1. Are the following matrices invertible? If yes, compute their inverses!

$$A=\begin{pmatrix}2&3\\5&7\end{pmatrix},\qquad B=\begin{pmatrix}1&1&1\\2&2&2\\3&2&1\end{pmatrix},\qquad C=\begin{pmatrix}1&2&3\\2&3&4\\3&4&6\end{pmatrix},\qquad D=\begin{pmatrix}1&-1&1\\-1&1&1\\1&1&-1\end{pmatrix}$$

2. Solve the following matrix equations (A, B, C and D are as above)!

a) $CX=D$, b) $BX=C$, c) $XB=\begin{pmatrix}1&2&3\\5&3&1\end{pmatrix}$, $XB=A\begin{pmatrix}1&2&3\\5&3&1\end{pmatrix}$

3. Compute the rank factorization of the $A=\begin{pmatrix}1&0&1\\2&-1&0\\0&1&2\end{pmatrix}$, and write A as the sum of $\mathrm{rk}(A)$ dyadic matrices!

4. Compute the LU decomposition of the matrix $A=\begin{pmatrix}1&2&4\\3&8&14\\2&5&13\end{pmatrix}$!

5. Show that the matrix $\begin{pmatrix}0&1\\1&0\end{pmatrix}$ has no LU decomposition!

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6. Compute the following determinants with row operations:

a) $\begin{vmatrix}3&1\\4&-3\end{vmatrix}$ b) $\begin{vmatrix}2&2\\6&9\end{vmatrix}$ c) $\begin{vmatrix}2&1&3\\1&-1&5\\5&3&1\end{vmatrix}$ d) $\begin{vmatrix}0&\dots&\dots&0&1\\0&\dots&0&1&0\\\vdots&&&&\vdots\\0&1&0&\dots&0\\1&0&\dots&\dots&0\end{vmatrix}_{(n\times n)}$

7. Compute the following determinants with inversion numbers:

a) $\begin{vmatrix}0&0&0&1\\0&0&1&0\\0&1&0&0\\1&0&0&0\end{vmatrix}$ b) $\begin{vmatrix}2&0&0&0\\0&0&-1&0\\0&0&0&3\\0&4&0&0\end{vmatrix}$ c) $\begin{vmatrix}0&3&0\\2&0&0\\0&0&5\end{vmatrix}$

8. Let $A\in\mathbb{F}^{5\times5}$ such that $\det(A)=3$. What is the determinant of a) $2A^{-1}$, b) $(2A)^{-1}$ and c) $A^2\cdot A^T\cdot A^{-1}$?

9. What is the value of the following determinant?

$$\begin{vmatrix}1&2&2&\dots&2\\2&2&2&\dots&2\\2&2&3&\dots&2\\\vdots&\vdots&\vdots&\ddots&\vdots\\2&2&2&\dots&n\end{vmatrix}$$

10. Compute

$$\begin{vmatrix}b&a&a\\a&b&a\\a&a&b\end{vmatrix}!$$

How can this be generalized to $n\times n$ matrices.
What is the rank of the matrix (depending on n, a and b)?

The problem sheets are available on the homepage of the lecturer: `www.math.bme.hu/~merdelyi/bevalg1/`