1. Find the roots of the following polynomials and decompose to product of irreducibles over $\mathbb{R}$, $\mathbb{C}$ and $\mathbb{Z}_5$!
   a) $2x^3 - 7x^2 + 2$,
   b) $x^6 - 2x^5 - x^4 + 4x^3 - 5x^2 + 6x - 3$ and
   c) $x^5 + 1$.
2. Determine the monic polynomials of lowest degree a) in $\mathbb{C}[x]$ and b) in $\mathbb{R}[x]$, for which i is a double root and 1 is a triple root!
3. How many irreducible factors does the polynomial $-6x^3 + 6x^2 - 12$ have in $\mathbb{Q}[x]$, $\mathbb{Z}[x]$, $\mathbb{R}[x]$ and $\mathbb{C}[x]$?
4. What is $\gcd(-6x^3 + 6x^2 - 12, 3x^2 - 3x - 6)$ in $\mathbb{Q}[x]$ and $\mathbb{Z}[x]$?

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5. For which integers c does the polynomial $x^3 + 2x^2 + cx + 4$ have a rational root?
6. Consider the polynomial $x^4 - 6x^3 + 9x^2 + 3$. Is it irreducible over $\mathbb{R}$, $\mathbb{Q}$ and $\mathbb{Z}_2$?
7. Decompose the polynomial $2x^6 - x^5 - 9x^4 + 4x^3 - 6x^2 + 5x + 5$ as products of irreducibles in $\mathbb{Q}[x]$ and $\mathbb{Z}_5[x]$.
8. Show that if $p \in \mathbb{Q}[x]$ is irreducible, then it has no multiple roots in $\mathbb{C}$!

The problem sheets are available on the homepage of the lecturer: `www.math.bme.hu/~merdelyi/bevalg1/`