Introduction to Algebra 1 **Problem sheet 9.** **2023.11.02.**

1. Solve the following systems of linear equations!

a)
$$\begin{aligned} x+ y+ z &= 10 \\ x+ 2y+ 3z &= 23 \\ x+ 4y+ 9z &= 59 \end{aligned}$$

b)
$$\begin{aligned} x- y- z &= 1 \\ x+ y+ 2z &= 2 \\ 5x+ y+ 4z &= 3 \end{aligned}$$

c)
$$\begin{aligned} x- y- z &= 1 \\ x+ y+ 2z &= 2 \\ 5x+ y+ 4z &= 8 \end{aligned}$$

2. Solve the following systems of linear equations in $\mathbb{Z}_5$!

a)
$$\begin{aligned} x+ 2y+ z &= 4 \\ x+ 3y+ 4z &= 3 \\ 2x- y+ 5z &= 1 \\ y+ 7z &= 3 \end{aligned}$$

b)
$$\begin{aligned} -y+ 2z+ 3w &= 1 \\ 2x+ 3y+ 4z+ 5w &= 2 \\ 2x+ 2y+ z- 2w &= 2 \end{aligned}$$

3. Which of the following are in (reduced) row echelon form? Compute the solutions of the corresponding equations (in parametric and in vectorial form)!

a) $\left(\begin{array}{cccc|c} 2 & 0 & 1 & 1 & 3 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 2 \end{array}\right)$ b) $\left(\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -1 \end{array}\right)$ c) $\left(\begin{array}{cccc|c} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 2 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{array}\right)$

d) $\left(\begin{array}{cccc|c} 1 & 0 & 0 & 2 & 1 \\ 0 & 0 & 1 & 3 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$ e) $\left(\begin{array}{ccc|c} 1 & 0 & 2 & 1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right)$

4. Solve the following systems of linear equations!

a)
$$\begin{aligned} x+ y+ z &= 4 \\ -x+ y- z &= 2 \\ 2x+ y+ 2z &= 1 \\ 4x+ 4y+ 4z &= 1 \end{aligned}$$

b)
$$\begin{aligned} 7x+ 14y- 21z &= 7 \\ x+ 2y- 3z &= 1 \\ 5x+ 10y+ 15z &= 1 \\ 3x+ 6y- 9z &= 3 \end{aligned}$$

Can we leave some of the equations such that the system remains equivalent? Which can be left?

5. Does there exist a system of linear equations such that
a) the number of equations is 5, the number of variables is 6 and there is a unique solution;
b) the number of equations is 6, the number of variables is 5 and there is a unique solution;
c) the number of equations is 5, the number of variables is 6 and there is no solution;
d) the number of equations is 5, the number of variables is 5 and there are exactly 5 solutions (over $\mathbb{F} = \mathbb{R}$ and over any field)?

6. How many solutions does the following system of linear equations have depending on the values of a and b? Solve the problem over $\mathbb{R}, \mathbb{Z}_2$ and $\mathbb{Z}_3$!

$$\begin{aligned} x+ y \quad &= 1 \\ x+ 2y- az &= b \\ x+ 3y+ az &= 0 \end{aligned}$$

The problem sheets are available on the homepage of the lecturer: `www.math.bme.hu/~merdelyi/bevalg1/`