1. What is the matrix of the following linear maps $\mathbb{R}^3 \to \mathbb{R}^3$:
   a) Reflection to the plane $x = y$,
   b) Rotation around the axis y with angle $45°$ CCW and
   c) Rotation around the axis $x = y = z$ with angle $120°$.

2. Describe geometrically the linear maps corresponding to the following matrices:
   a) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}$ b) $\begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ c) $\begin{pmatrix} 1/\sqrt{2} & 0 & -1/\sqrt{2} \\ 0 & 0 & 0 \\ 1/\sqrt{2} & 0 & 1/\sqrt{2} \end{pmatrix}$

3. Consider a linear map $\varphi : \mathbb{R}^3 \to \mathbb{R}^3$ such that $0 \neq \operatorname{Im}(\varphi) \leq \operatorname{Ker}(\varphi)$. What is $\operatorname{rk}(\varphi)$?

4. What is the rank of the matrix A and B, where $a_{jk} = j + k$ and $b_{jk} = j^2 + k^2$?

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5. Let $A = \begin{pmatrix} 2 & 3 \\ 4 & 5 \end{pmatrix}, B = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}, C = \begin{pmatrix} 0 & 1 & 1 \\ 2 & 3 & 5 \end{pmatrix}, D = \begin{pmatrix} 1 & 1 & -1 \\ 0 & 2 & 5 \\ 0 & 0 & 7 \end{pmatrix}$.

   a) Compute those, which are defined:

   $A + B, A + C, A + D, B + A, B + D, AB, BA, AC, CA, CD, DC, A^2, B^2, C^2, D^2$.

   b) What is the rank of the above matrices?

6. What is the row and column operations corresponding to the following matrices?

   $$\begin{pmatrix} 3 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 1 \end{pmatrix} \qquad \begin{pmatrix} 1 & 2 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 1 \end{pmatrix} \qquad \begin{pmatrix} 0 & 1 & \dots & 0 \\ 1 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 1 \end{pmatrix}$$

7. Do there exists matrices C, D in $\mathbb{R}^{2\times2}$ and $\mathbb{R}^{3\times3}$ such that
   a) $C \neq 0$ and $C^2 = 0$;   b) $D^2 \neq 0$ and $D^3 = 0$?

8. Do the following equalities hold for all matrices $A, B \in \mathbb{F}^{n\times n}$?
   a) $(A + B)(A - B) = A^2 - B^2$ b) $(A + I)(A - I) = A^2 - I^2$
   c) $(A + B)^2 = A^2 + 2AB + B^2$ d) $(AB)^T = A^T B^T$

The problem sheets are available on the homepage of the lecturer: `www.math.bme.hu/~merdelyi/bevalg1/`