a \in \mathbb{R}$ such that $(x+1)^2 | x^5 - ax^2 - ax + 1$?
4. Show that all polynomial in $\mathbb{R}[x]$ of odd degree has at least one real root!
5. What is $\gcd(-6x^3 + 6x^2 - 12, 3x^2 - 3x - 6)$ in $\mathbb{Q}[x]$ and $\mathbb{Z}[x]$?
6. Determine the roots of the following polynomials and write them as products of irreducibles over $\mathbb{C}[x]$, $\mathbb{R}[x]$ and $\mathbb{Z}_5[x]$.
   a) $2x^3 - 7x^2 + 2$
   b) $x^5 + 1$

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7. What is the greatest common divisor of $a(x) = x^3 - 2x^2 + x - 1$ and $b(x) = x^2 + 2$. With the help of the extended Euclidean algorithm find polynomials p and q such that $\gcd(a, b) = pa + qb$.
8. Divide $x^4 - 2x + 5$ with remainder by
   a) $x^2 - x + 2$,
   b) $x + 1$,
   c) $(x+1)^2$ and
   d) $x^2 - 1$!
9. What is the gcd and lcm of $(x-2)^2(x+i)^5(x-3)(x-4)^2$ and $(x-2)(x+i)^2(x-3)^3$?
10. Determine the monic polynomials of lowest degree a) in $\mathbb{C}[x]$ and b) in $\mathbb{R}[x]$, for which i is a double root and 1 is a triple root!
11. How many irreducible factors does the polynomial $-6x^3 + 6x^2 - 12$ have in $\mathbb{Q}[x]$, $\mathbb{Z}[x]$, $\mathbb{R}[x]$ and $\mathbb{C}[x]$?

The problem sheets are available on the homepage of the lecturer: `www.math.bme.hu/~merdelyi/bevalg1/`