1. Solve the following systems of linear equations simultaneously.

$$\begin{cases} 2x- & y+ & z= & 1 \\ x+ & y+ & 2z= & 2 \\ x- & 2y- & z= & -1 \end{cases} \qquad \begin{cases} 2x- & y+ & z= & 1 \\ x+ & y+ & 2z= & 2 \\ x- & 2y- & z= & 0 \end{cases}$$

   What does it mean from the point of view of rows/columns?

2. Which of the following subsets of $\mathbb{R}^3$ is a subspace? For those which are affine subspaces, find a vector $\underline{u}$ and a subspace V such that they are equal to $\underline{u}+V$!

   a) $\{\underline{v} \in \mathbb{R}^3 \mid |\underline{v}| = 1\}$, b) $\{(x,y,z) | x+2y+z=0\}$ and c) $\{(x,y,z)|x+2y+z=1\}$.

3. Verify that
   a) the intersection of two subspaces is a subspace, but
   b) the union of two subspaces is a subspace if and only if one contains the other.

4. What is the intersection of the planes $x+3y+z=2$ and $x+2y+2z=5$? If the intersection is a line, compute its paramteric and vectorial form!

5. What is the explicit equation of the plane $2x-y+z=1$ and its vectorial form?

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6. Let $B = \{(1,0,1)^T, (0,2,-1)^T, (1,1,0)^T\}$.
   a) Show that B is a basis in $\mathbb{R}^3$!
   b) What is the coordinate vector of $(1,0,0)^T$ with respect to B?
   c) For which vector $\underline{w} \in \mathbb{R}^3$ is $[w]_B = (5,1,-2)^T$?

7. Choose a maximal linearly independent system of the columns of the following matrix A! Write the other columns as linear combinations of the previous ones. Compute a basis of $\mathcal{N}(A)$ (where $\mathcal{N}(A)$ is the nullspace of A)!

$$\begin{pmatrix} 1 & 0 & 1 & 0 & 2 \\ 2 & -1 & 0 & 1 & 3 \\ 0 & 1 & 2 & 1 & 3 \\ 1 & -1 & -1 & 1 & 1 \end{pmatrix}$$

8. What is the reduced row echelon form of $A = (\underline{a}_1, \underline{a}_2, \underline{a}_3) \in \mathbb{R}^{4\times 3}$ if $\underline{a}_3 = \underline{a}_1 - 2\underline{a}_2$ and $a_1 \notin \mathrm{Span}(\underline{a}_3)$?

9. Let $\underline{a} = (1,0,-1,2)^T$, $\underline{b} = (1,1,0,1)^T$ and $\underline{c} = (0,2,1,0)^T \in \mathbb{R}^4$. Show that $\underline{a},\underline{b},\underline{c}$ are linearly independent! Which of $\underline{v} = (1,0,0,0)^T$ and $\underline{w} = (1,1,1,1)^T$ is the linear combination of $\underline{a},\underline{b}$ and $\underline{c}$? For those which are, compute the coefficients!

10. Find a basis in those subsets of problem 4 which are subspaces!

The problem sheets are available on the homepage of the lecturer: `www.math.bme.hu/~merdelyi/bevalg1/`