Introduction to Algebra 1 **Problem sheet 8.** **2023.10.26-31.**

1. Determine the roots of $x^3 + 3x^2 + 9x + 5$ with the help of Cardano's formula!
2. Show that if p is an odd prime, then $\Phi_{2p}(x) = \Phi_p(-x)$. Show that this polynomial is irreducible!
3. a) Verify that $\Phi_{12}(x) = x^4 - x^2 + 1$.
   b) Show that $\Phi_{12}(ax + b)$ does not satisfy the conditions of the Schönemann-Eisenstein criterion for any $a, b \in \mathbb{Z}$, $a \neq 0$ and prime p!
   c) Prove that $\Phi_{12}(x) \mod p$ is reducible in $\mathbb{Z}_p[x]$ for all prime p.
   d) "By hand" show that $\Phi_{12}(x)$ is irreducible in $\mathbb{Z}[x]$!
4. a) Show that if $p \in \mathbb{Z}[x]$ and $a, b \in \mathbb{Z}$, then $a - b | p(a) - p(b)$!
   b) Find a polynomial in $\mathbb{Z}[x]$ such that $\{f(-2), f(1), f(3)\} = \{2, 6, 11\}$ (maybe not in this order)!
5. Let α, β and γ be the complex roots of the polynomial $x^3 - 2x^2 + 4x + 6$. What is the monic polynomial which has roots $\alpha + \beta, \beta + \gamma$ and $\gamma + \alpha$?
   (Hint: don't compute the roots, use Vieta's formulas!)

The problem sheets are available on the homepage of the lecturer: `www.math.bme.hu/~merdelyi/bevalg1/`