Sample midterm

While solving the problems of the midterm empty A4 sized papers, writing instruments can be used. No other exam aids (e. g. calculators, notes) are permitted! Also any communcation with other students will be forbidden.

There will be 6 problems for 20 points in the midterm, to pass you need to get at least 10 points. You will have 60 minutes to work.

You find below the problems from a midterm few years ago. I plan keep the same difficulty.

1. Prove that $\sum_{k=1}^{n} f_k^2 = f_n f_{n+1}$, where f_n is the n-th Fibonacci number ($f_1 = f_2 = 1$ and $f_n = f_{n-1} + f_{n-2}$ for $n > 2$)! (3 points)

2. Using the extended Euclidean algorithm find integers x and y such that $47x + 19y = 4$ (3 points)!

3. What are the integer solutions of the following system of congruences? (3 points)

$$x \equiv 2 \pmod 4$$
$$x \equiv 1 \pmod 5$$
$$x \equiv -2 \pmod 7$$

4. What are the third roots of $-i$ and what is their order? (4 points)

5. Represent the set $|\bar{z} - 1| = \operatorname{Im} z$ on the plane! (4 points)

6. Let $p > 0$ be a prime. What is the largest power of p which divides $\binom{p^3}{p}$? Here $\binom{n}{k} = \dfrac{n!}{k!(n-k)!}$ is the binomial coefficient n choose k. (3 points)