Introduction to Algebra 1 **Problem sheet 13.** **2023.12.04-07.**

1. Solve the following systems of linear equations using Cramer's rule!

$$\begin{array}{rrrl} x+ & y+ & z= & 10 \\ x+ & 2y+ & 3z= & 23 \\ x+ & 4y+ & 9z= & 59 \end{array}$$

2. Compute the inverse of the following matrix with the help of determinants!

$$\begin{pmatrix} 1 & 0 & 2 \\ 0 & 3 & 1 \\ -1 & 1 & 0 \end{pmatrix}$$

3. Compute the following determinants (as short as possible)!

a) $\begin{vmatrix} 1 & 2 & -1 \\ 1 & 2 & 4 \\ 2 & 3 & 1 \end{vmatrix}$ b) $\begin{vmatrix} 1 & 1 & 0 & 1 \\ 1 & 2 & 3 & 0 \\ 0 & 2 & 3 & 0 \\ 1 & 0 & 0 & 4 \end{vmatrix}$ c) $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c+d & c^2+d^2 \end{vmatrix}$ d) $\begin{vmatrix} 2 & 1 & 0 & \dots & 0 \\ 1 & 2 & 1 & \dots & 0 \\ 0 & 1 & 2 & \ddots & 0 \\ 0 & \vdots & \ddots & \ddots & 1 \\ 0 & 0 & \dots & 1 & 2 \end{vmatrix}_{(n\times n)}$

4. Determine the rank of the following matrix. Find a maximal nonzero subdeterminant!

$$\begin{pmatrix} 1 & 2 & -1 & 0 \\ 1 & 2 & -1 & 1 \\ 2 & 3 & 1 & 1 \end{pmatrix}$$

The problem sheets are available on the homepage of the lecturer: `www.math.bme.hu/~merdelyi/bevalg1/`